# Clustering

Clustering Presented by David John Baker December 30th, 2019



## Outline

#### What is Clustering?

Definitions of Clustering
When To Use
Case Study: Marketing Segmentation

#### K-Means Clustering

Selecting K Advantages/Disadvantages

#### Hierarchical Clustering

Dendrograms
Deciding Where to Cut

# Dangers of Unsupervised Learning

When ML Goes Wrong!!

## Outline

**Springer Texts in Statistics** 

Gareth James Daniela Witten Trevor Hastie Robert Tibshirani

# An Introduction to Statistical Learning

with Applications in R



#### What is Clustering?

Definitions of Clustering
When To Use
Case Study: Marketing Segmentation

#### K-Means Clustering

Selecting K Advantages/Disadvantages

#### Hierarchical Clustering

Dendrograms
Deciding Where to Cut

# Dangers of Unsupervised Learning

When ML Goes Wrong!!

In what type of situation would you want to find subgroups within a larger population???

(don't say market segmentation...)

What is Clustering???

Set of techniques for finding subgroups or clusters in a larger population

In order to find out what subgroups ARE you need to know how similar something is...?

#### **Discussion Question:**

Which genres of music are closer to one another?

Rock Jazz Bluegrass Country

#### **Discussion Question: Answer?**

In order to find similarity between groups, you need to define what you even mean by similar. Your definition of similar depends on what variables you have and how you measure them.

#### **PCA vs Clustering**

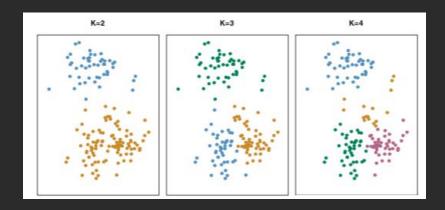
 PCA: Low Dimensional Representation of Data that explains portion of the data

Clustering: Find homogeneous subgroups

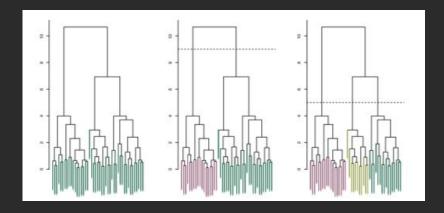
How are these different?

#### **Types of Clustering**

**K-Means Clustering** 



**Hierarchical Clustering** 



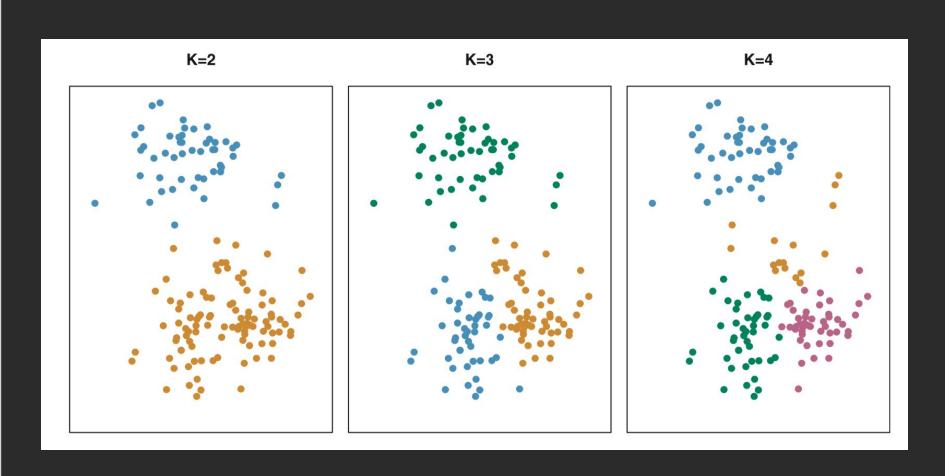
#### **Types of Clustering**

K-Means Clustering

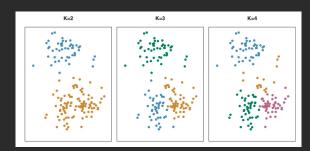
K=2 K=3 K=4

**Hierarchical Clustering** 





#### **K Means Basics**

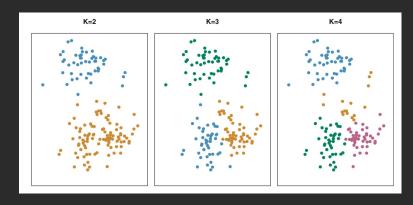


Each observation belongs to a cluster

No overlapping clusters!

