Statistical Distributions

Presented by David John Baker
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FLATIRON SCHOOL

Statistical Distributions ?

- Distributions are to statistics as data structures are to programming
- Reflect assumptions about the underlying processes
- Allow us to make generalizations about data that we have not yet seen

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Lesson Goals

- Name and Describe Distributions
- Explain Difference between Continuous and Discrete Distributions
- Know PMF, PDF, CDF
- Describe Normal Distribution + (Standard Normal)
- Give examples of distributions

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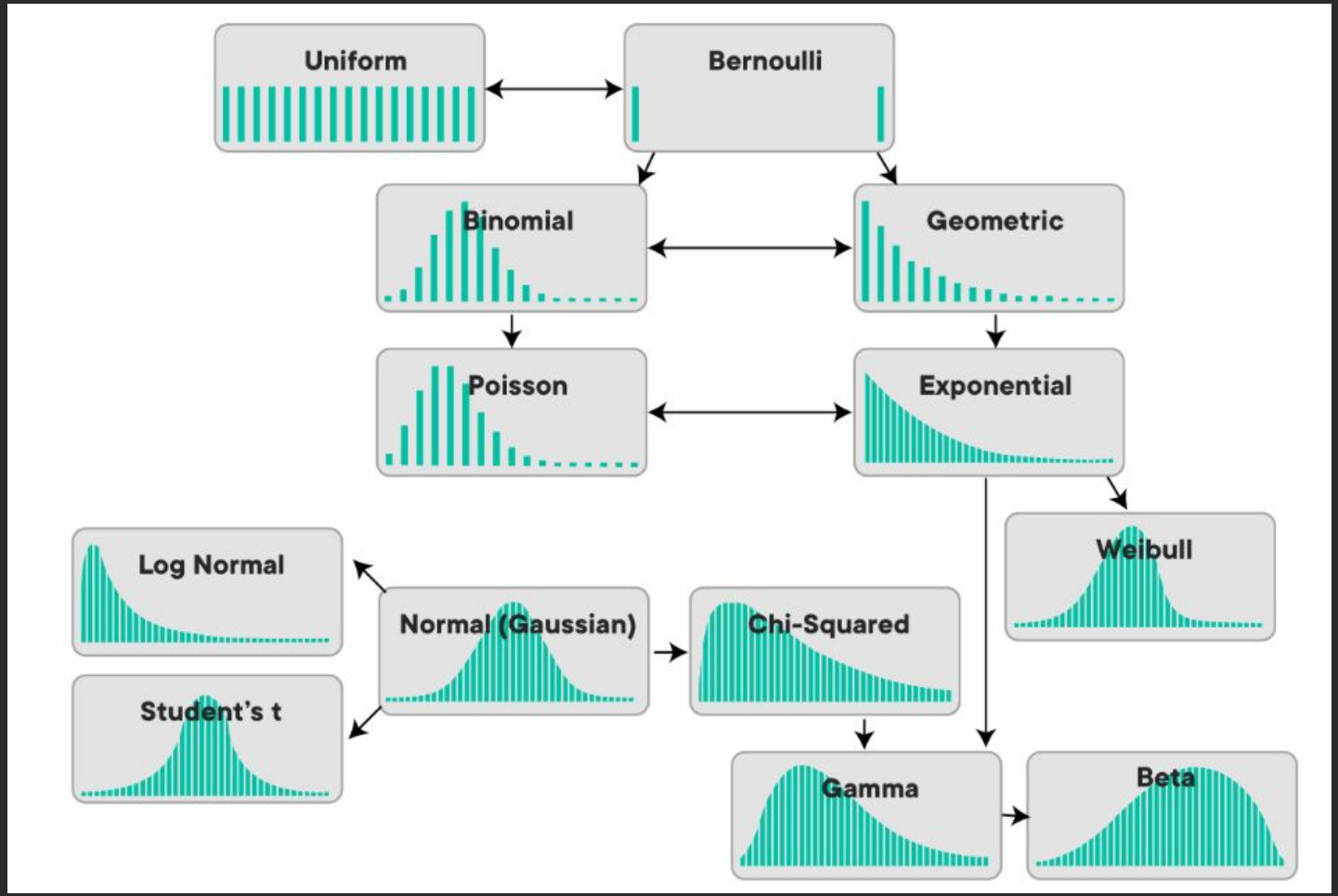
If I measure some quantity, what is the probability that I get one value rather than another?

If I flip a fair coin, what is the probability that I get heads rather than tails? If I measure the height of someone randomly selected from the U.K. population, what is the probability that I get someone who's 5'9"? What is the probability that I get someone who's 7'9"?


Uniform
Bernoulli
Binomial
Geometric
Poisson
Exponential
Weibull
Log Normal
Normal (Gaussian)
Chi-Squared
Student's t
Gamma
Beta

Discrete vs Continuous

- Discrete distributions have finite event space (ex: dice, categories of drawing balls out of an urn, species of animals)
    - Bernoulli
    - Poisson
    - Uniform
- Continuous distributions have infinite event space (ex: heights, time, financial data)
- Normal (Standard Normal)

