Effect, Power, ANOVA, and NHST Review

Presented by David John Baker
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FLATIRON SCHOOL

Power, Effects, ANOVA, Review

- Power tells us how likely are we to find an effect if there is one
- In practice, this means allocating the appropriate amount of resources (time and money!)
- Power relates to the other concepts this week and we will dive into all those relationships while reviewing others this week
- Introduce ANOVA and problems of multiple comparison
- Review Terms Together

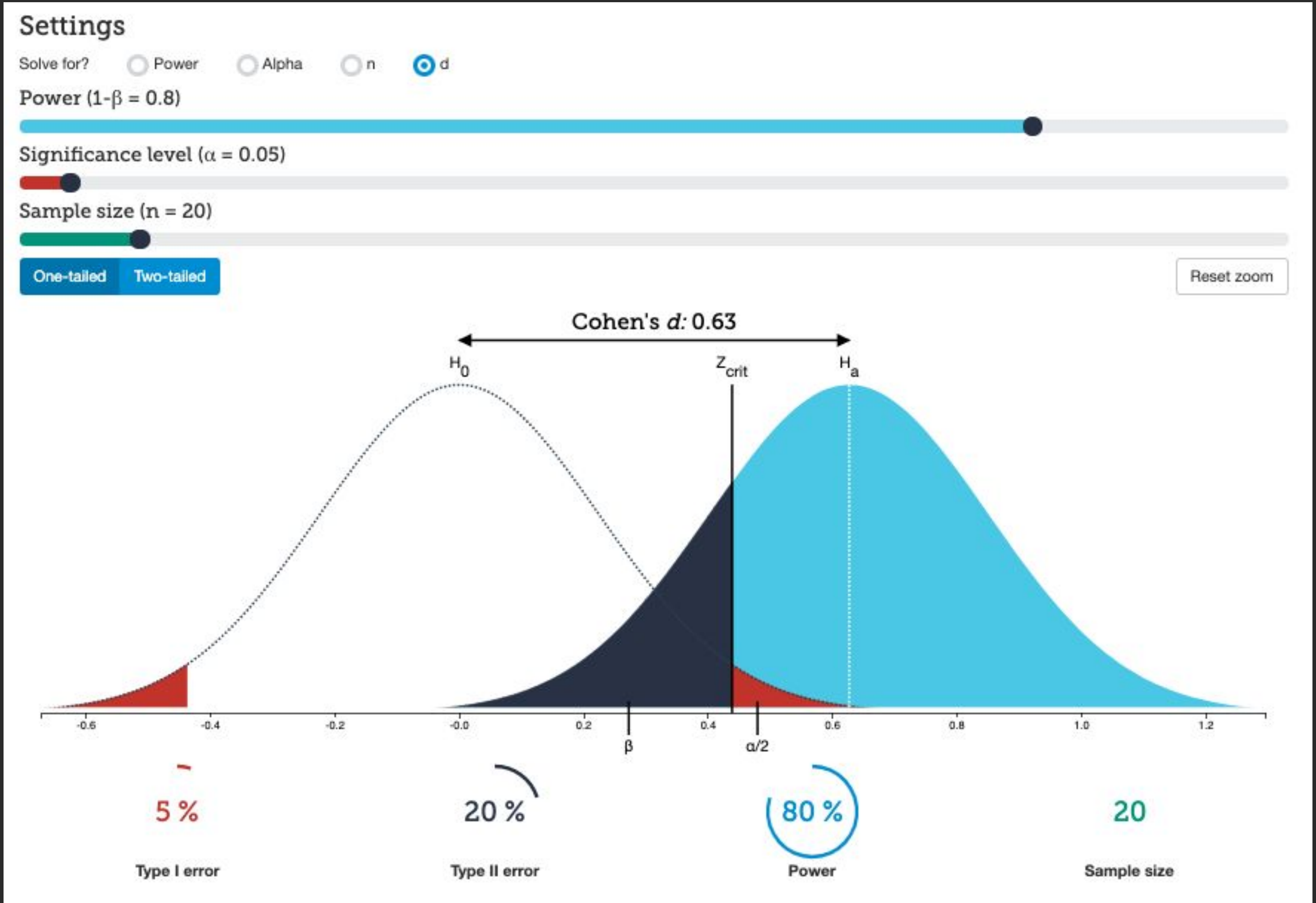
Types of Errors

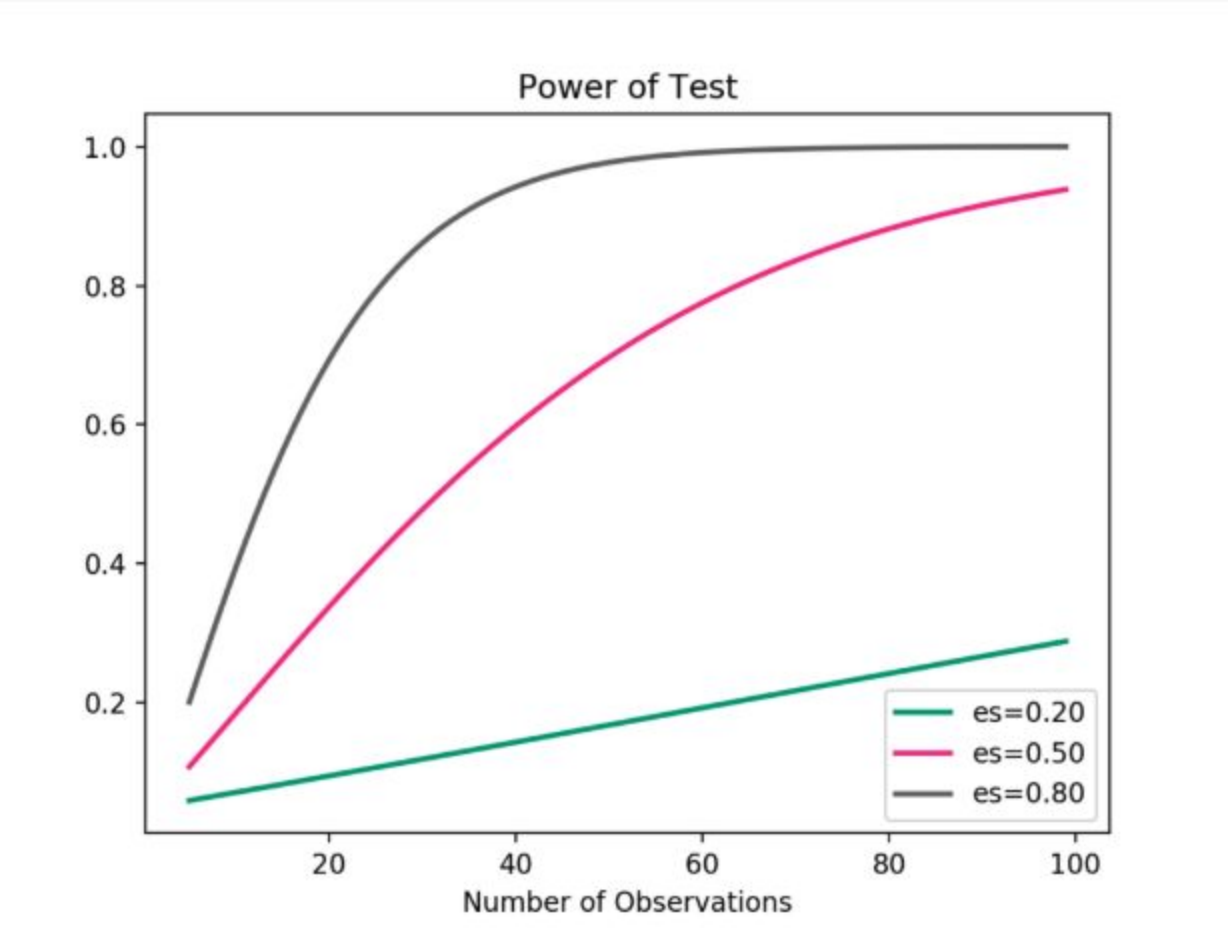
	H0 True	H1 True
Significant Finding	False Positive	True Positive
Non-Significant Finding	True Negative	False Negative

Review (Interview Questions)