We want to MINIMIZE within cluster variation

#### Within Cluster Variation

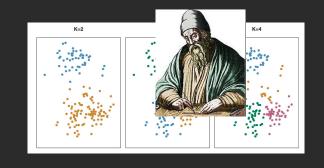


Most common is Squared Euclidean Distance

- Sum all squared pairwise distances
- Divide by number of observations
- But how to do it?!

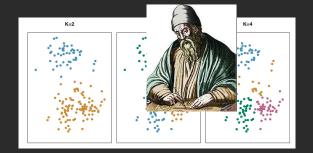


#### **Computation Problem**



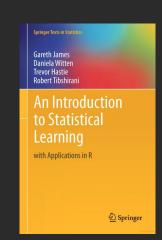
- There are  $K^n$  ways to partition n observations into K clusters!!
- Where do you even begin !?

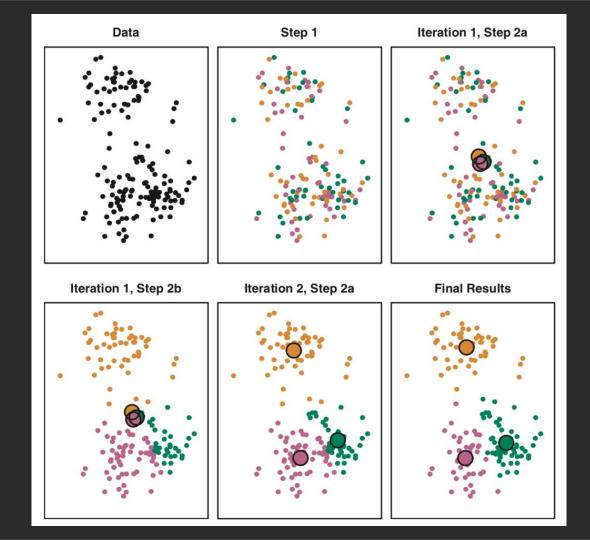
#### **Computation Problem**



#### Algorithm 10.1 K-Means Clustering

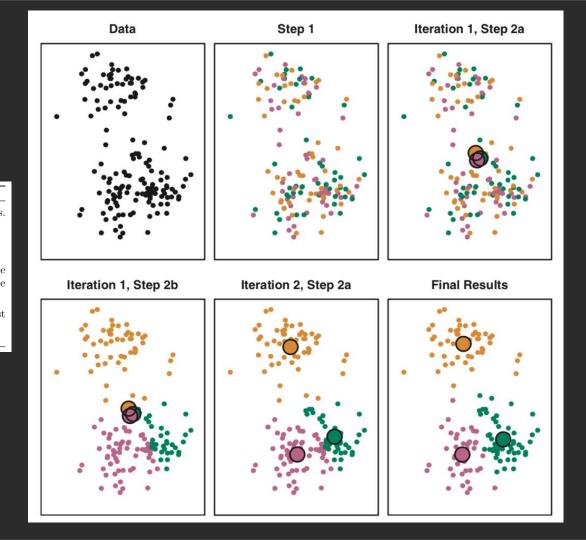
- 1. Randomly assign a number, from 1 to K, to each of the observations. These serve as initial cluster assignments for the observations.
- 2. Iterate until the cluster assignments stop changing:
  - (a) For each of the K clusters, compute the cluster centroid. The kth cluster centroid is the vector of the p feature means for the observations in the kth cluster.
  - (b) Assign each observation to the cluster whose centroid is closest (where *closest* is defined using Euclidean distance).





