

Dirichlet

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1) $a(u, \delta u) = 0 \Rightarrow u$

$$a(u, \delta u) := -\frac{1}{|\Omega|} \int_{\Omega^{cp}} J(u) \cdot \nabla \delta u \, d\Omega - \frac{1}{|\Omega|} \int_{\Gamma_{ITE}} J(u) \cdot \frac{\partial \delta u}{\partial s} \, d\Gamma$$

Taylor w. homogenized ITE

2) $\bar{J}_{||} = -a(u, \underline{e}_{||} \cdot [x - \bar{x}]) = \frac{1}{|\Omega|} \int_{\Omega^{cp}} J_{||} \, d\Omega + \int_{\Gamma_{ITE}} J_{||} \, d\Gamma$

Discrete

$$K u = 0$$

$$\bar{J}_{||} = \underline{e}_{||}^T K u$$

Taylor w. homogenized (surface) ITE

1) $u_i = e_i \cdot [x - \bar{x}]$

2) $\bar{J}_{||} = -a(u_i, u_i)$

Continuum Taylor

Alt.

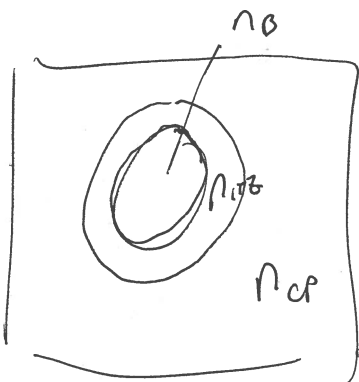
$$\bar{D}^V = n_{cp} \bar{D}^{cp} + n_{ITE} \bar{D}^{ITE}$$

$$\frac{A_{ITE} t_{ITE} D_{ITE}}{V_{SVE} n_{ITE}} = A_{ITE} t_{ITE}$$

$$V_{SVE} n_{ITE} = A_{ITE} t_{ITE}$$

$$A_{ITE} = \text{surface integral} \quad [m^2/m^3]$$

$$A_{ITE} = \frac{a_{ITE}}{V_{OAC}}$$



$$\bar{D}_{ITE} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \bar{D}_{ITE}(\varphi) \, d\varphi$$

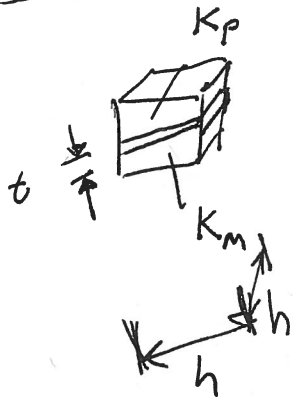
Taylor or homogenized ITC



$$\bar{J}_{\sim} = n_{CP} \bar{D}^{CP} + n_{ITZ} \left[\frac{1}{2\pi^2} \int_{-\pi}^{\pi} \int_0^{\pi} D^{ITZ}(\varphi, \theta) d\varphi d\theta \right]$$

$A_{ITZ} t_{ITZ}$

1TE-modeling



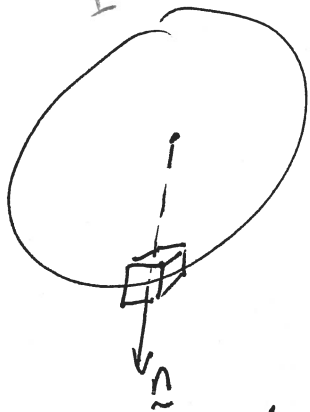
Isotropic Voigt assumption

$$\bar{K} \approx \frac{1}{2}(K_p + K_m) + \frac{\hat{K}_{1TE} \cdot h^2}{h^3} = \frac{1}{2}(K_p + K_m) + \frac{\hat{K}_{1TE}}{h}$$