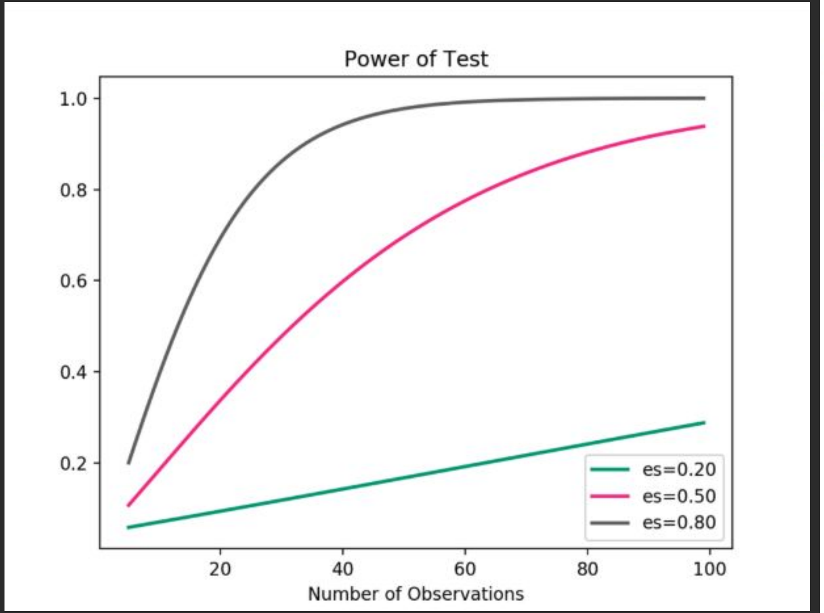
- What is hypothesis testing and how is it used in research?
- Why do we set up a null hypothesis and an alternative hypothesis?
- What are Type I errors?
- What are Type II errors?
- What can be done in order to lower your error rates?
- Describe the differences between frequentist and Bayesian schools of thought on data and its relation to the world.

https://rpsychologist.com/d3/NHST/


Settings
Solve for?
Power
Alpha
n
d
Power (1-β = 0.8)
Significance level (α = 0.05)
Sample size (n = 20)
One-tailed
Two-tailed
Reset zoom
Cohen's d: 0.63
H0
Zcrit
Ha
-0.6
-0.4
-0.2
-0.0
0.2
0.4
0.6
0.8
1.0
1.2
β
α/2
5 %
Type I error
20 %
Type II error
80 %
Power
20
Sample size



Power of Test
1.0
0.8
0.6
0.4
0.2
20
40
60
80
100
Number of Observations
es=0.20
es=0.50
es=0.80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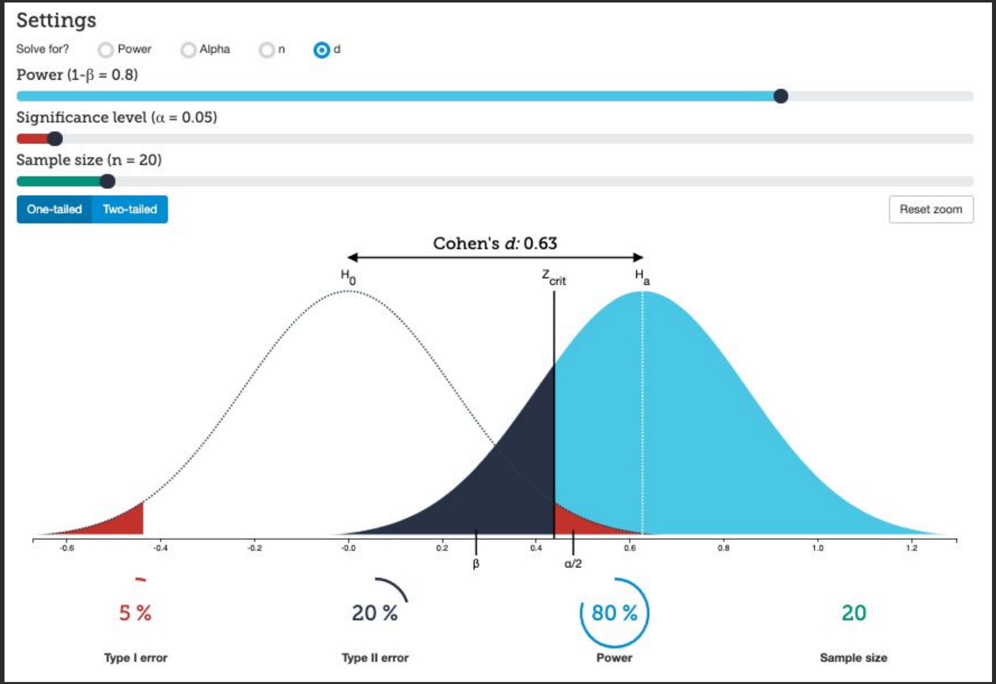
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Special Relationships

- There is a special relationship between effect size, sample size, alpha, and beta. If you know three, you can always calculate the fourth.
- In groups, come up with a situation in which you have three and would need to calculate the fourth. Be prepared to justify your answers. You should have four scenarios.

