UNIVERSIDADE DO SUL DE SANTA CATARINA
Curso de Engenharia civil e Computação
Disciplina: Integrais
Aluno (a): Roberto Abna Bento

Avaliação

1) Resolva as integrais definidas:

a) $\int_0^2 (x^3 + 2x^2 + 1)dx =$

b) $\int_0^3 \frac{1}{\sqrt{5x+1}} dx =$

c) $\int_{-1}^2 (x+2)^4 dx =$

2) Calcule a área da região limitadas pelas curvas dadas :

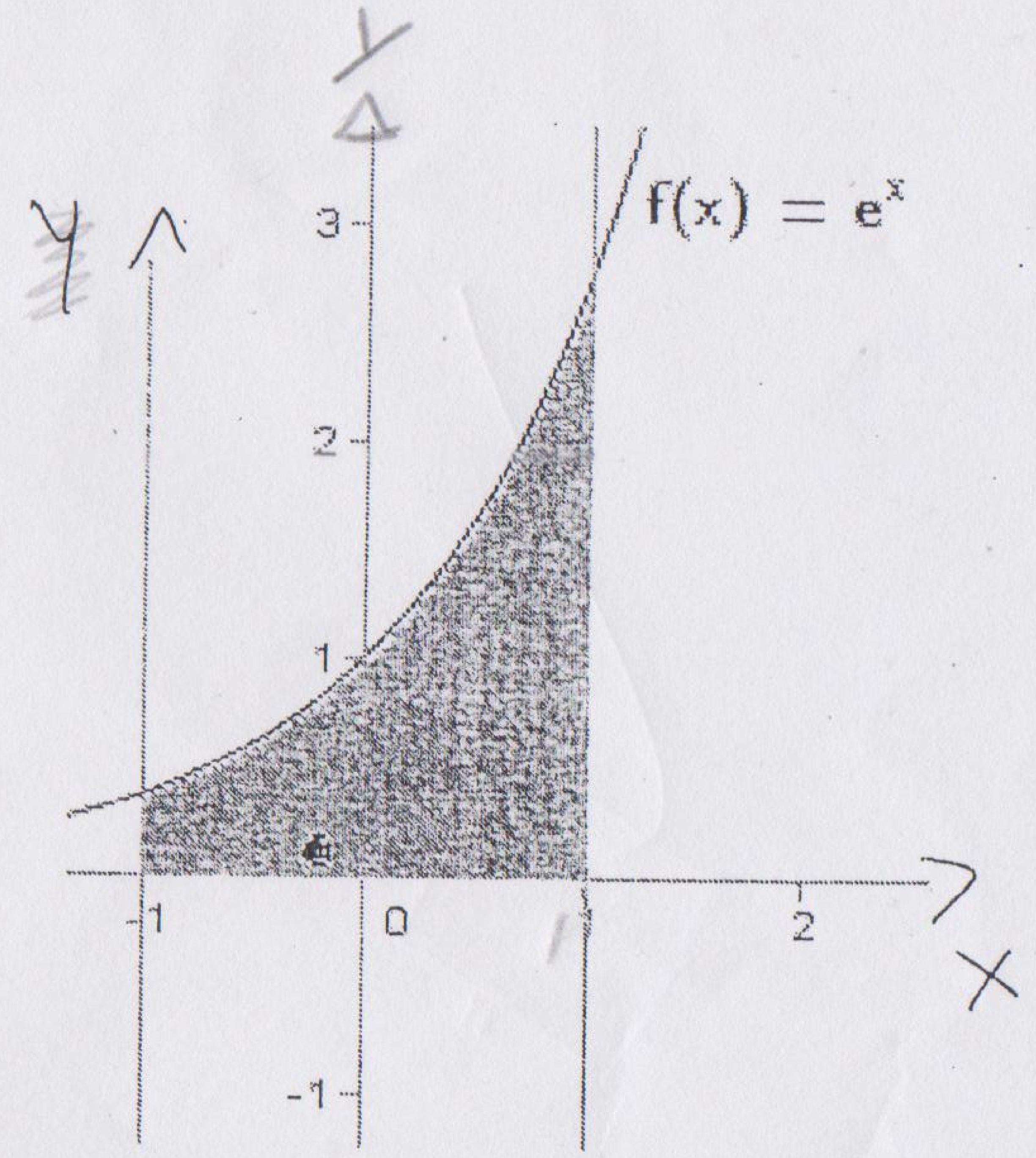
a) $y = x^2$, $y = x + 6$

b) $y = -5 + x^2$, $y = 4$