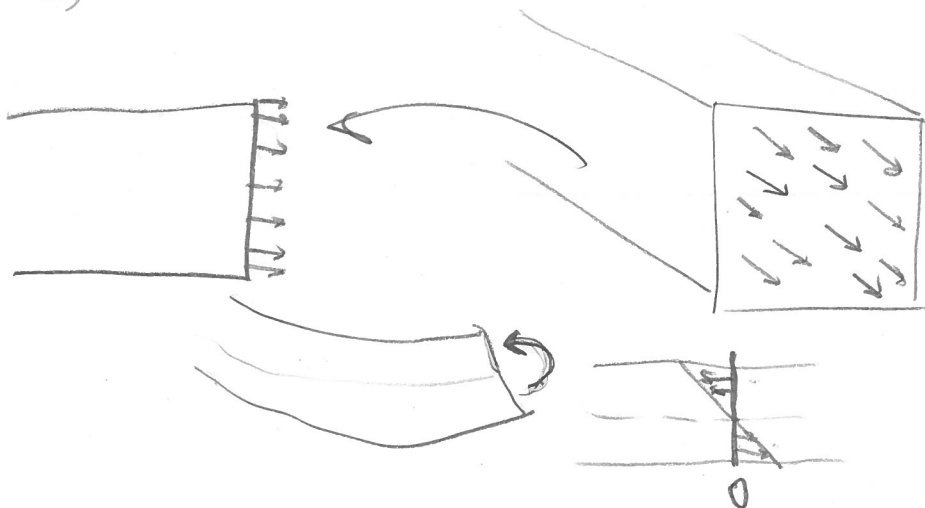
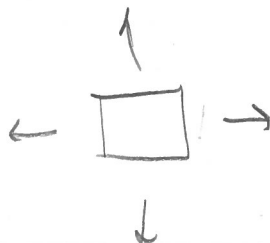
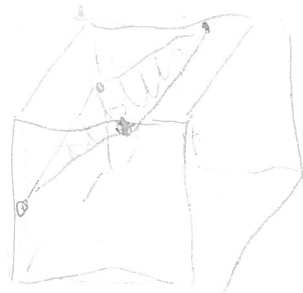
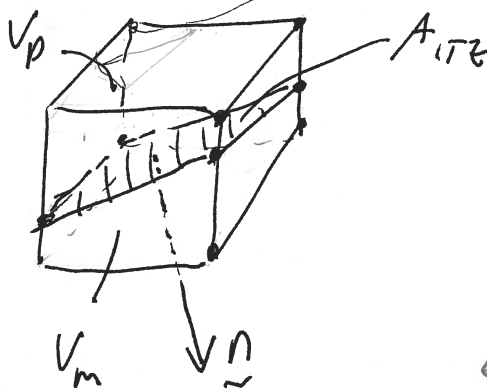
$$\hat{K}_{1TE} = K_{1TE} t \quad \text{2D-permeability}$$

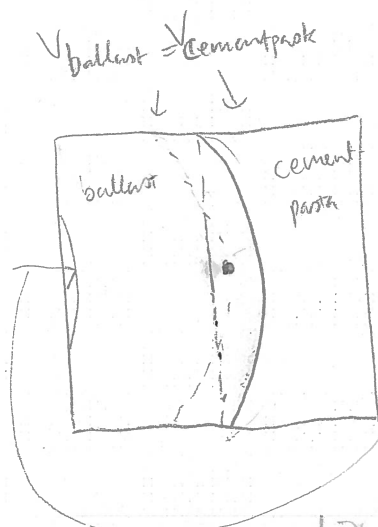
Anisotropic Voigt (known normal)

$$\bar{V} = \frac{M}{I} z$$



$$\bar{K} = \frac{V_p K_p + V_m K_m}{V_p + V_m} \underline{\underline{I}} + \frac{A_{1TE} \hat{K}_{1TE}}{V_p + V_m} [\underline{\underline{I}} - \underline{\underline{n}} \underline{\underline{n}}]$$





No ITZ (old) går efter tyngdpunkt
 \Rightarrow mängden ballast underskattas alltid
 Isotropic Voigt räknar alltid 50% var
 \Rightarrow mängden ballast underskattas lite
 Anisotropisk Voigt räknar efter ^{punkt} streckade linjen
 \Rightarrow mängden ballast underskattas

Mesh sensitivity (ITZ)