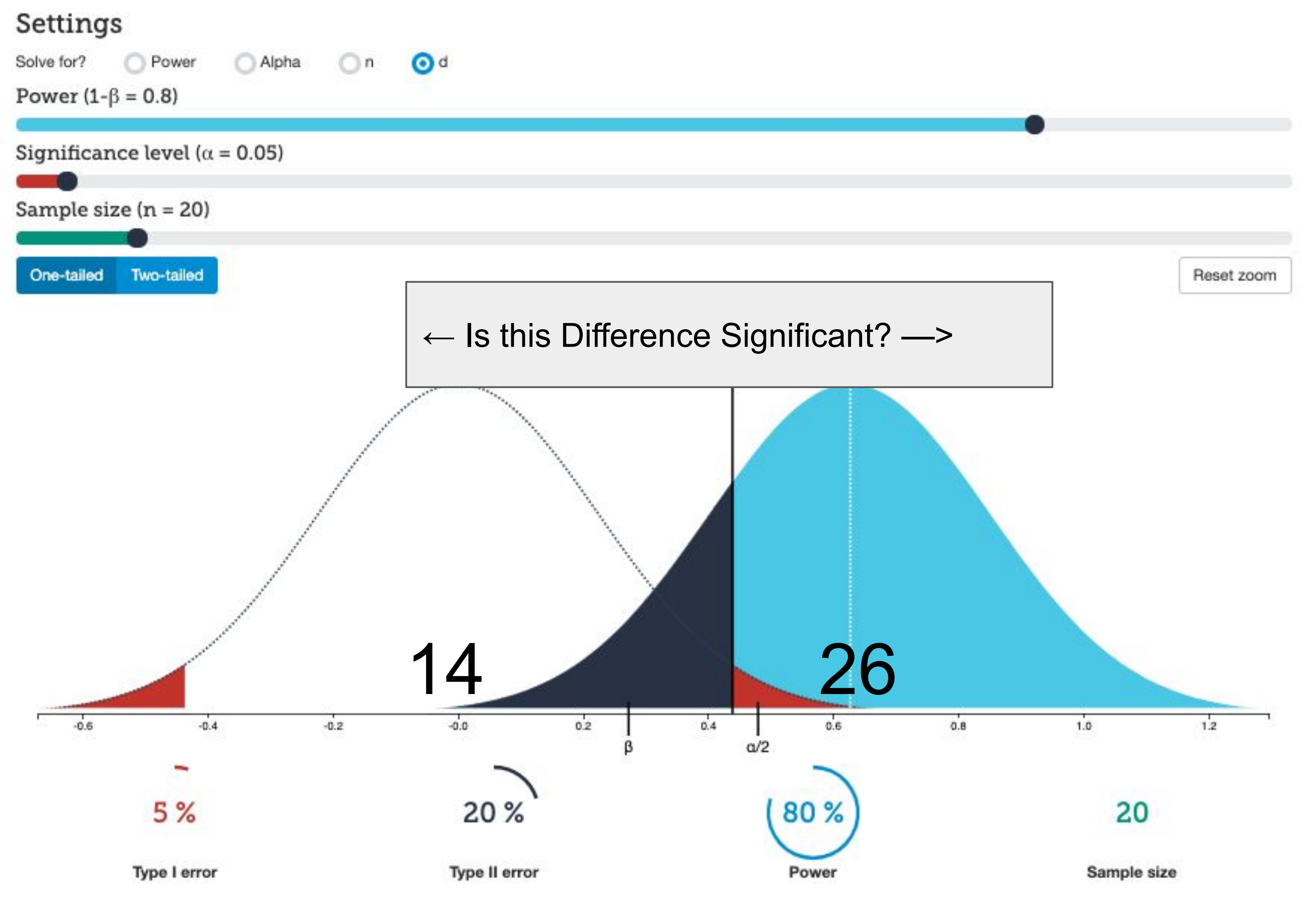
Discussion Questions

- Describe the relationship between the power of a test and the size of its effect
- What affects effect size? ***

Calculating Effect Size

Cohen's d = (Mean 1 - Mean 2) / SD*

$*SD_{pooled} = \sqrt{((SD_1^2 + SD_2^2)/2)}$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Significance level (α = 0.05)
Sample size (n = 20)
One-tailed
Two-tailed
Reset zoom
← Is this Difference Significant? —>
14
26
-0.6
-0.4
-0.2
-0.0
0.2
0.4
0.6
0.8
1.0
1.2
β
α/2
5 %
Type I error
20 %
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Power
20
Sample size