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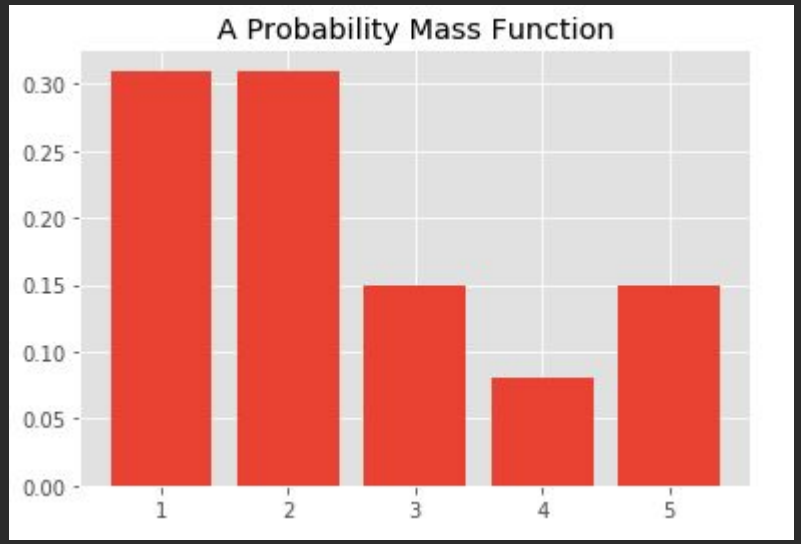
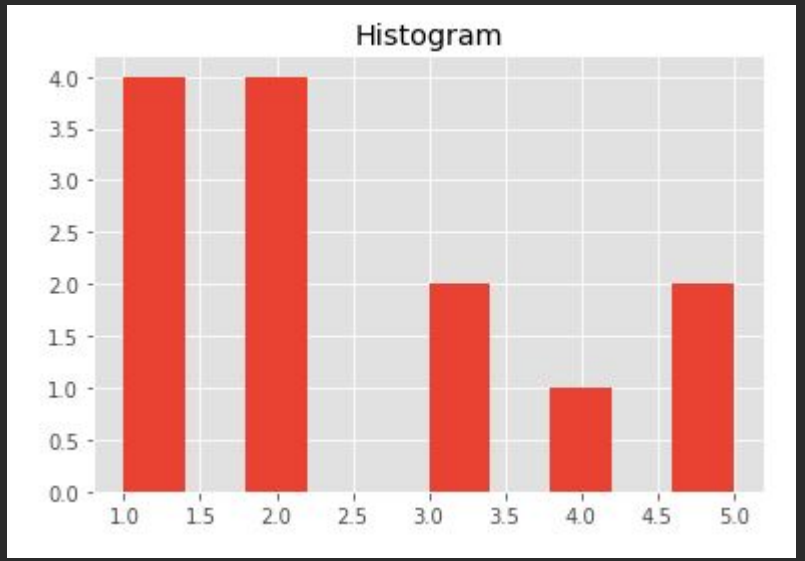
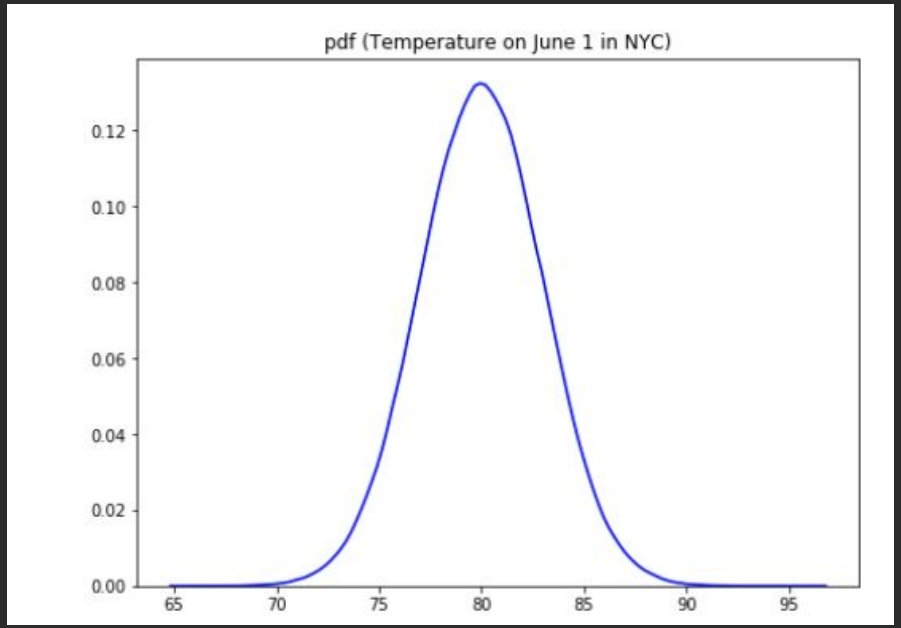
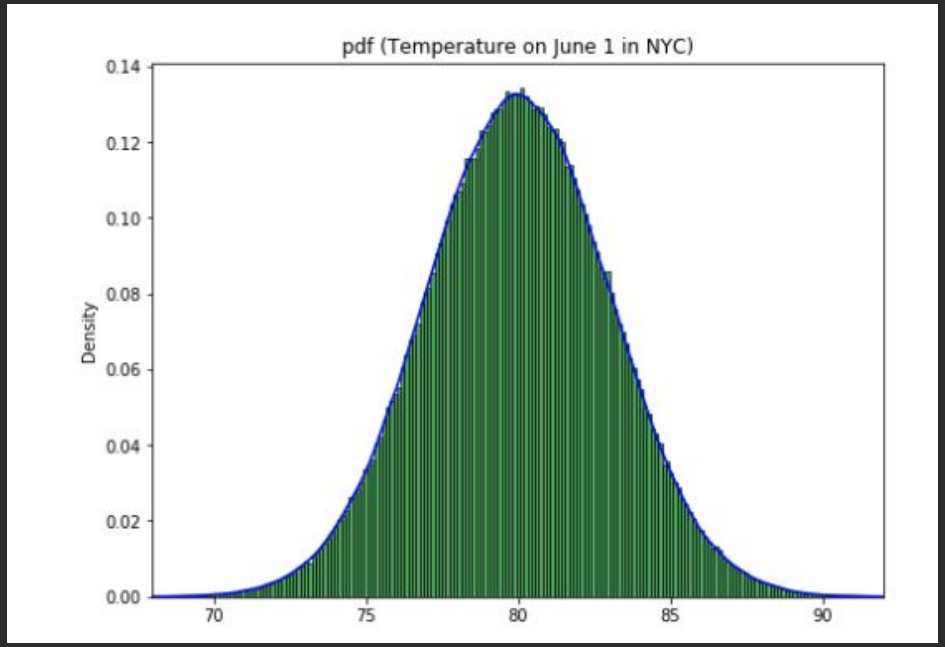
Probability Mass Functions (PMF)
(~A Normalized Barchart~)





Probability Density Functions (PDF)
(A Normalized Histogram)