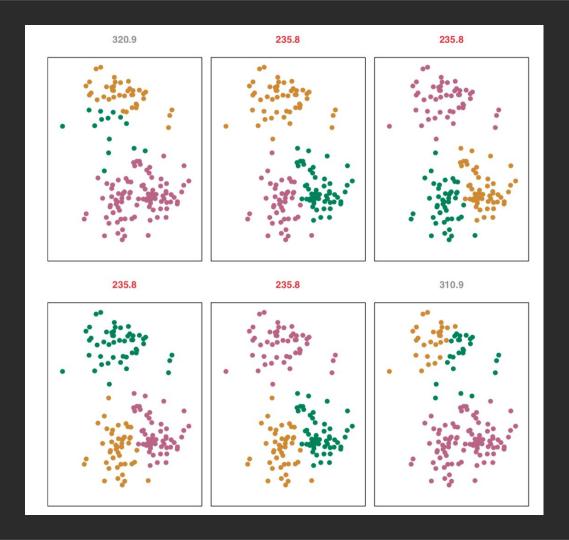
#### Algorithm 10.1 K-Means Clustering

- 1. Randomly assign a number, from 1 to K, to each of the observations. These serve as initial cluster assignments for the observations.
- 2. Iterate until the cluster assignments stop changing:
  - (a) For each of the K clusters, compute the cluster centroid. The kth cluster centroid is the vector of the p feature means for the observations in the kth cluster.
  - (b) Assign each observation to the cluster whose centroid is closest (where *closest* is defined using Euclidean distance).



# Starting State Problems...

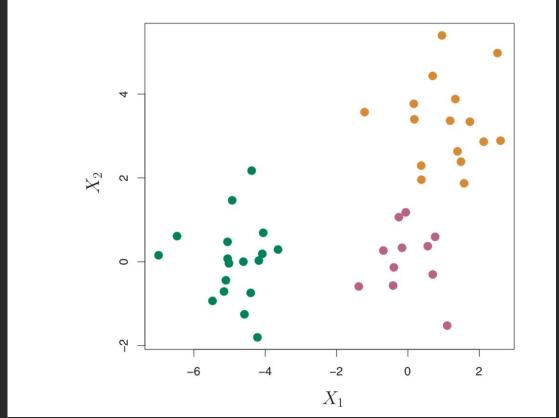
Need lower Objective!! (red)



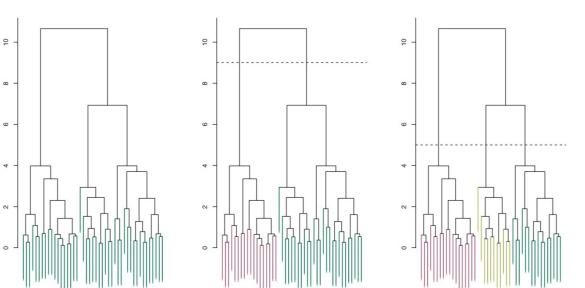
#### **Hierarchical Clustering**

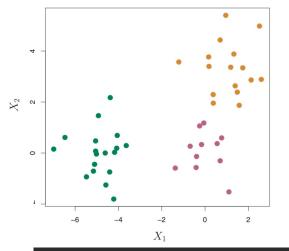
- What if you don't know K? (Top down)
- What if there are clusters within clusters worth writing home about?!
- Gotta go bottom (agglomerative) up!
- More common form
- Build it bottom up!

How do we recover these clusters?