c) $y = x^2$ e $y = -x^2 + 8$

d) $y = x + 5$, $y = 5$ e $x = -6$

3) Encontrar a área da região na figura:



Folha de respostas — Roberto Abreu Bento 8,2

1-a) $\int_0^2 (x^3+2x^2+1)\,dx = \frac{x^4}{4}+\frac{2x^3}{3}+x \Big]_0^2 =$

$= \frac{2^4}{4}+\frac{2.2^3}{3}+2-\left(\frac{0^4}{4}+\frac{2.0^3}{3}+0\right) = \frac{16}{4}+\frac{16}{3}+\frac{2}{1} =$

$= \frac{48+64+24}{12} = \frac{136}{12} = \boxed{\frac{34}{3}}$

1-b) $\int_0^3 \frac{1}{\sqrt{5x+1}}\,dx = \int_0^3 \frac{1}{(5x+1)^{\frac{1}{2}}}\,dx = \int_0^3 u^{-\frac{1}{2}}\frac{du}{5} = \frac{1}{5}\int_0^3 u^{-\frac{1}{2}}\,du =$

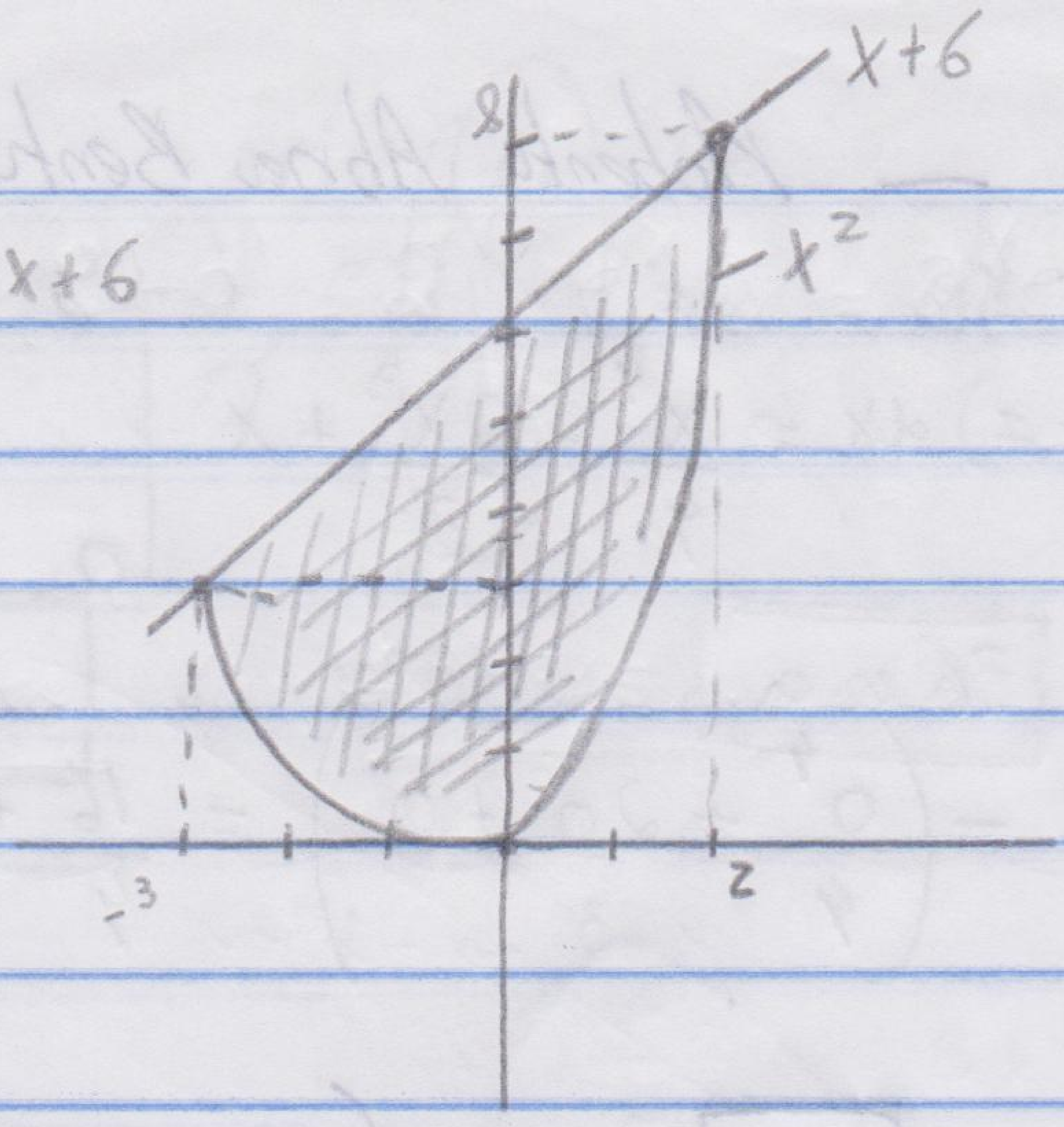
$\boxed{\begin{array}{l} u=5x+1 \\ du=5dx \\ \frac{du}{5}=dx \end{array}}$ $= \frac{1}{5}\left(\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right)\Big]_0^3 = \frac{1}{5}\left(2u^{\frac{1}{2}}\right)\Big]_0^3 = \frac{1}{5}\left(2(5x+1)^{\frac{1}{2}}\right)\Big]_0^3$