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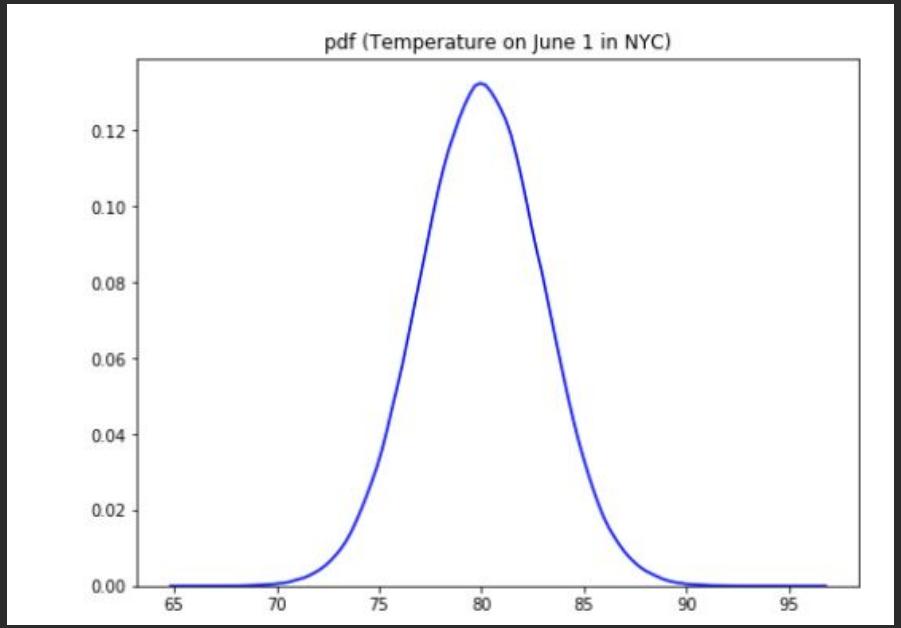

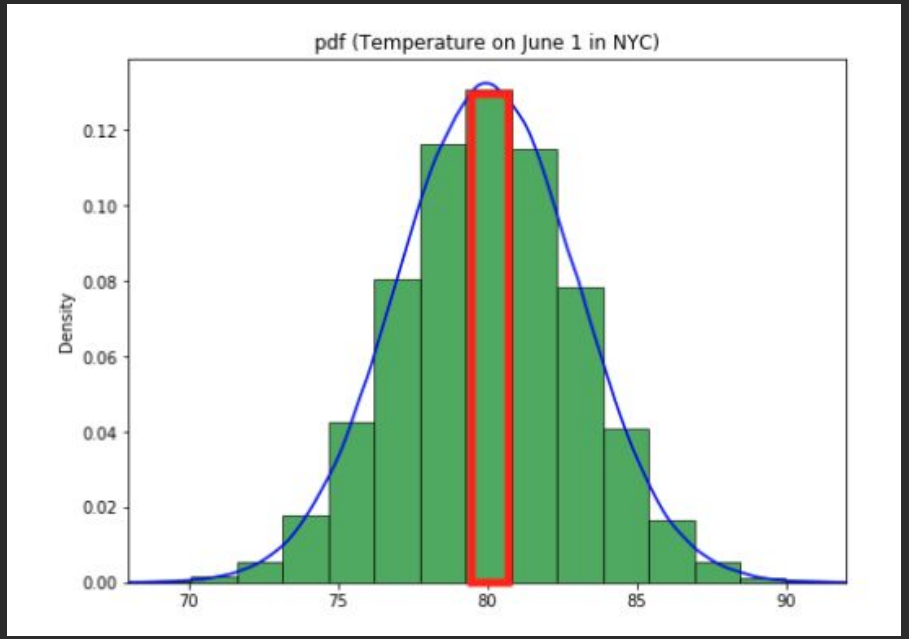

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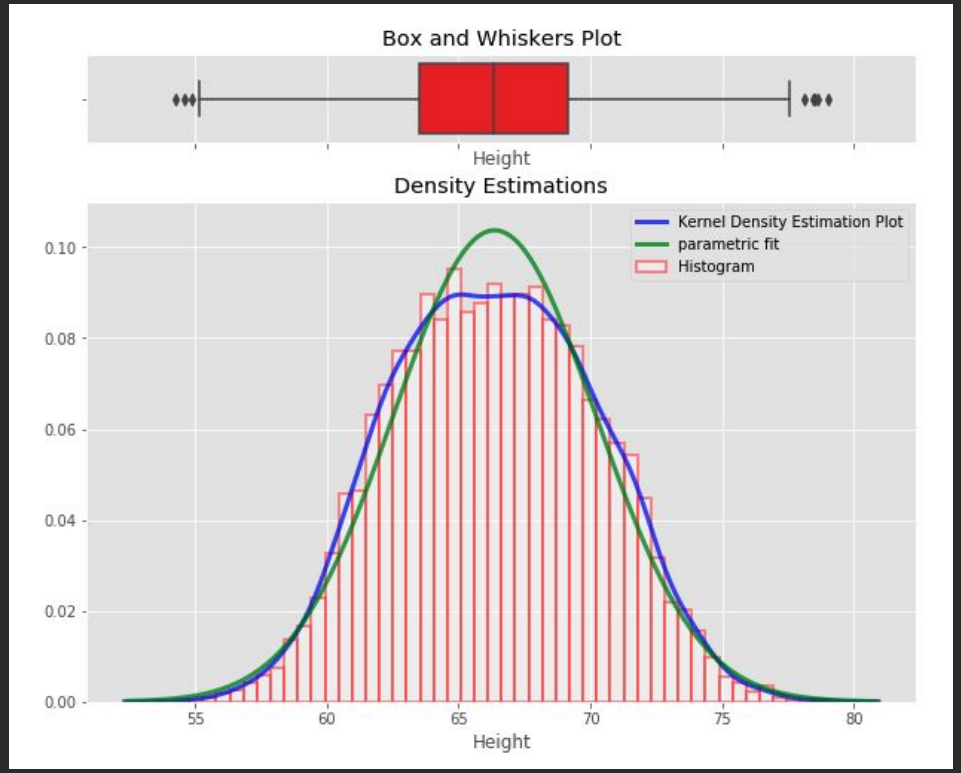

