Computer Laboratory Exercises in Optimization 2020
Lab 1: Line Search and Multidimensional Search

The lab is run with the MATLAB package, version 2014 or later, which you can install freely in your computer; see `http://program.ddg.lth.se/en/`. You should work in a group of 3 students.

Preparations to do before the lab session in order to pass

- If you are not familiar with MATLAB you should examine this interactive program with the command `demo`. A faster way may be to type `help demos` and choose a demo from the list, e.g., type `intro`. You might find it useful to look at the MATLAB links on the course homepage `https://canvas.education.lu.se/courses/7321`

- Read in the textbook and be sure that you understand the line search methods and the multidimensional methods Steepest Descent, Newton and Modified Newton.

- Implement in MATLAB the following line search methods: Golden Section, Dichotomous, Bisection and Newton. Write one m-function file per method, for example, starting like

  ```
  function [X,N]=goldensection(F,a,b,tol)
  % For example F = @(x) 1-x*exp(-x)
  % [a,b] interval
  % tol = max ratio of final to initial interval lengths
  % X output matrix containing final a and b, b-a
  % N = number of function evaluations
  ```

  Solve the following exercises in the textbook with your programs: 2.1a, 2.3, 2.6.

- Download the file `lab1.m` from the course web page (see the link above) and type `lab1` in the MATLAB window to start the program.

- A new window appears. Select 'Golden section'. The function to be investigated is called `fun1` defined on the interval $-0.25 \leq x \leq 0.5$. Select an initial interval by clicking two points in the window. For each next click, anywhere in the window, a new smaller interval of uncertainty will be shown. A new initial interval can be defined after you have clicked the restart button. When you have tried many different initial intervals, click the change button and the interval becomes $-10 \leq x \leq 10$. Continue your experimentation. Investigate also the function `fun2` on the interval $-10 \leq x \leq 10$.

During the laboratory session

1. Start MATLAB. Be ready to present your implemented line search programs and to test them with a function given by the teacher.

2. Start `lab1` again. Repeat the investigations (made above for Golden Section) for Newton's method. Try different initial points all over the interval $0 \leq x \leq 2$ for `fun1`.

3. Questions to be answered for Golden Section and Newton's method:

   - Does the method always converge?
   - What kind of points are found by this method? Global minimum, local minimum or something else?

- Would you say that it is a reliable method?
- How far away from a minimum can you start without losing the convergence to it?

4. Investigate Armijo's method. Note that you need not press the restart button. A single click in the window selects an initial point, then some possible iteration points that *may occur* (depending on where you start). Interpret and explain the red and blue asterisks.

5. Try two MATLAB methods for minimization of `fun1` by typing

   `options=optimset('display','iter');`

   `fminbnd(@lab1,-1,1,options)` (the interval is here $[-1, 1]$; try also $[-10, 10]$)

   `[x,fval,exitflag,output]=fminunc(@lab1,-1)` (initial point is -1; try other)

   Do you see any similarities with Golden Section or Newton's methods? Is the MATLAB result satisfactory? (convergence? global minimum?)

6. Continue with the multidimensional search by clicking on the button 'multidim'. In a new window you can minimize functions of two variables. Available methods are:

   Steepest Descent with Armijo's line search

   Newton's method without line search

   Modified Newton's method without line search

   All methods can be switched on at the same time but you are advised to begin with Steepest Descent: click out an initial point and continue clicking. After you have examined the behaviour for many different starting points, try Newton's method in the same way and finally the modified Newton for different values of ε (in the box to the right of EPSILON). Investigate also the functions `f2`, `f5` and `f6`.

7. Questions to be answered for each of these three search methods:
   - Does the method always converge? Is it fast or slow? For what kind of functions does it work best?
   - What kind of points are found by the method? Global minimum, local minimum or something else?
   - How does the size of ε in the Modified Newton method affect the performance?

8. Choose 'Strategy'. This allows you to change the method as well as the value of ε in Modified Newton at each step. This is useful for finding a mixed strategy that works well for all starting points. Try to figure out such a strategy and test it on the function `f2`.

   For the functions `f3` and `f4` no contours are shown. Try to find the global minimum for these functions with as few steps as possible. Can you be sure that you have found the global minimum?

9. Compare with MATLAB's `fminsearch`. Type

   `[point,val]=fminsearch(@(x)lab1(x,'f3'),[5 5],options)`

   `[point,val]=fminsearch(@(x)lab1(x,'f4'),[5 5],options)`

   Here [5 5] is the initial point (try several different).

   Which method does MATLAB use when there is no information about the regularity of the input function? Search for animations of this method on the internet and watch how it proceeds in two dimensions.