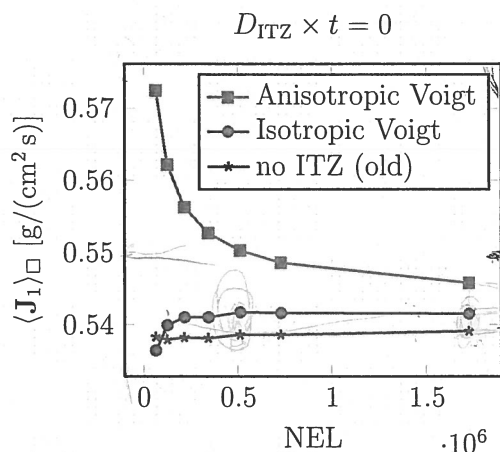
Filip Nilenius

March 12, 2013

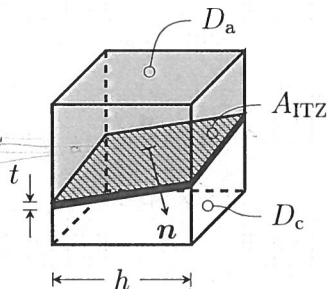
$\approx 55\%$ cement

$$0.2 \text{ cm} \rightarrow h = \underline{0.029 \text{ cm}}$$

$$\Rightarrow \frac{0.2}{0.029} \approx 10$$



(a) Mesh sensitivity.

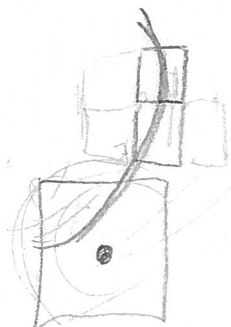


(b) Interface voxel.

$$\text{Anisotropic Voigt: } \bar{D} = \frac{V_a D_a + V_c D_c}{V_a + V_c} \mathbf{I} + \frac{A_{ITZ} D_{ITZ} t}{V_a + V_c} (\mathbf{I} - \mathbf{n} \otimes \mathbf{n})$$

$$D_{ITZ} = 0$$

$$\text{Isotropic Voigt: } \bar{D} = \frac{1}{2}(D_b + D_c) + \frac{t}{h} D_{ITZ}$$



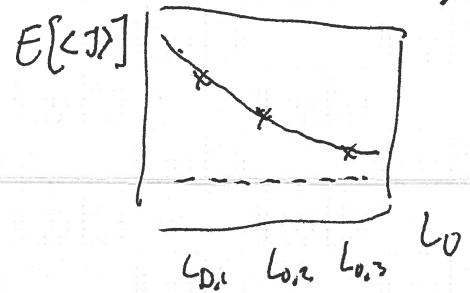
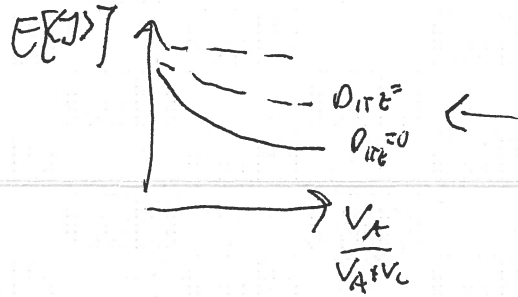
$$\frac{V_a}{V_c}$$