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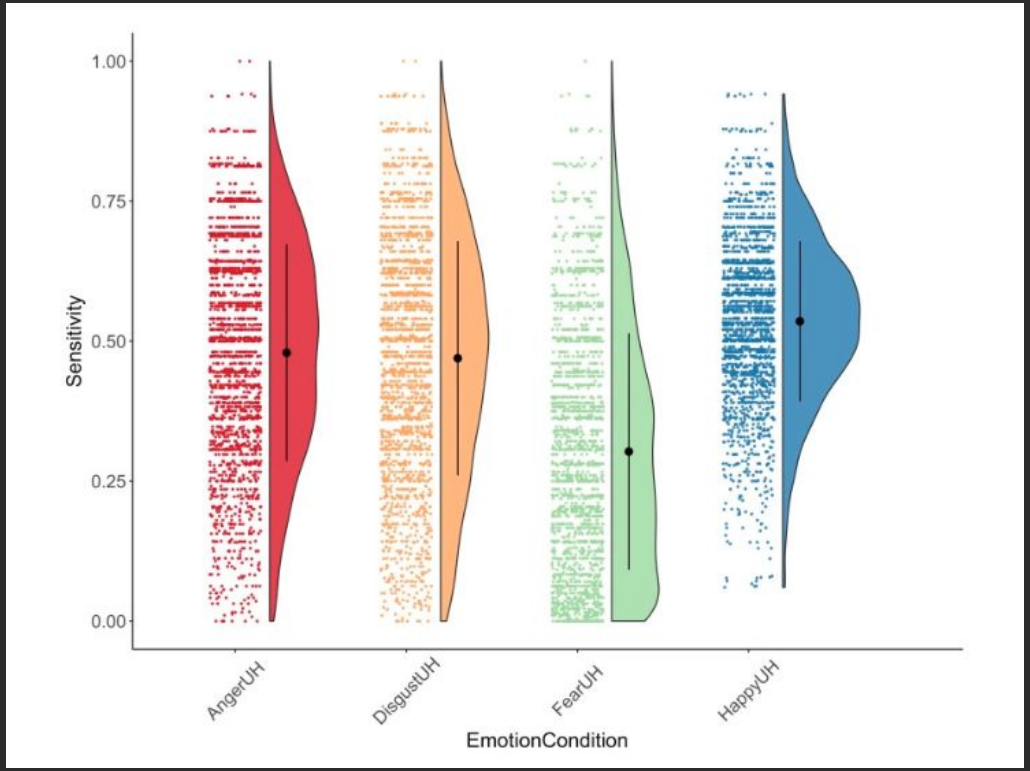
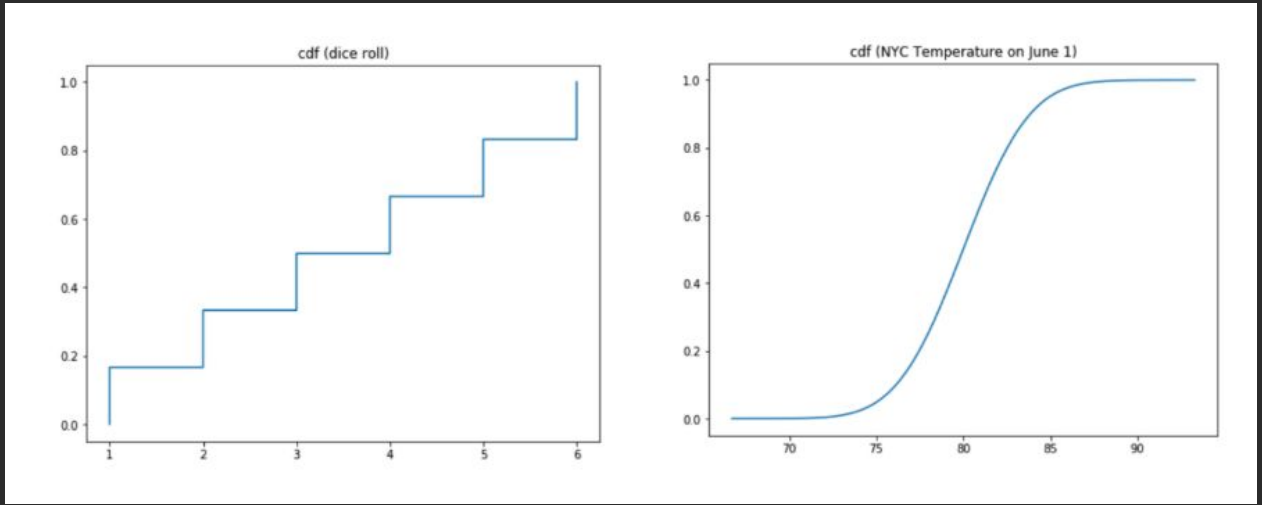


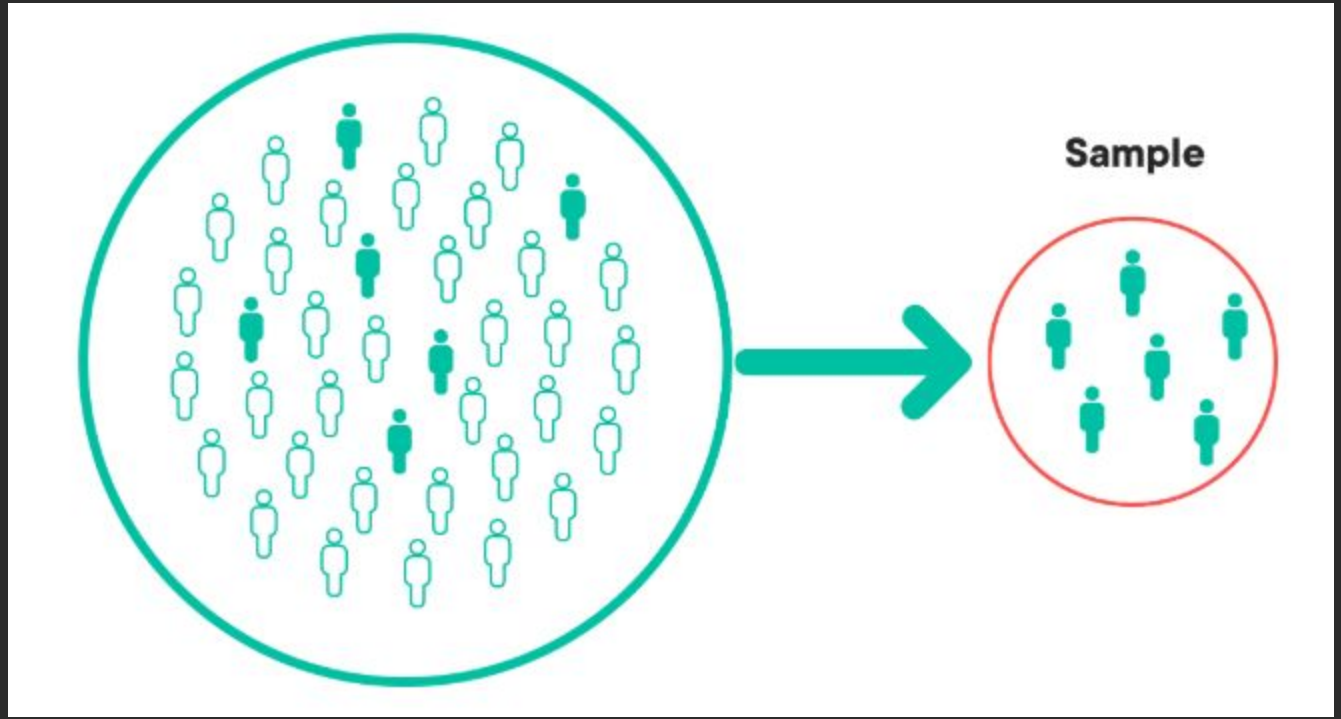
Problems with PMF and PDF?