Calculating Effect Size

Scenario: On a standardized anagram task, $\mu = 26$ anagrams solved with a $\sigma = 4$. A researcher tests whether the arousal from anxiety is distracting and will decrease performance. A sample of $n = 14$ anxiety patients is tested on the task. There average performance is 23.36 anagrams.

c. **Step three**: Select the sample and collect your data.

d. **Step four**: Locate the region of rejection and the critical value(s) of your test statistic. Again, directionality is important to consider.

Cohen's d = (Mean 1 - Mean 2) / SD*

Cohen's d = (26 - 14) / 4 = 3

$*SD_{pooled} = \sqrt{((SD_1^2 + SD_2^2)/2)}$

https://rpsychologist.com/d3/cohend/

Effect size	*d*	Reference
Very small	0.01	Sawilowsky, 2009
Small	0.20	Cohen, 1988
Medium	0.50	Cohen, 1988
Large	0.80	Cohen, 1988
Very large	1.20	Sawilowsky, 2009
Huge	2.0	Sawilowsky, 2009

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What other factors might contribute to a measure of effect size?

Where else have we seen an an effect size?

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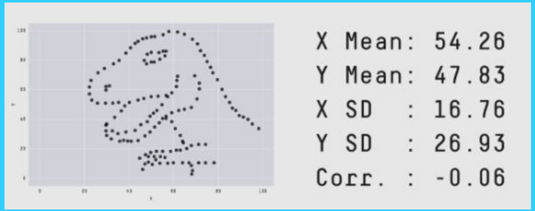
What other factors might contribute to a measure of effect size?

Correlation coefficients are effect sizes!!!

$$r_P = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2}\sqrt{\sum_i (y_i - \bar{y})^2}}$$

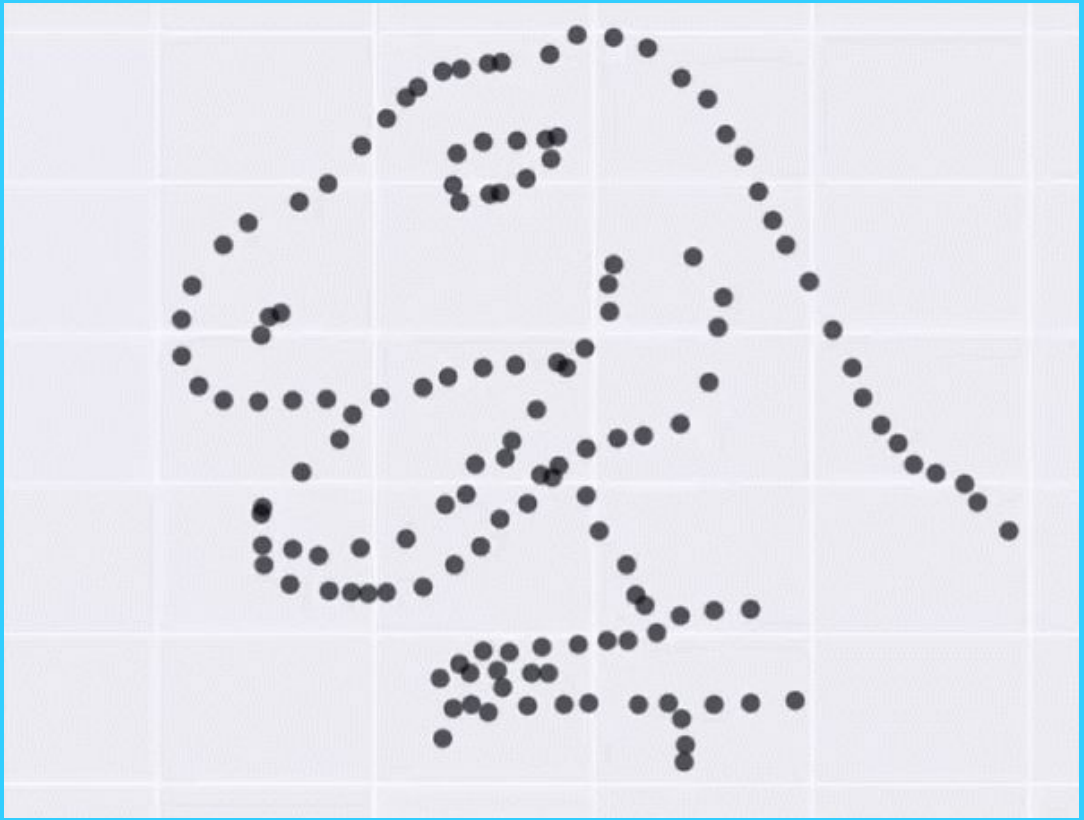
$$\beta_1 = r_P \frac{\sigma_y}{\sigma_x}$$