Andelen
 Mängden gränselement minskar för ökande antal element

Converged values
~ given L_0

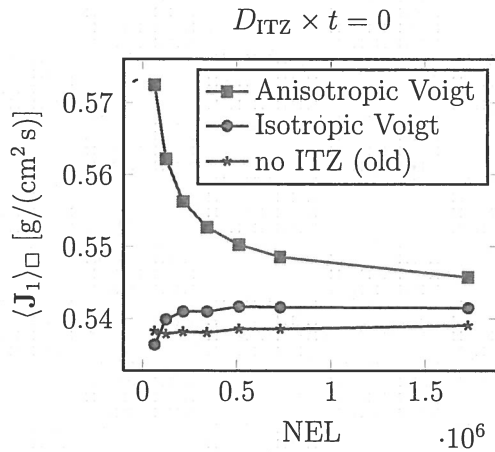
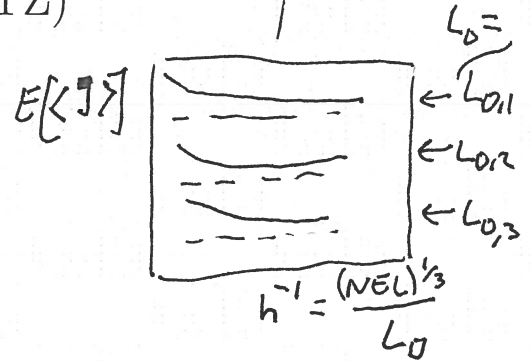
Converged values
~ given h



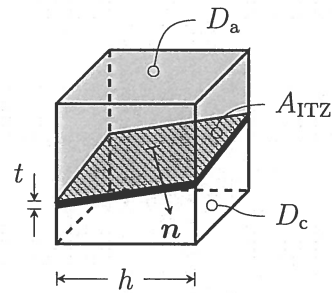
Mesh sensitivity (ITZ)

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(a) Mesh sensitivity.

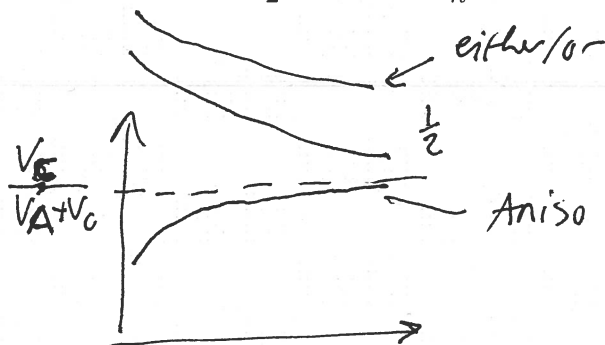


(b) Interface voxel.

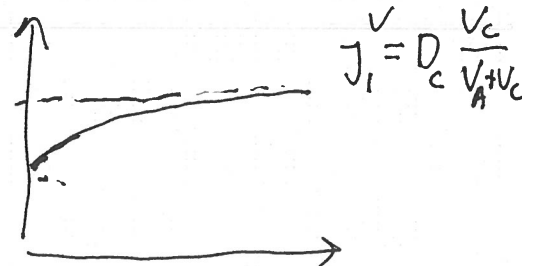
Anisotropic Voigt: $\bar{D} = \frac{V_a D_a + V_c D_c}{V_a + V_c} \mathbf{I} + \frac{A_{ITZ} D_{ITZ} t}{V_a + V_c} (\mathbf{I} - \mathbf{n} \otimes \mathbf{n})$

Isotropic Voigt: $\bar{D} = \frac{1}{2}(D_b + D_c) + \frac{t}{h} D_{ITZ}$

RVE-Voigt bound $D_A = 0$



~ GL

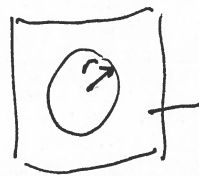


Voigt ($L_0 \rightarrow 0$) with ITZ

$$D = \frac{V_A D_A + V_C D_C + A_{ITZ} \bar{t} D_{ITZ}}{V_A + V_C} = D^V + \frac{\bar{t}}{\bar{r}} D_{ITZ}$$

\uparrow Voigt w/o ITZ \nwarrow "average" radius

$$\frac{1}{\bar{r}} = \frac{A_{ITZ}}{V_A + V_C}$$



$$V_A + V_C = L^3$$

$$\frac{V_C}{V_A + V_C} = n \Rightarrow V_A + V_C = \frac{V_C}{n}$$

$$V_C = \frac{4}{3} \pi r^3$$

$$A_{ITZ} = 4\pi r^2$$

$$\Rightarrow \frac{1}{\bar{r}} = \frac{n 4\pi r^2}{\frac{4}{3} \pi r^3} = \frac{3n}{r}$$

$$\Rightarrow \bar{r} = \frac{r}{3n}$$