Sampling, The Central Limit Theorem, Confidence Intervals

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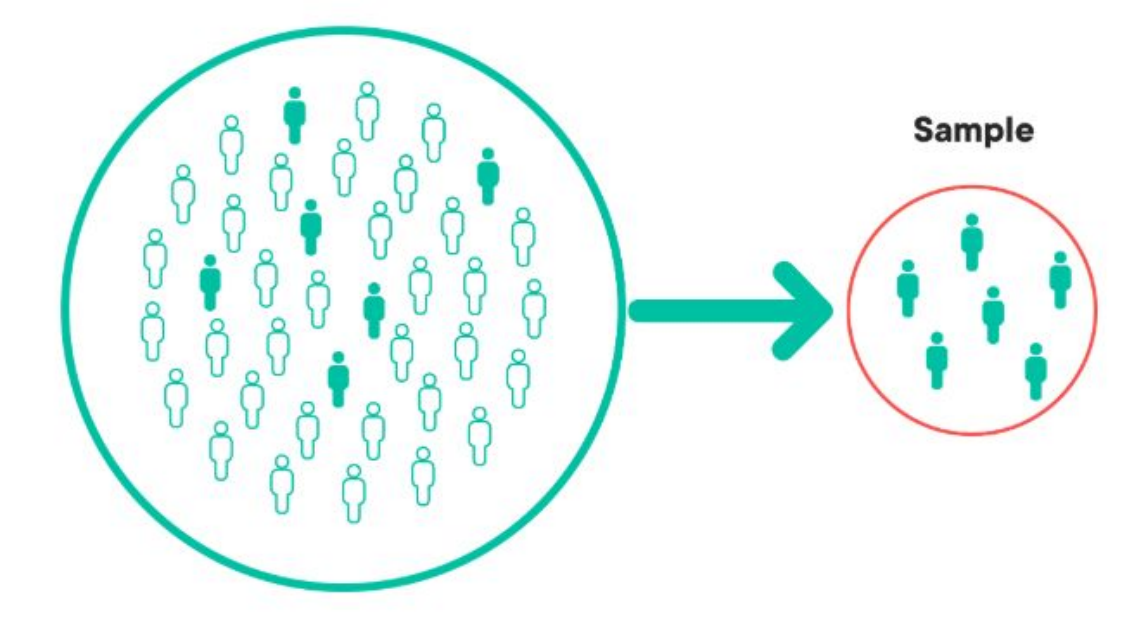
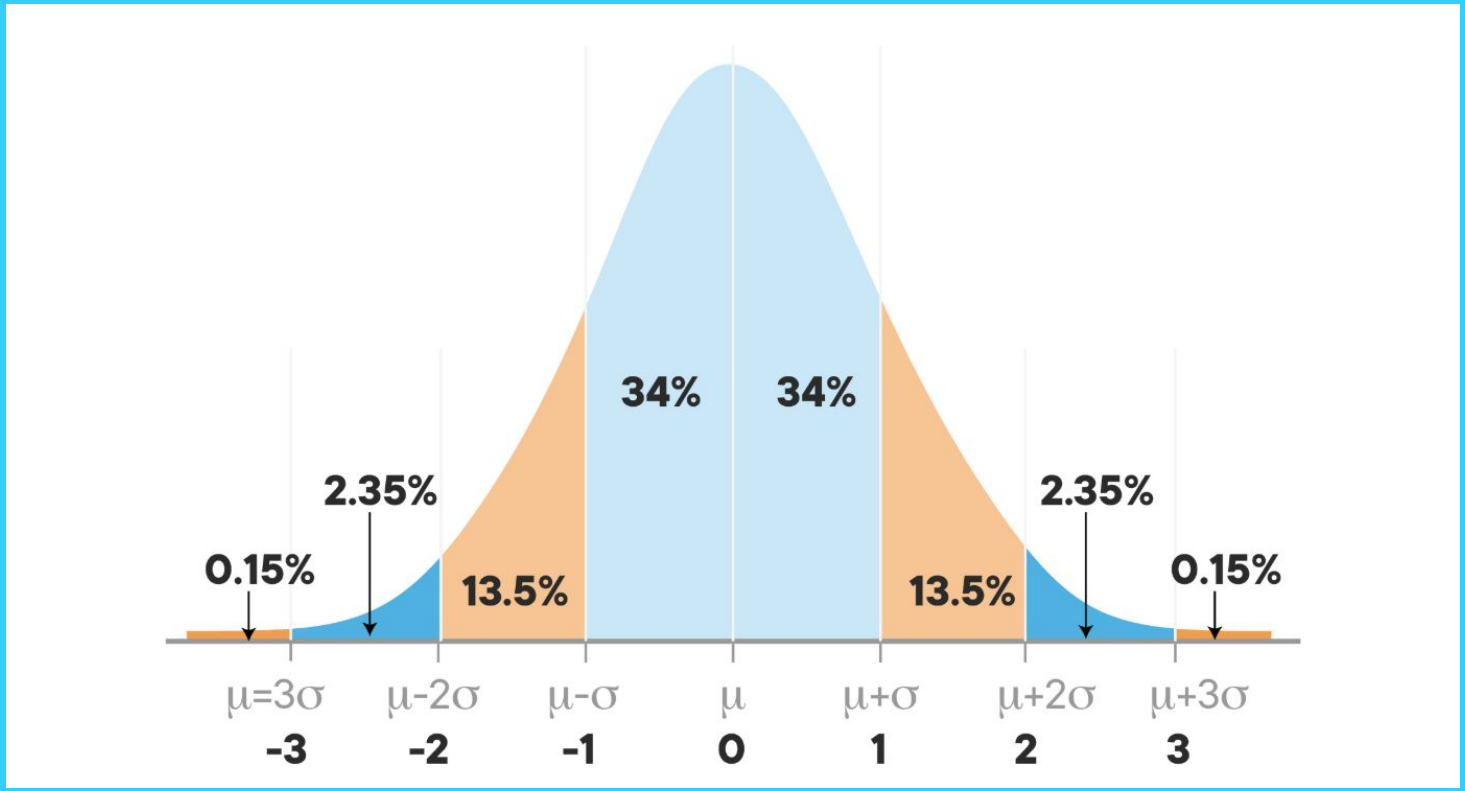
August 2019