$$\beta_0 = \bar{y_1} - \beta_1 \bar{x}$$



Correlation coefficients are effect sizes!!!

But make sure to always look at your data



https://www.autodeskresearch.com/publications/samestats https://rpsychologist.com/d3/correlation/

```
#Load pwr package to easily calculate the statistical power
if(!require(pwr)){install.packages('pwr')}
library(pwr)
#Disable scientific notation (1.05e10)
options(scipen=999)
#Set number of simulations
nSims <- 100000 #number of simulated experiments

M<-106 #Mean IQ score in the sample (will be compared with 100 in a one-sample t-test)
n<-26 #set sample size
SD<-15 #SD of the simulated data
#With a mean difference of 6, and SD of 15, and a sample size of 26, the test has 50% power)

p <-numeric(nSims) #set up empty variable to store all simulated p-values
bars<-20
#Run simulation
for(i in 1:nSims){ #for each simulated experiment
  x<-rnorm(n = n, mean = M, sd = SD) #Simulate data with specified mean, standard deviation, and sample size
  z<-t.test(x, mu=100) #perform the t-test against mu (set to value you want to test against)
  p[i]<-z$p.value #get the p-value and store it
}

#Check power by summing significant p-values and dividing by number of simulations
(sum(p < 0.05)/nSims) #power
#Calculate power formally by power analysis
power<-pwr.t.test(d=(M-100)/SD, n=n,sig.level=0.05,type="one.sample",alternative="two.sided")$power #determines M when power > 0. When power = 0, will set  M = 100.

#Plot figure
#png(file="P-valueDist.png",width=4000,height=3000, , units = "px", res = 500)
op <- par(mar = c(5,7,4,4)) #change white-space around graph
hist(p, breaks=bars, xlab="P-values", ylab="number of p-values\n", axes=FALSE,
     main=paste("P-value Distribution with",round(power*100, digits=1),"% Power"),
     col="grey", xlim=c(0,1),  ylim=c(0, nSims))
axis(side=1, at=seq(0,1, 0.1), labels=seq(0,1,0.1))
axis(side=2, at=seq(0,nSims, nSims/4), labels=seq(0,nSims, nSims/4), las=2)
abline(h=nSims/bars, col = "red", lty=3)
#dev.off()

#Š Daniel Lakens, 2016.
# This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License. https://creativecommons.org/licenses/by-nc-sa/4.0/
```

ANOVA

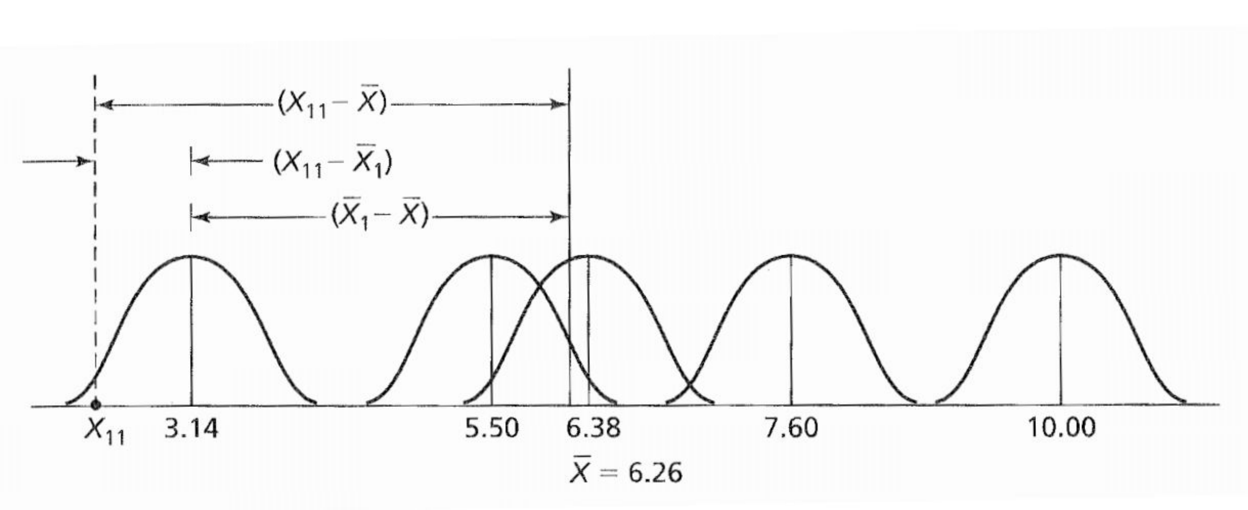
- For the past two lectures, we have been focusing on differences between 2 distributions, quantifying our uncertainty, but we can't just run tons of t-tests our whole lives
- Knowing what we know about Type I error rates, what problem would there be in running t-test after t-test after t-test?