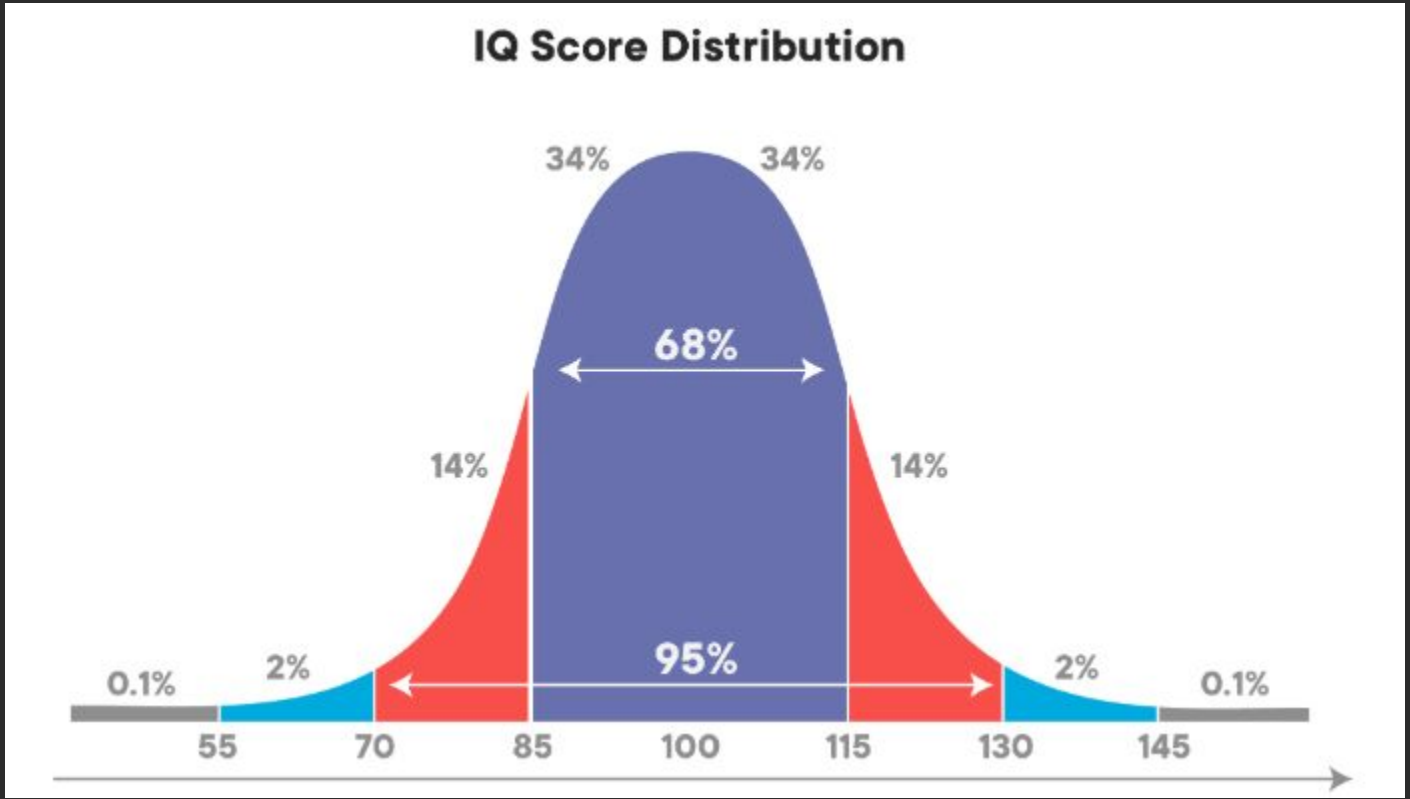
- Interpreting PMF, not to so bad?
- But what about if you wanted to say what is the probability of getting an exact value from a PDF? (a point probability)
- CDF!