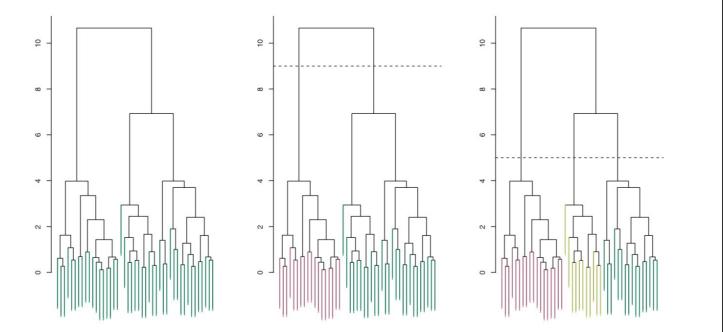


# How do we recover these clusters? Build tree from leaves to branches!



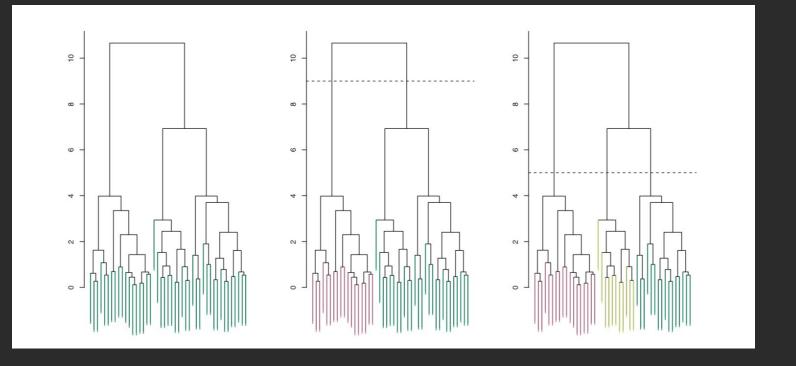


#### **Dendrogram**

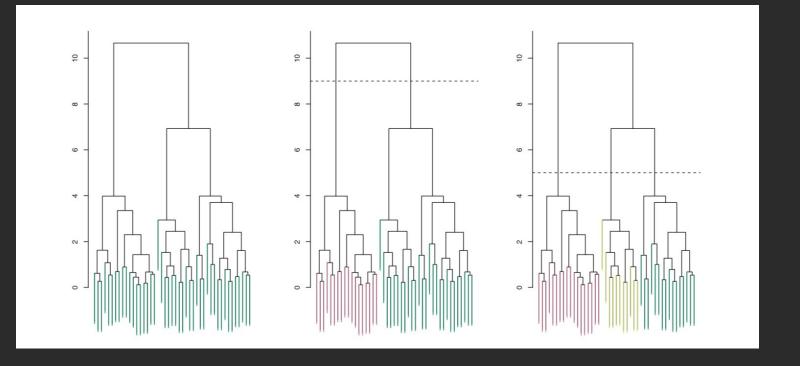


As we go up tree, observations become more different from one another

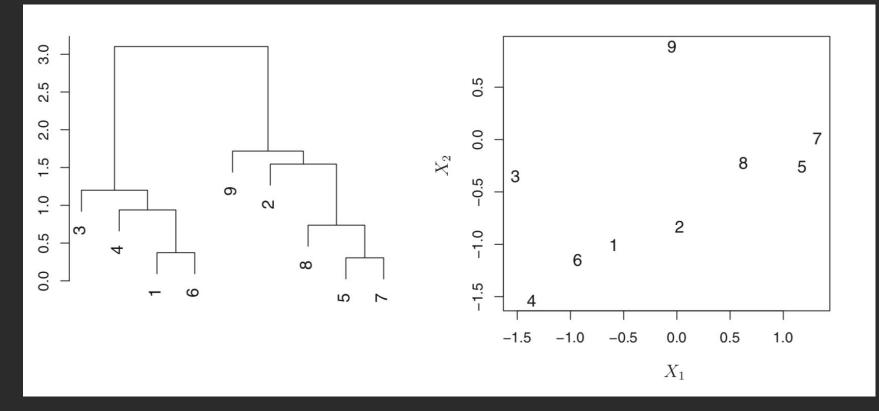
**Lower in tree == More Similar to each other** 



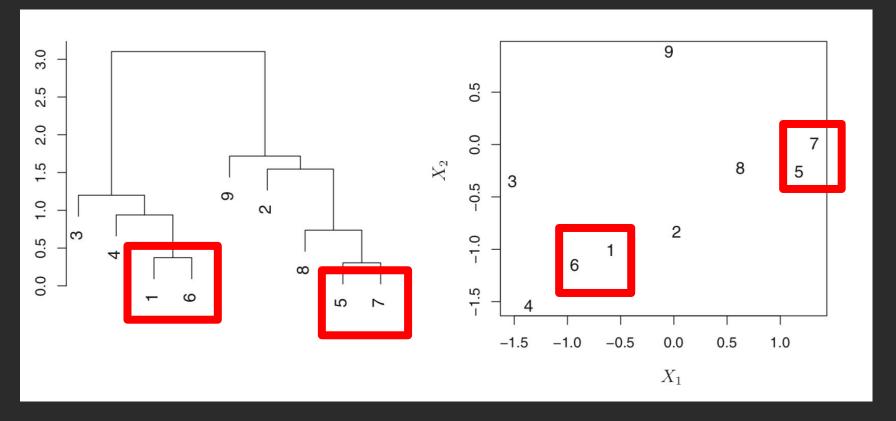
Height of fusion per vertical indicates similarity