ANOVA II

- Performing multiple t-tests increases our Type I error rate!!
- In order to not fool ourselves in the long run, we need to set up a way to protect ourselves against Type I error rates
- Experimentwise Type I Error Rate
  - $1-(1-\text{alpha})^c$
  - Where c = number of independent t tests
  - For example, if I had three groups, my experiment type I error rate would be $1 - (1-.05)^3 = .142$

ANOVA III

- ANOVA or ANalysis Of VAriance or F test attempts to fix this problem



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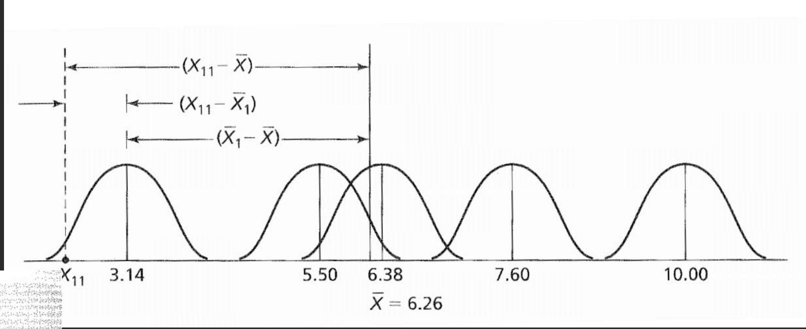
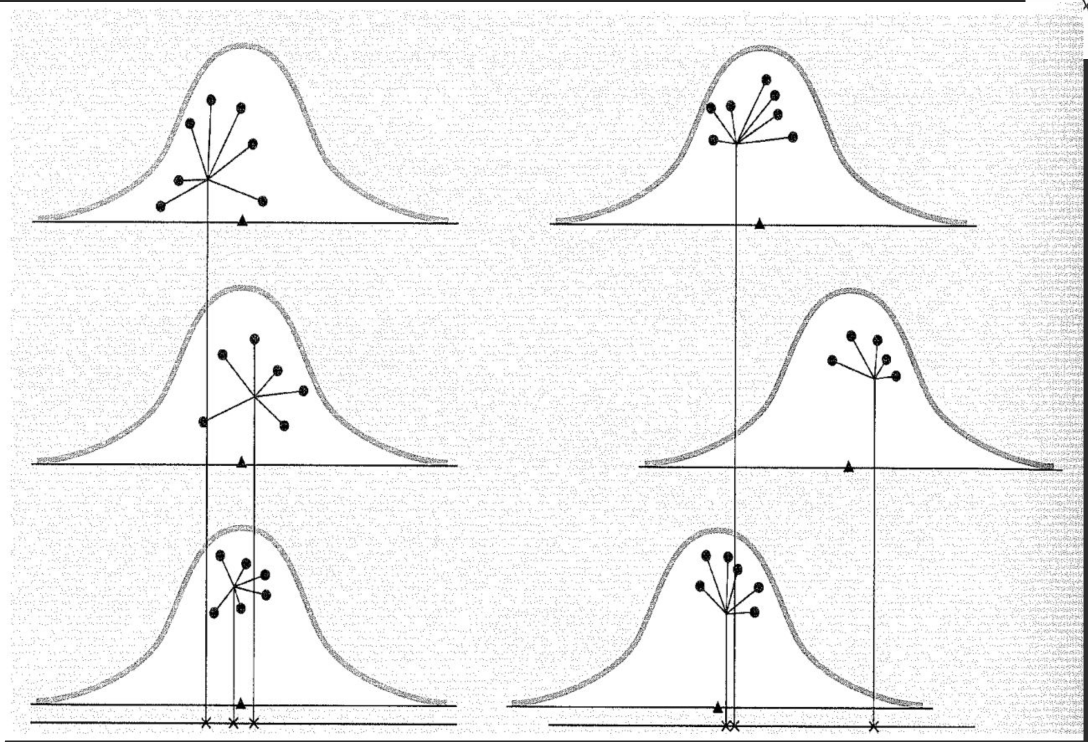
ANOVA IV