FLATIRON SCHOOL

Review?

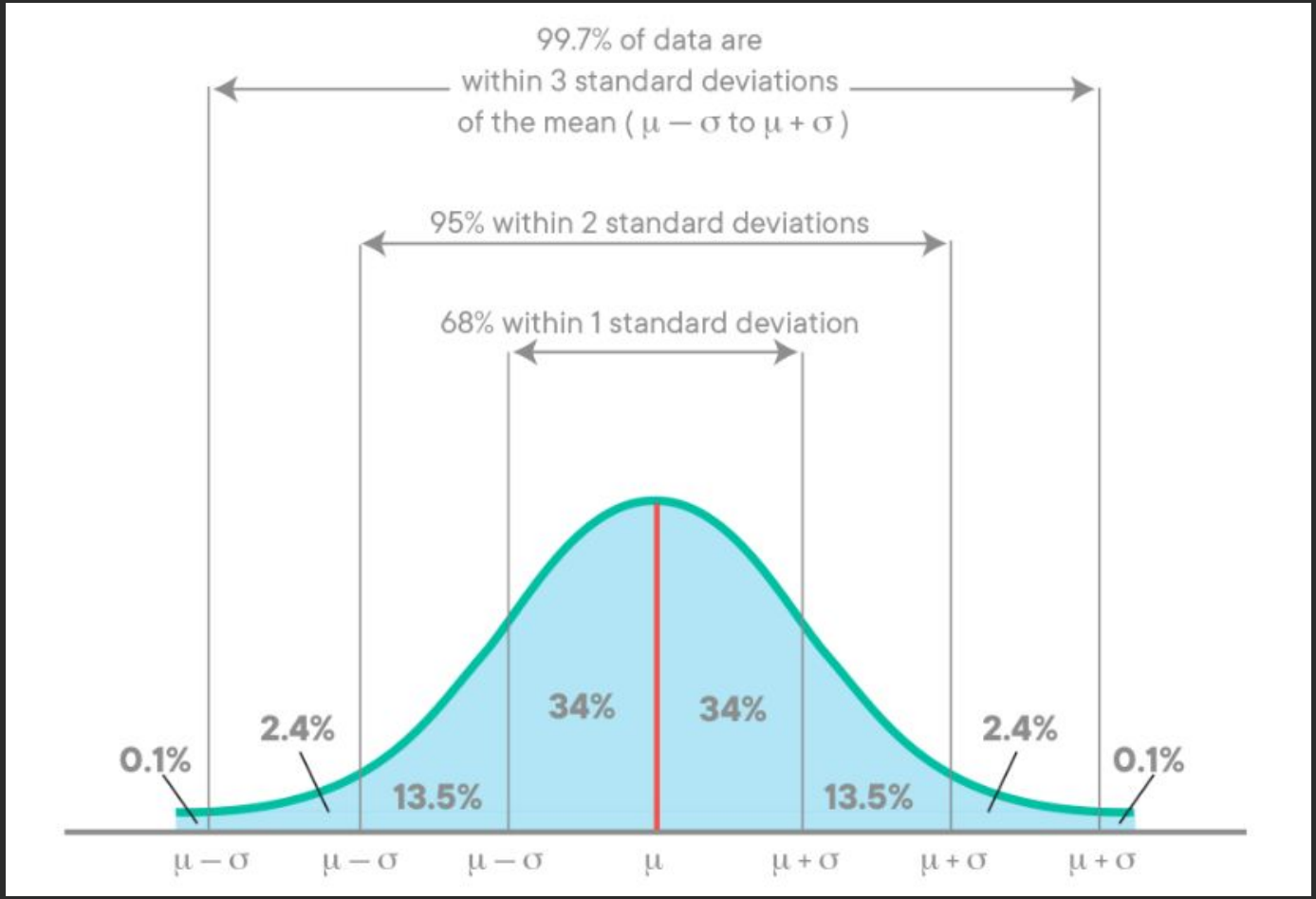
- What are distributions?
- Why do we care about them?
- What are the two types of distributions?
- How would you describe the difference between the two?
- What is a PDF? CDF? PMF?
- What is the normal distribution?
- What are some properties of the normal distribution?


Sample


34%
34%
2.35%
2.35%
0.15%
13.5%
13.5%
0.15%
μ=3σ
μ-2σ
μ-σ
μ
μ+σ
μ+2σ
μ+3σ
-3
-2
-1
0
1
2
3

Sampling

- Rarely do we want a point estimate
- In principle, know that never going to get a point, better to have some sort of range for the long run
- There is uncertainty, we must quantify, we must report!
- Today talk about confidence intervals, how to calculate them, and how to interpret them


99.7% of data are
within 3 standard deviations
of the mean (μ − σ to μ + σ)
95% within 2 standard deviations
68% within 1 standard deviation
0.1%
2.4%
13.5%
34%
34%
13.5%
2.4%
0.1%
μ − σ
μ − σ
μ − σ
μ
μ + σ
μ + σ
μ + σ

Central Limit Theorem

- When you add a large number of independent random variables, irrespective of the original distribution of these variables, their sum tends towards a normal distribution.

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Confidence Intervals

- Range of values above and below a point estimate
- Captures a true population parameter
- Typically set at ~95% (though nothing special about this)
- Higher confidence levels result in wider confidence intervals

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Recipe for Confidence Intervals

- Sigma (population standard deviation)
- N (Sample Size)
- Z (Critical Value)
- “The z-critical value is the number of standard deviations you’d have to go from the mean of the normal distribution to capture the proportion of data associated with the desired confidence interval”

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What is a confidence interval?

- Confidence intervals are a statement about the percentage of confidence intervals that contain the true parameter value
- Parameter: The “true” value of point estimate
- Can be means, effect size, other point estimates?