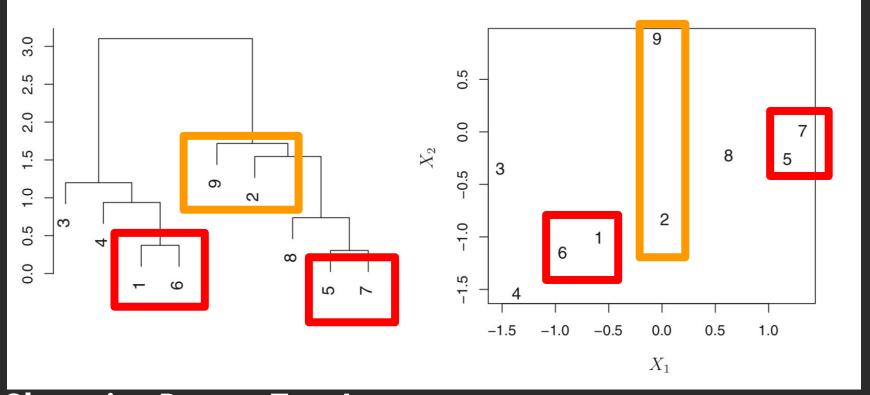
Height of fusion per vertical indicates similarity



### Clustering Danger Zone!

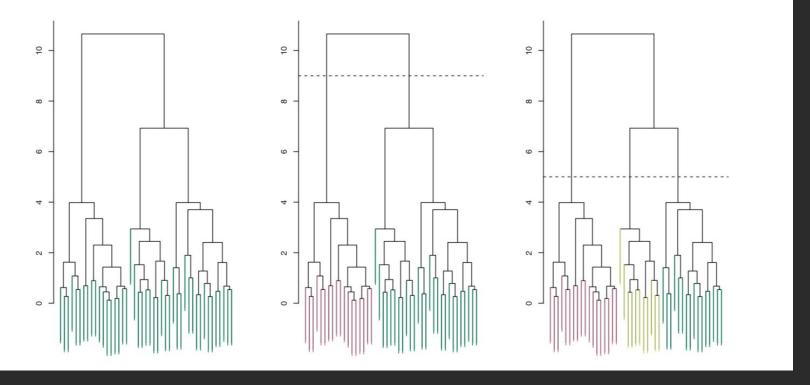


Clustering Danger Zone!



Clustering Danger Zone!

DO NOT MAKE CONCLUSIONS ON HORIZONTAL AXIS!!



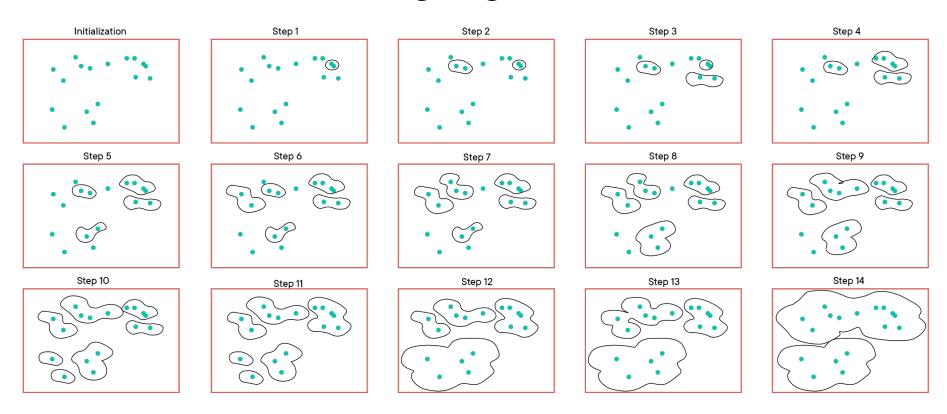
Make horizontal cut, get distinct clusters
One dendrogram, many clusters!!

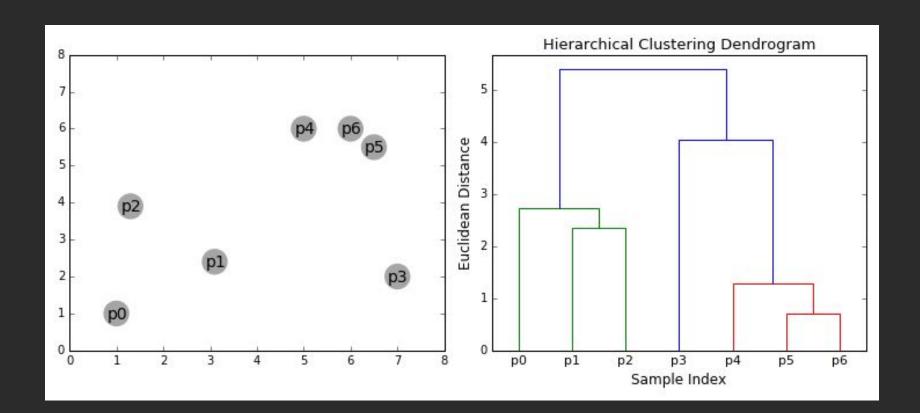
#### **Hierarchical Clustering Algorithm**