$(X_{11} - \bar{X})$
$(X_{11} - \bar{X}_1)$
$(\bar{X}_1 - \bar{X})$
X_{11}
3.14
5.50
6.38
7.60
10.00
$\bar{X} = 6.26$



Scenario: Does the ethnicity of a defendant affect the likelihood that he is judged guilty? People were given transcripts of a trial and asked to judge the likelihood that a defendant was guilty, on a 0 – 10 scale. The transcript was identical, but across 3 conditions, the reported ethnicity of the defendant varied. The results were as follows (study based on Stephen, 1975):

White	African-American	Hispanic
6	10	10
7	9	6
2	4	5
3	10	5
5	10	2
0	3	10

Step one: null and alternative hypotheses

Null	Alternative
$\mu_W = \mu_{AA} = \mu_H$	$\mu_i \neq \mu_k$, for some groups i and k
	$\mu_W < \mu_{AA} = \mu_H$
	$\mu_W = \mu_{AA} \neq \mu_H$
	$\mu_W = \mu_H \neq \mu_{AA}$

Step two: select the test and significance level

F test, $\alpha = .05$

Step three: select samples and collect data

Step four: locate region of rejection (i.e., critical value)

F table with *df* in numerator = $K - 1$, *df* in denominator = $N - K$

$$F_{2,15;\alpha=.05} = 3.68$$

Step five: calculate the test statistic

	White	A-A	Hispanic	
	6	10	10	
	7	9	6	
	2	4	5	
	3	8	5	
	5	10	2	
	0	3	8	
n_k	6	6	6	N = 18
s^2	6.97	9.47	7.60	
$\bar{X}$	3.83	7.33	6.00	5.72

when group n is NOT equal:

$$MS_W = \frac{\sum (n_i - 1)s_i^2}{\mathrm{df}_W}$$

$$MS_W = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + (n_3 - 1)s_3^2 + \cdots + (n_k - 1)s_k^2}{N_T - k}$$

$$MS_W = \frac{\sum s^2}{k} = \frac{6.97+9.47+7.60}{3} = 8.01, \text{ when group } n \text{ is equal}$$

$$MS_{\mathrm{B}} = \frac{\sum n_i(\bar{X}_i - \bar{X}_G)^2}{\mathrm{df_B}} = \frac{6(3.83-5.72)^2 + 6(7.33-5.72)^2 + 6(6.00-5.72)^2}{3-1}$$

$$MS_{\mathrm{B}} = \frac{21.43 + 15.55 + 0.47}{2} = 18.73$$

$$F = \frac{MS_B}{MS_W} = \frac{18.73}{8.01} = 2.34$$

Scenario: On a standardized anagram task, $\mu = 26$ anagrams solved with a $\sigma = 4$. A researcher tests whether the arousal from anxiety is distracting and will decrease performance. A sample of $n = 14$ anxiety patients is tested on the task. There average performance is 23.36 anagrams.

e. **Step five**: Compute the appropriate test statistic. σ is known, so we use the z test.

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{14}} = 1.07 \qquad z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{23.36 - 26}{4/\sqrt{14}}$$

$$z = \frac{-2.64}{1.07} = -2.47$$

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ANOVA summary table

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	37.45	2	18.73	2.34	0.130761	3.682317
Within Groups	120.17	15	8.01			
Total	157.61	17				

Review Questions

- What is statistical power?
- How does it relate to effect size?
- What are some common effect size measures?
- What contributes to effect size?
- What parameters would change when you manipulate…
    - Sample size?
    - Alpha Levels?
    - Beta Levels?

Review Questions II — True, False, and Why?

- A significant p value indicates the degree of evidence for the alternative hypothesis
- Collecting larger samples will result in larger effect sizes
- The null hypothesis and the alternative hypothesis should be probabilistically mutually exclusive
- Confidence intervals capture the range where the point estimate is most likely to occur