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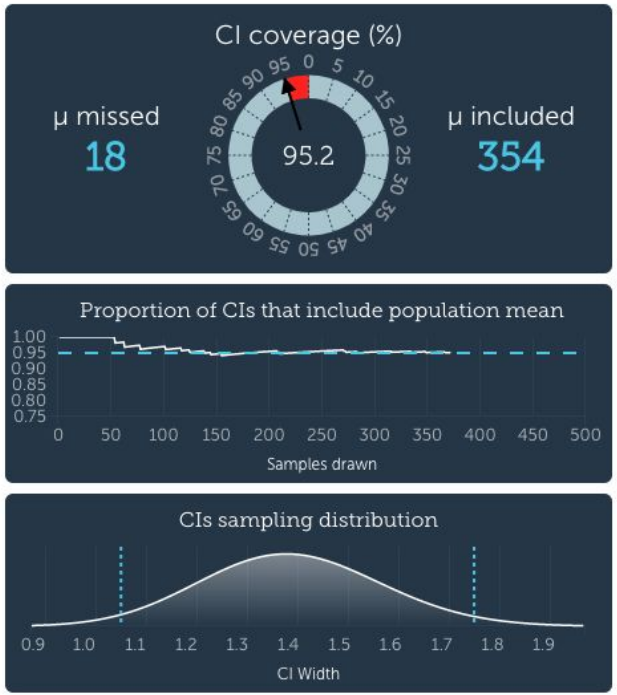
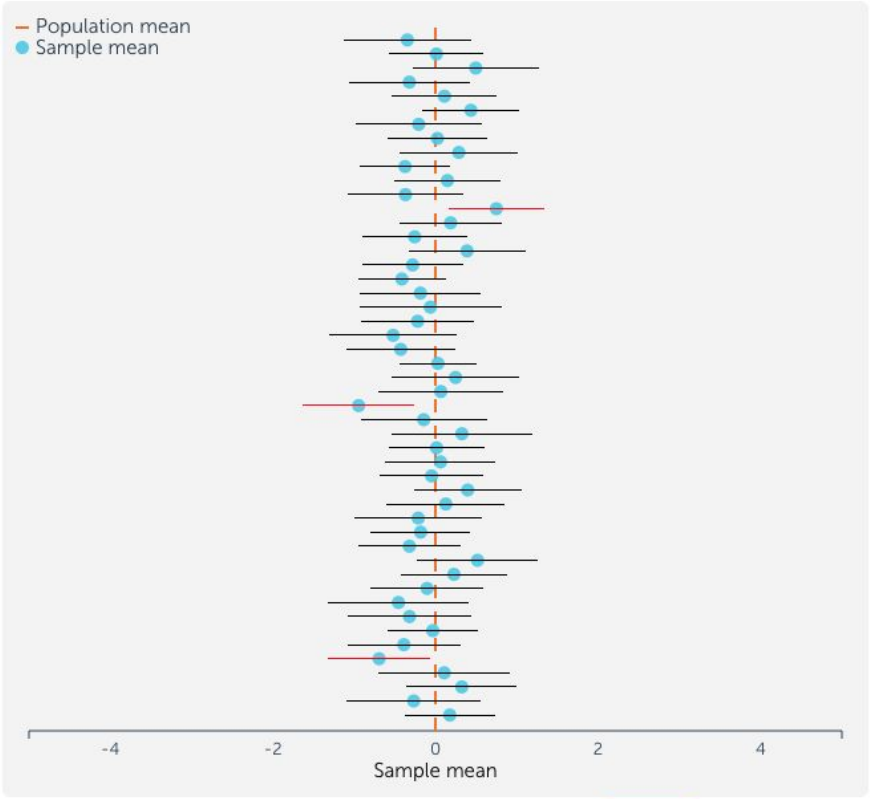
Slide me

Simulation statistics



95% confidence intervals



Sample size - 33 +

Speed - pause +

Spend the next five minutes exploring the following tool. Be prepared to share what you learn from playing with the demo's parameters with the class.

https://rpsychologist.com/d3/CI/

What affects confidence interval?