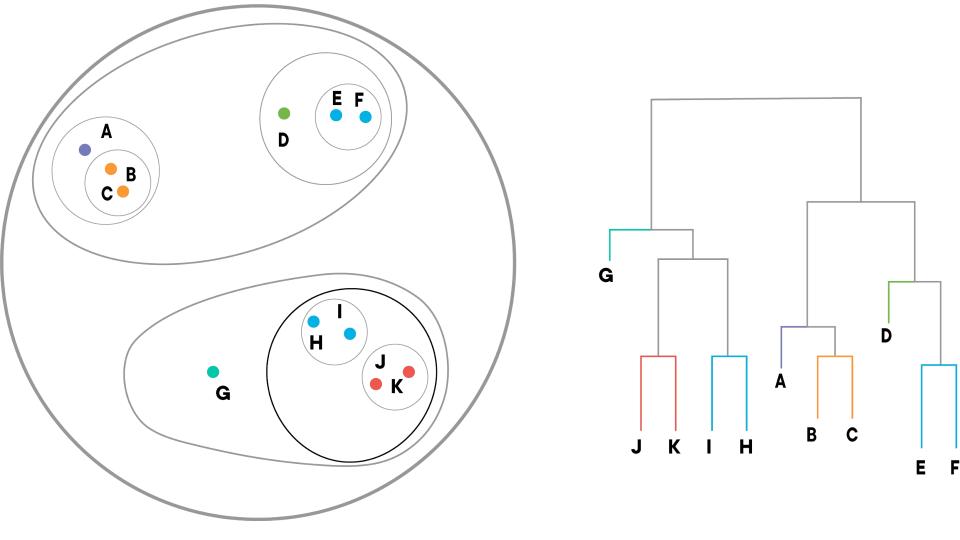
#### Algorithm 10.2 Hierarchical Clustering

- 1. Begin with n observations and a measure (such as Euclidean distance) of all the  $\binom{n}{2} = n(n-1)/2$  pairwise dissimilarities. Treat each observation as its own cluster.
- 2. For  $i = n, n 1, \dots, 2$ :
  - (a) Examine all pairwise inter-cluster dissimilarities among the *i* clusters and identify the pair of clusters that are least dissimilar (that is, most similar). Fuse these two clusters. The dissimilarity between these two clusters indicates the height in the dendrogram at which the fusion should be placed.
  - (b) Compute the new pairwise inter-cluster dissimilarities among the i-1 remaining clusters.

#### **Hierarchical Clustering Algorithm**





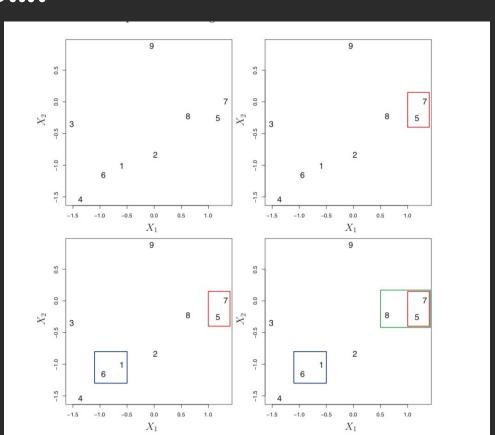


#### **Sneaky Problems....**

Why {5,7} with {8}?

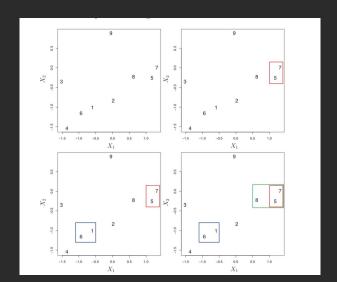
Unequal # obs?