$= \frac{1}{5}\left(2.(5.3+1)^{\frac{1}{2}}\right) = \frac{1}{5}.8 = \boxed{\frac{8}{5}}$

1-c) $\int_{-1}^2 (x+2)^4\,dx = \int_{-1}^2 u^4\,dx = \frac{u^5}{5}\Big]_{-1} = \frac{(x+2)^5}{5}\Big]_{-1}^2 =$

$\boxed{\begin{array}{l} u=x+2 \\ du=dx \end{array}}$ $= \frac{4^5}{5}-\frac{1^5}{5} = \frac{1024}{5}-\frac{1}{5} = \boxed{\frac{1023}{5}}$

2-a) $y = x^2$, $y = x+6$



$x^2 = x+6$

$x^2 - x - 6 = 0$

$\Delta = 1+24 = 25$

$x = \frac{-1 \pm 5}{2}$

$x' = \frac{4}{2} = 2$

$x'' = -\frac{6}{2} = -3$

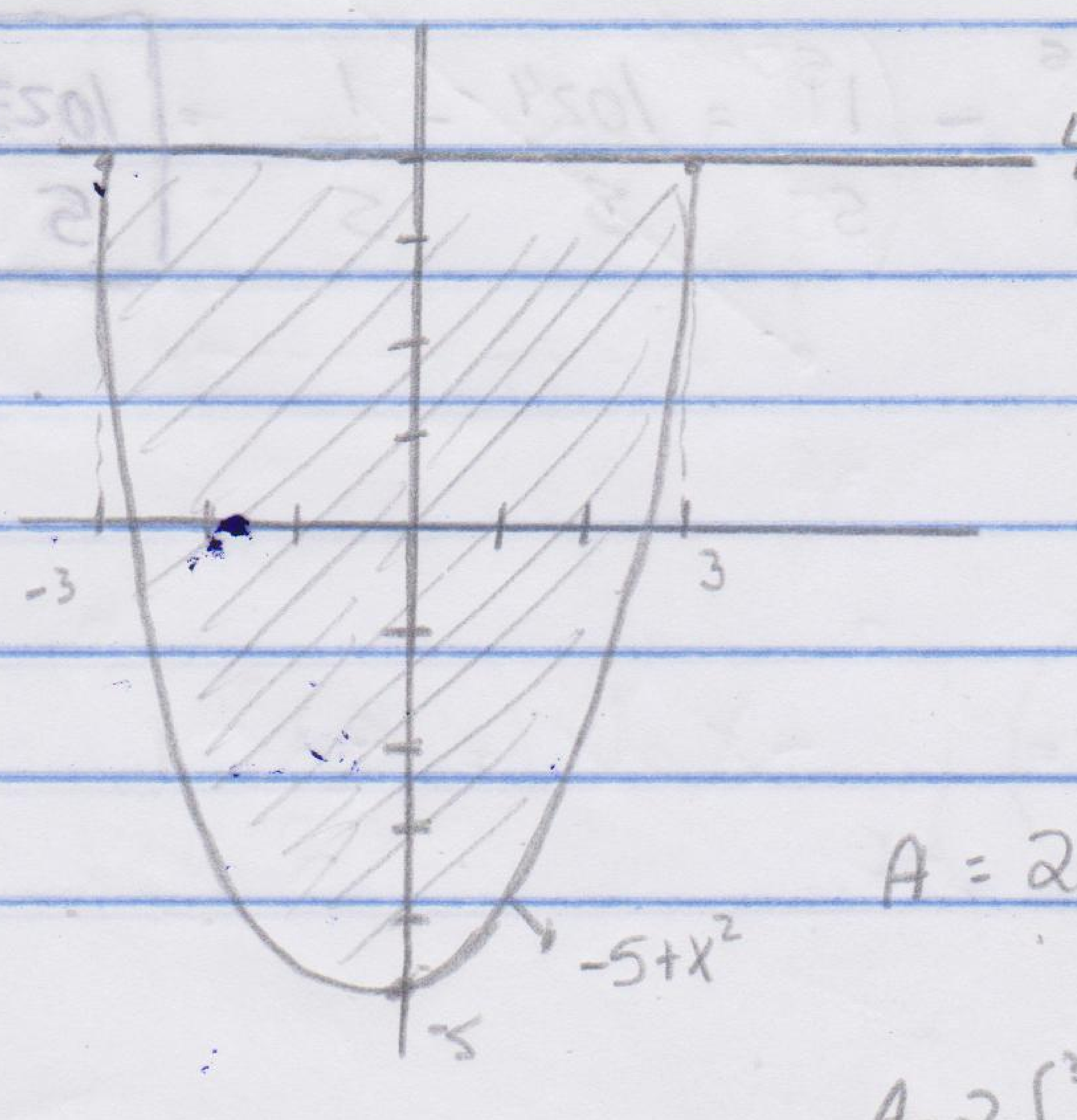
$$A = \int_{-3}^{2} (x+6-x^2) = \left[\frac{x^2}{2}+6x-\frac{x^3}{3}\right]_{-3}^{2} =$$

$$= \frac{2^2}{2}+6.2-\frac{2^3}{3} - \left(\frac{(-3)^2}{2}+6.(-3)-\frac{(-3)^3}{3}\right) = 2+12-\frac{8}{3} -$$

$$\left(\frac{9}{2}-18+\frac{27}{3}\right) = 14-\frac{8}{3}-\frac{9}{2}+18-\frac{27}{3} = 32-\frac{9}{2}-\frac{35}{3} =$$