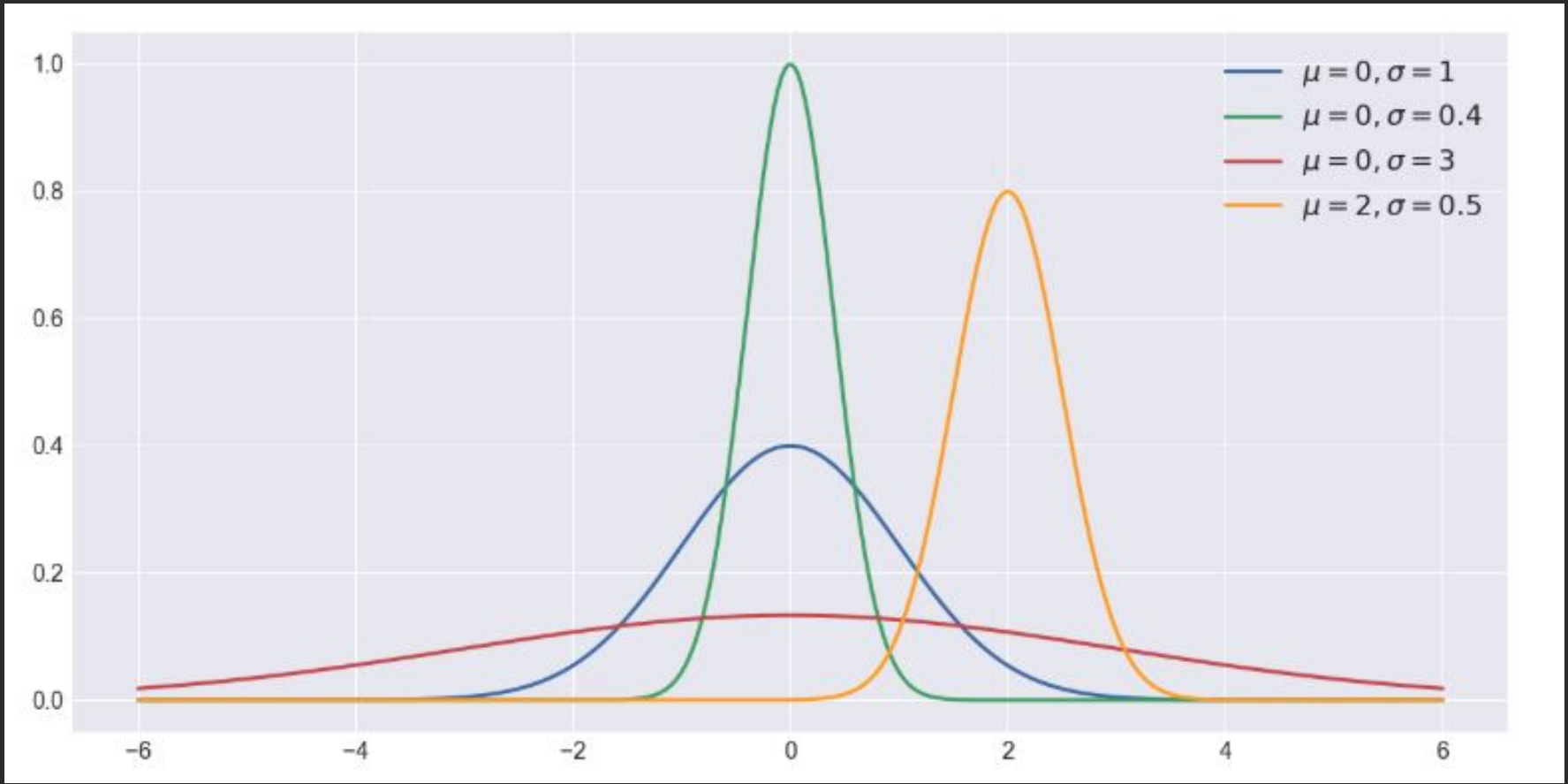


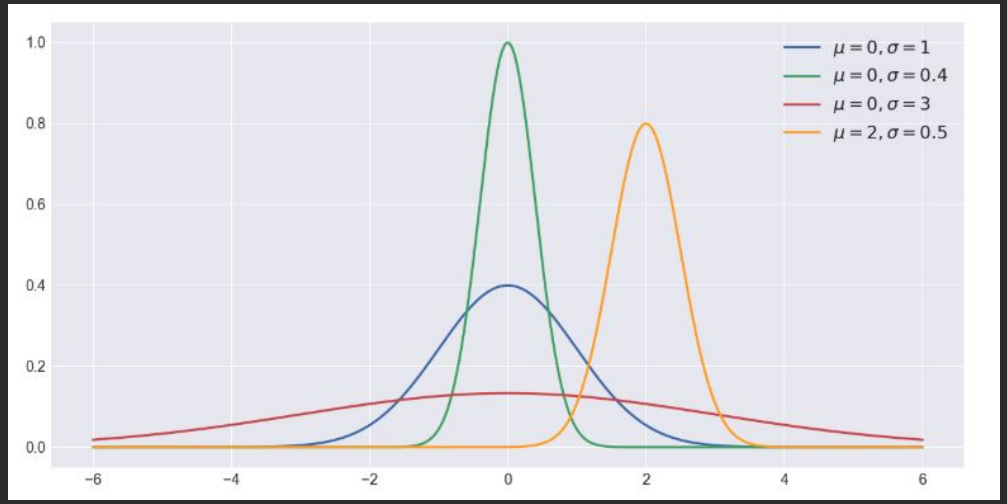
Normal Distributions

- One of most common distributions found in natural world
- Normal is not a value judgment
- Defined by a mean (mu) and standard deviation (sigma) and general symmetry
- Understanding the properties of the normal distribution will allow us to generalize about data we have not yet seen!
- Reflect on relationship between population, sample, and sampling distribution.


Sample


IQ Score Distribution
34%
34%
68%
14%
14%
95%
0.1%
2%
2%
0.1%
55
70
85
100
115
130
145