- Point estimate (obvs)
- Sample Size (n)
- Standard Error
- Critical Value (found from critical value table)

Calculating an Effect Size

Mean +/- critical value * standard error of the mean

Standard error of mean = standard deviation / sqrt(n)

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Calculating a Confidence Interval

Mean +/- critical value * standard error of the mean

Standard error of mean = standard deviation / sqrt(n)

Calculate a 90% confidence interval for a mean score of 17 taken from a sample of 34 people where the standard deviation is 4.3 assuming a t distribution.

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Calculate a 90% confidence interval for a mean score of 17 taken from a sample of 34 people where the standard deviation is 4.3 assuming a t distribution.

Mean/mu: 17

N: 34

SD/sigma: 4.3

CI: 90% (find critical value from table!)

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Mean/mu: 17

N: 34

SD/sigma: 4.3

CI: 90% (find critical value from table!)

Step 1: List out what we know (check!)

Mean/mu: 17

N: 34

SD/sigma: 4.3

CI: 90% (find critical value from table!)

Step 1: List out what we know (check!)

Step 2: Figure out critical value!

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Calculating Critical Value

1. Calculate degrees of freedom (df = n -1)
2. Subtract CI from 1 then divide by 2 for alpha level
3. Use df and alpha level to look up critical value in table
4. Plug and Chug

CHECK OUT HERE FOR MORE INFORMATION ON DEGREES OF FREEDOM:
https://www.youtube.com/watch?v=rATNoxKg1yA

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1. DF: 34 - 1 = 33 Degrees of Freedom
2. Alpha: 1-90 = .10; .10 / 2 tails = .05
3. Use DF of 34 and alpha of .05 in table = 2.032
4. PLUG AND CHUG

t Distribution: Critical Values of *t*

Degrees of freedom	*Two-tailed test:* / *One-tailed test:*	*Significance level* 10% / 5%	5% / 2.5%	2% / 1%	1% / 0.5%	0.2% / 0.1%	0.1% / 0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
34		1.691	2.032	2.441	2.728	3.348	3.601
38		1.686	2.024	2.429	2.712	3.319	3.566
40		1.684	2.021	2.423	2.704	3.307	3.551
42		1.682	2.018	2.418	2.698	3.296	3.538
44		1.680	2.015	2.414	2.692	3.286	3.526
46		1.679	2.013	2.410	2.687	3.277	3.515
48		1.677	2.011	2.407	2.682	3.269	3.505
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
120		1.658	1.980	2.358	2.617	3.160	3.373
150		1.655	1.976	2.351	2.609	3.145	3.357
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
∞		1.645	1.960	2.326	2.576	3.090	3.291

Mean/mu: 17 N: 34 SD/sigma: 4.3 CI: 90% / critical value: 2.03

Mean +/- critical value * standard error of the mean

Standard error of mean = standard deviation / sqrt(n)

17 + 2.03 * (4.3 / sqrt(34)) = 18.49

17 - 2.03 * (4.3 / sqrt(34)) = 15.50 [15.50, 18.49]

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DANGER ZONE!

The most intuitive explanation of a confidence interval is not correct. 95% of future estimates WILL NOT fall into the range that you make. The confidence interval is not about future estimates.

At 95% CI, you get the population mean 84.3% of the time!!

THAT interpretation is closer to a concept in Bayes:

- Credible Intervals (95% of values that are most plausible)
- Highest Density Intervals (Parameter values most likely)

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CIs and Ps

CIs and P values also share relationship

If CI crosses 0, p value will be greater than .05!!

We will return to this concept tomorrow....

[-1.2, 2.3] will have p value greater than .05!!

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Think, Pair, Share:

In your pairs, prepare a one sentence, three sentence, and ten sentence summary to the question: What are confidence intervals and why do we need them?

After discussing for 10 minutes, I will ask each group to randomly share one of their three explanations of confidence intervals.

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