Filip

Úloha 1

Maxi

$$f(x,y) = \exp\left(-\frac{1}{x^2+y^2}\right) \qquad [x,y] \in \mathbb{R}^2 \setminus \{[0,0]\}$$

- dodefinujte spojitě v $(0,0)$, spočítejte parciální derivace a totální diferenciál ve všech bodech $\mathbb{R}^2$

$$\lim_{t \to 0^+} f(t,0) = \exp\left(-\frac{1}{t^2}\right) = e^{-\infty} = 0$$

$$\lim_{t \to 0^+} f(t,t) = \exp\left(-\frac{1}{t^2+t^2}\right) = 0$$

$$\lim_{(x,y) \to (0,0)} f(x,y) = 0$$

$$\frac{\partial}{\partial x} f = \exp\left(-\frac{1}{x^2+y^2}\right) \cdot \frac{1}{(x^2+y^2)^2} \cdot (2x + 0)$$

$$= \frac{2x \exp\left(-\frac{1}{x^2+y^2}\right)}{(x^2+y^2)^2}$$

Víme, že spojité parciální derivace:

$$\frac{\partial f}{\partial x}(0,0) = 0$$

$$\frac{\partial}{\partial y} f = \frac{2y \exp\left(-\frac{1}{x^2+y^2}\right)}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y}(0,0) = 0$$

totally differentiable :

$$df(x,y) = \frac{2x \cdot \exp\left(-\frac{1}{x^2+y^2}\right)}{(x^2+y^2)^2}\,dx + \frac{2y \exp\left(-\frac{1}{x^2+y^2}\right)}{(x^2+y^2)^2}\,dy$$

$\forall\ (x,y) \neq (0,0)$

~~hence~~

Úloha 2

Filip Maxim

Vyšetřete lokál a glob. extrémy funkcie:

$$f(x,y) = -x^4 + xy - y^4 \qquad [x,y] \in \mathbb{R}^2$$

$$\frac{\partial}{\partial x} f = -4x^3 + y = 0$$

$$\frac{\partial}{\partial y} f = -4y^3 + x = 0$$

$$\frac{\partial^2}{\partial^2 x} f = -12x^2$$

$$\frac{\partial^2}{\partial x \partial y} f = 1$$

$$\frac{\partial^2}{\partial^2 y} f = -12y^2$$

$$H(x,y) = \begin{pmatrix} -12x^2 & 1 \\ 1 & -12y^2 \end{pmatrix}$$

$(0,0)$: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\left.\begin{array}{l} \lambda^2 - 1 \\ \lambda = \pm 1 \end{array}\right\} \rightarrow (1\ 1\ 0)$