1.0
0.8
0.6
0.4
0.2
0.0
−6
−4
−2
0
2
4
6
$\mu = 0, \sigma = 1$
$\mu = 0, \sigma = 0.4$
$\mu = 0, \sigma = 3$
$\mu = 2, \sigma = 0.5$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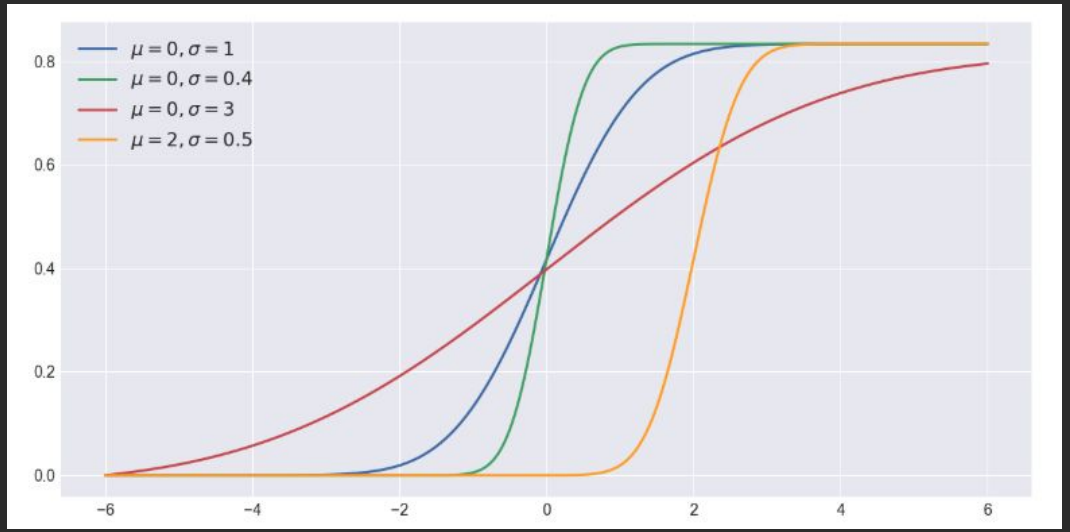
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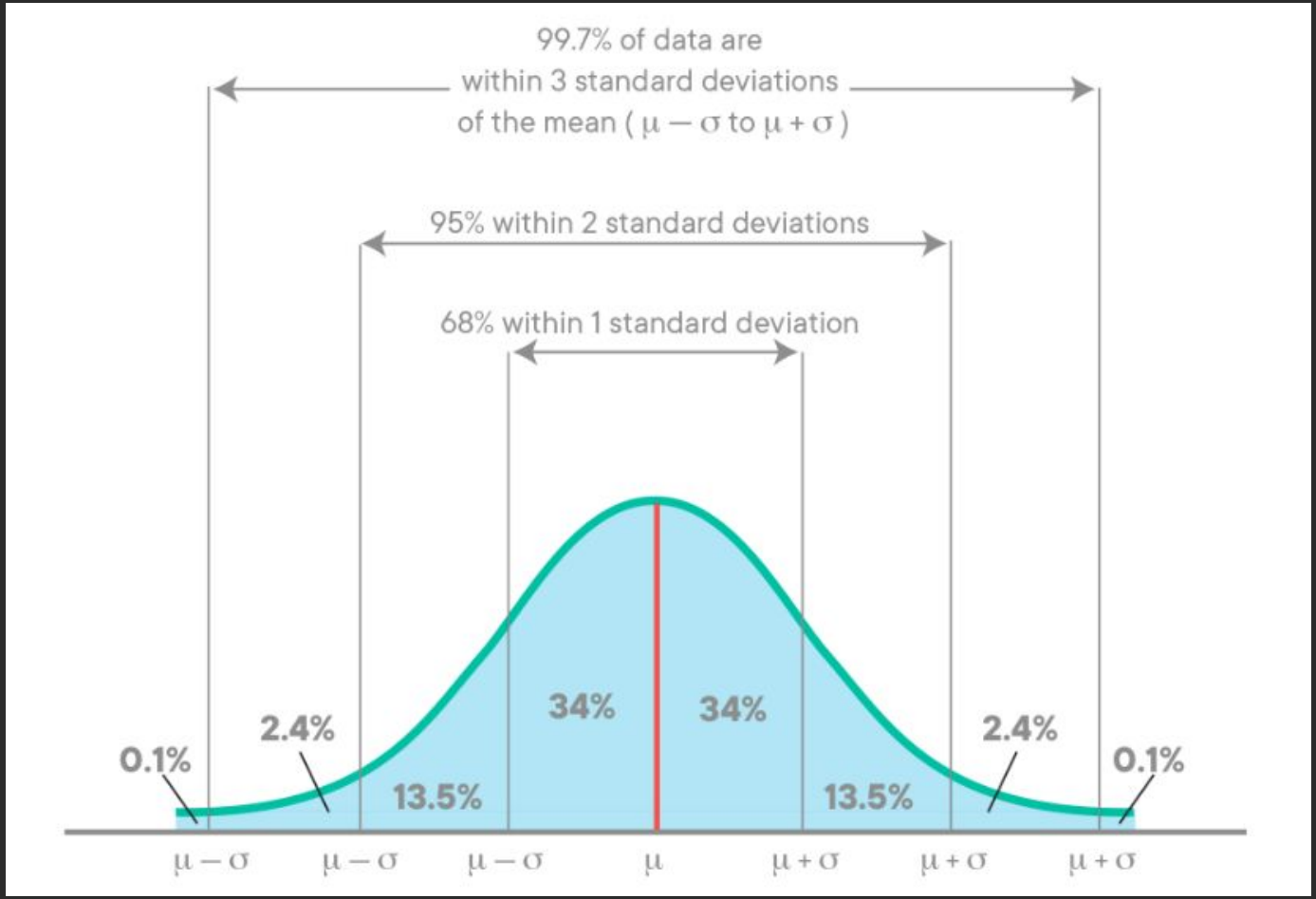
99.7% of data are
within 3 standard deviations
of the mean (μ − σ to μ + σ)
95% within 2 standard deviations
68% within 1 standard deviation
0.1%
2.4%
13.5%
34%
34%
13.5%
2.4%
0.1%
μ − σ
μ − σ
μ − σ
μ
μ + σ
μ + σ
μ + σ

Central Limit Theorem

- When you add a large number of independent random variables, irrespective of the original distribution of these variables, their sum tends towards a normal distribution.