$$= \frac{192-27-70}{6} = \boxed{\frac{95}{6}\ u.a}$$

2-b) $y=-5+x^2$, $y=4$



$-5+x^2=4$

$x^2=9$

$x=\sqrt{9}$

$x=\pm 3$

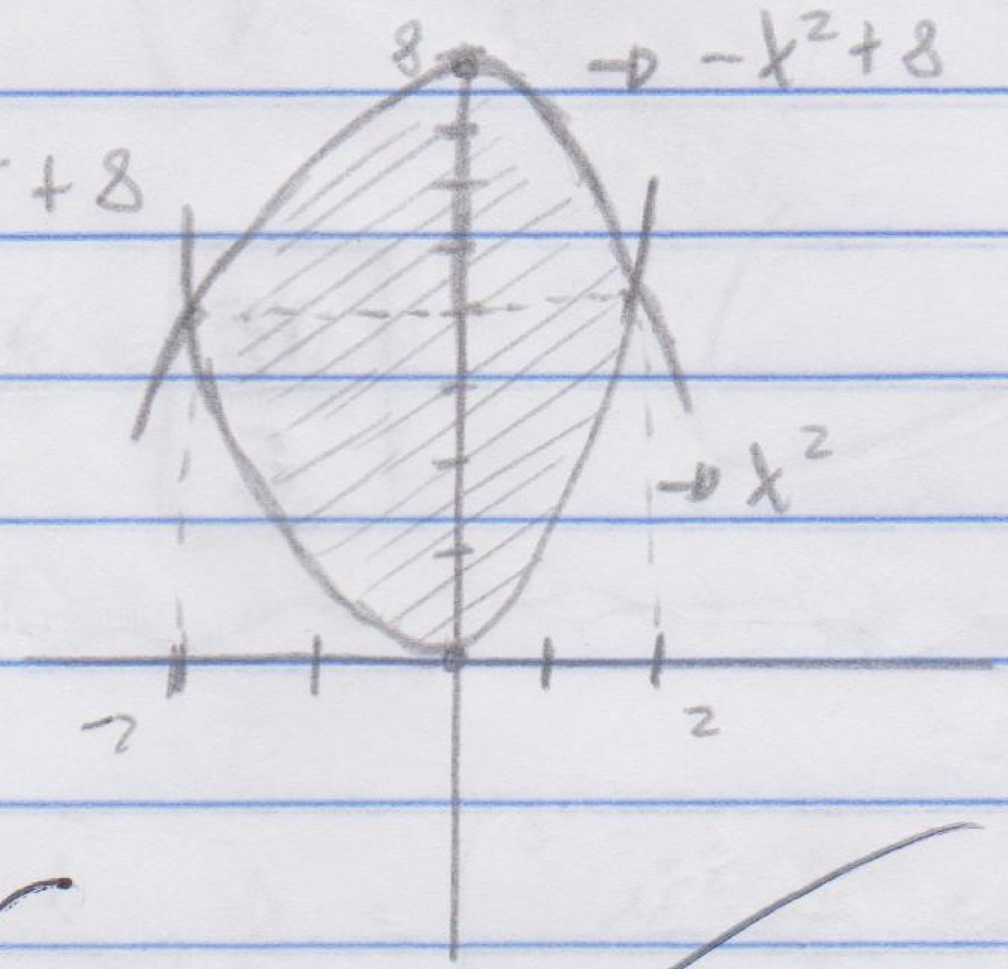
$$A = 2\int_0^3 (4+5-x^2) =$$

$$A = 2\int_0^3 (9-x^2) =$$

$$A = 2\int_0^3 (9-x^2) = 2\left(9x - \frac{x^3}{3}\right)\Big]_0^3 = 2\left(9.3 - \frac{3^3}{3}\right) = 2\left(27 - \frac{27}{3}\right) =$$