Obs = groups



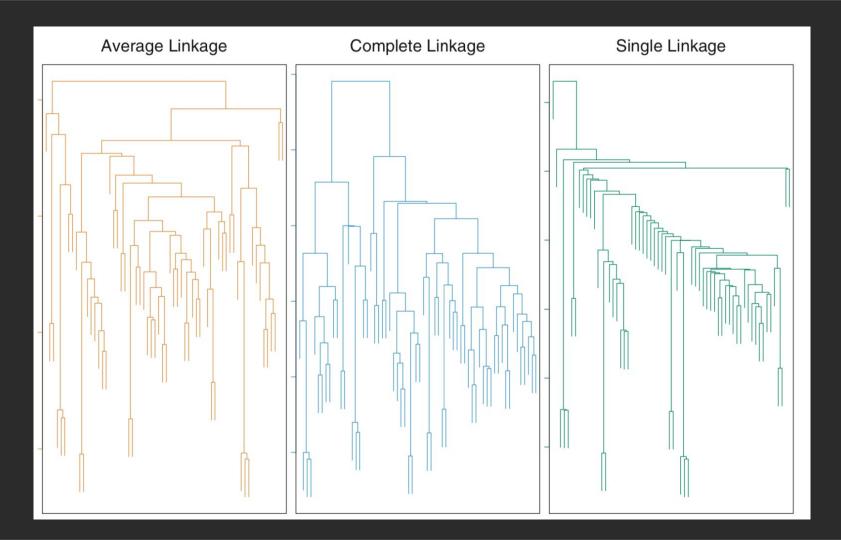
#### Linkage:

Defines dissimilarity between groups of observations!



Linkage	Description
Complete	Maximal intercluster dissimilarity. Compute all pairwise dissimilarities between the observations in cluster A and the observations in cluster B, and record the <i>largest</i> of these dissimilarities.
Single	Minimal intercluster dissimilarity. Compute all pairwise dissimilarities between the observations in cluster A and the observations in cluster B, and record the <i>smallest</i> of these dissimilarities. Single linkage can result in extended, trailing clusters in which single observations are fused one-at-a-time.
Average	Mean intercluster dissimilarity. Compute all pairwise dissimilarities between the observations in cluster A and the observations in cluster B, and record the average of these dissimilarities.
Centroid	Dissimilarity between the centroid for cluster A (a mean vector of length $p$ ) and the centroid for cluster B. Centroid linkage can result in undesirable $inversions$ .

Linkage	Description
Complete	Maximal intercluster dissimilarity. Compute all pairwise dissimilarities between the observations in cluster B, and revorb the largest of these dissimilarities.
Single	Minimal intercluster dissimilarity. Compare all pairwise dissimilarities between the observations in cluster A and the observations in cluster K and record the smallest of these dissimilarities. Single libkage can testile in extended, trailing clusters in which single observations are fused one-at-a-time.
Average	Mean interclister dissimilarity. Compute all pairwise dissimilarities between the observations in cluster A and the observations in cluster B, and record the average of these dissimilarities.
Centroid	Dissimilarity between the centroid for cluster A (a mean vector of length p) and the centroid for cluster B. Centroid linkage can result in undesirable <i>inversions</i> .

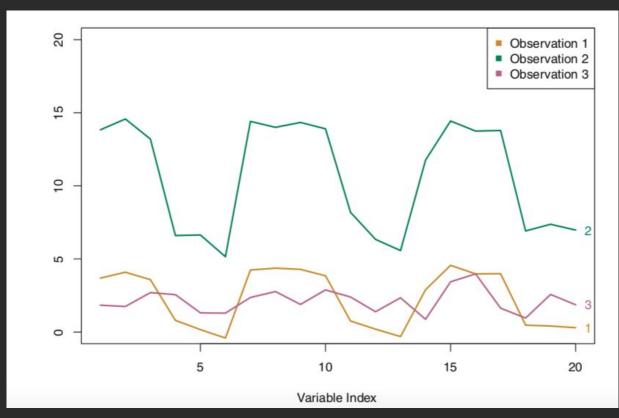


#### How dissimilar?

- Euclidean Distance
- Correlation
  - Shapes of profiles rather than magnitude

1 + 3 have small Euclidean Distance, weak correlation

1 + 2 have a similar shape so we can capture this with correlation!



#### **Practical Considerations**

- Standardize Variables?
- What dissimilarity metrics?
- What type of linkage?
- Where to cut dendrogram?
- How many clusters (for K means)?

Come up with a practical application of clustering analysis and decide how you would answer the the following questions and say why..

- Standardize Variables?
- What dissimilarity metrics?
- What type of linkage?
- Where to cut dendrogram?
- How many clusters (for K means)?

#### **Discussion Questions:**

Come up with a practical application of clustering analysis and decide how you would answer the the following questions and say why..

How would you go about validating your clusters?

#### **Future Reading**

https://scikit-learn.org/stable/modules/clustering.html#k-means