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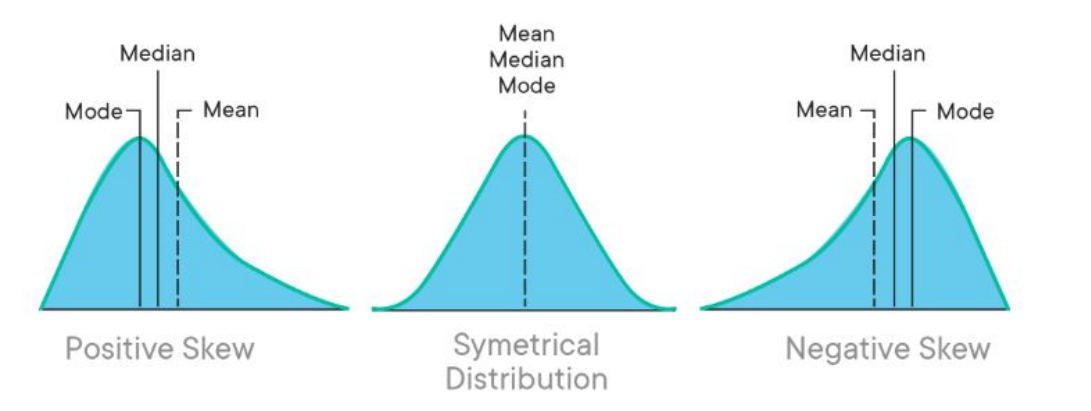
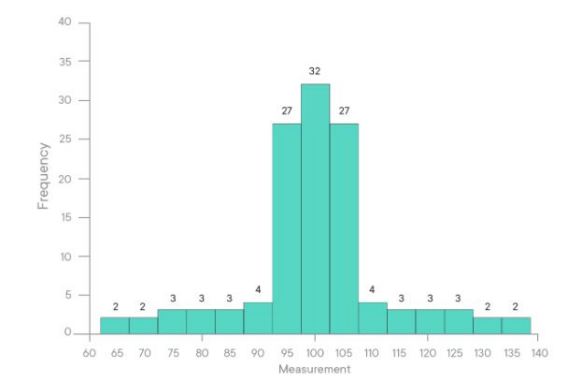
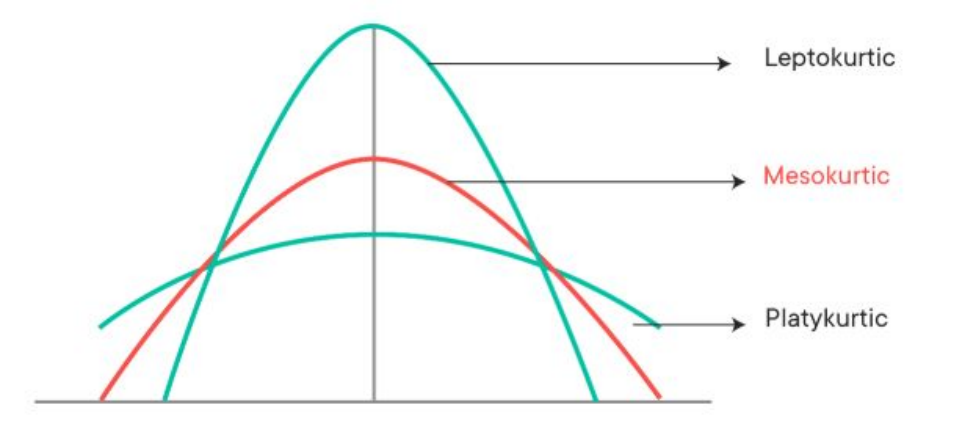
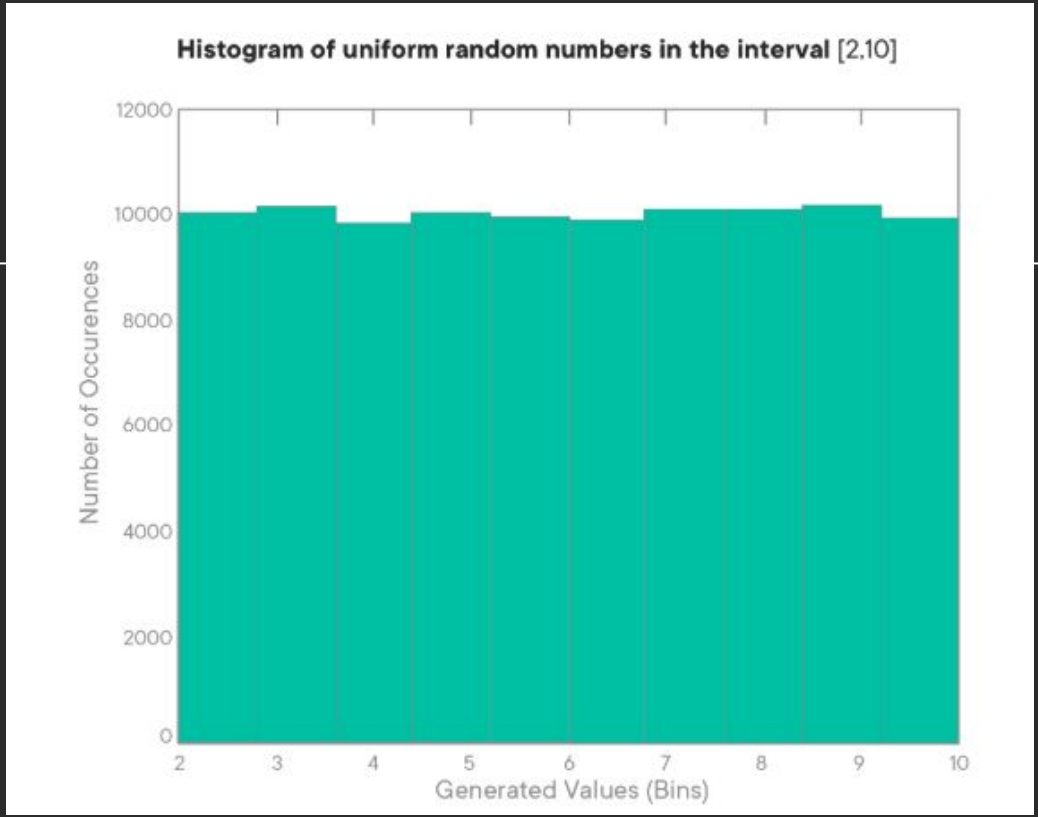
Skew and Kurtosis





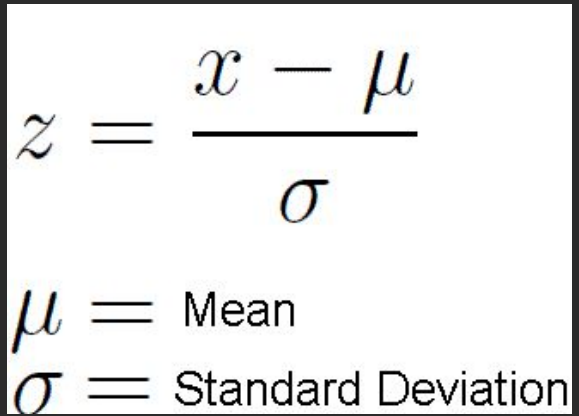


Uniform Distribution



Practice

- In the last portion of class, want to do a class activity
    - Collect heights of all people in class in CM
    - Create small analysis script that is going to import the data
    - First create a plot that plots the raw data
    - Check if that data is normal (visual inspection)
    - Convert all scores to z scores



Checking For Understanding

- Explain discrete vs continuous distributions
- Explain difference between PDF, PMF, CDF
- Explain normal distributions
- Standard Normal Distributions
- Z Scores