$$= 2\left(\frac{54}{3}\right) = \frac{108}{3} = \boxed{36\,u.a}$$

2-c) $y = x^2$ e $y = -x^2+8$



$x^2 = -x^2+8$

$2x^2 = 8$

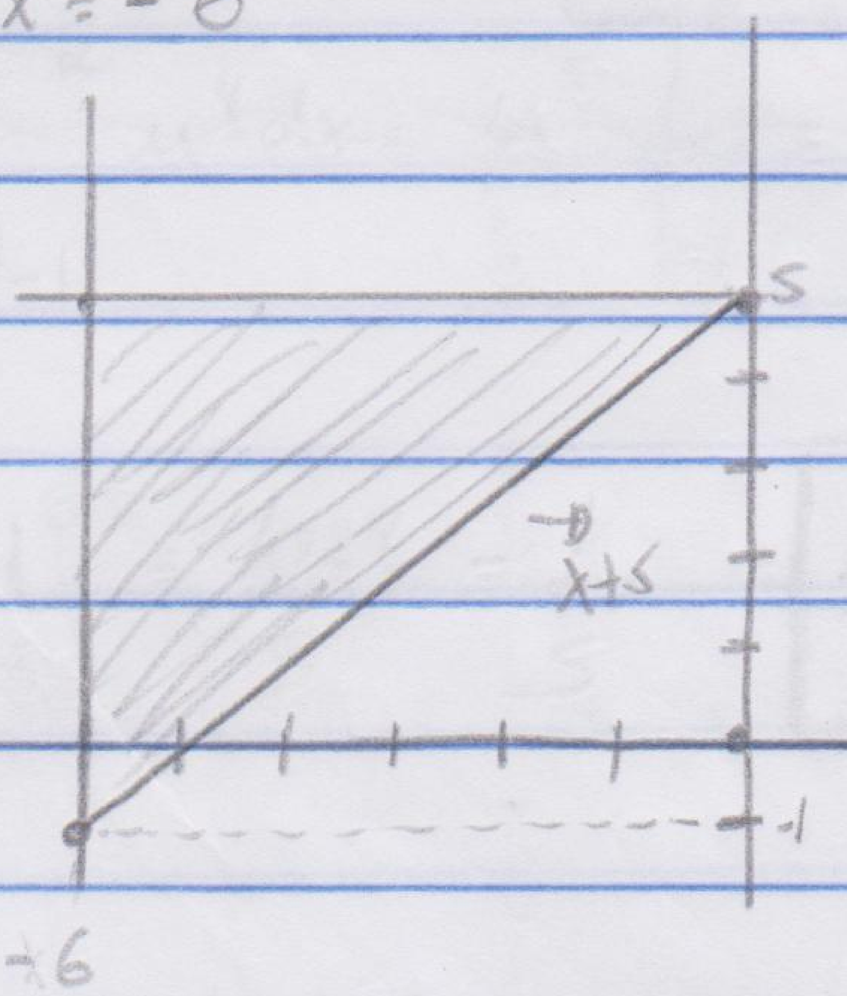
$x = \sqrt{4}$

$x = \pm 2$

$$A = 2\int_0^2 (-x^2+8-x^2) = 2\int_0^2 (-2x^2+8) = 2\left(-\frac{2x^3}{3} + 8x\right)\Big]_0^2 =$$

$$= 2\left(-\frac{2.2^3}{3} + 8.2\right) = 2\left(-\frac{16}{3} + 16\right) = 2.\frac{32}{3} = \boxed{\frac{64}{3}\,u.a}$$

2-d) $y = x+5$, $y = 5$ e $x = -6$



$x+5 = 5$

$x = 0$

$$A = \int_{-6}^0 (5-x-5) = \int_{-6}^0 -x\,dx = -\frac{x^2}{2}\Big]_{-6}^0 = 0 - \left(-\frac{(-6)^2}{2}\right) = \boxed{18\,u.a}$$

3) $A = 2\int_0^1 (e^x - 0) = 2\int_0^1 e^x\,dx = 2(e^x)\Big|_0^1 = 2.e^1 - 2.e^0 =$

$= \boxed{2e^1 - 2\ u.a}$ ou $\cong 3{,}4365\ u.a$