NOVA SCHOOL OF SCIENCE & TECHNOLOGY

Project Part II

Cyber-Physical Control Systems 2025-2026

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1 Satellite Attitude

Attitude control for satellites is required for pointing instruments or sensors for the correct targets. We consider a rigid-body spacecraft fitted with a body-fixed frame $\mathcal{F}_b$, whose origin is at the spacecraft's center of mass, and whose orientation relative to an inertial reference frame $\mathcal{F}_0$ is described by the rotation matrix $\mathbf{R} \in \mathrm{SO}(3)$. The rotational motion of the spacecraft is governed by

$$\begin{aligned} \dot{\mathbf{R}} &= \mathbf{R}\mathbf{S}(\boldsymbol{\omega}), \\ \dot{\boldsymbol{\omega}} &= \mathbf{J}^{-1}\mathbf{S}(\mathbf{J}\boldsymbol{\omega})\boldsymbol{\omega} + \mathbf{J}^{-1}\boldsymbol{\tau}, \end{aligned} \tag{1}$$

with the intertia matrix given by

$$\mathbf{J} = \begin{bmatrix} 125.734 & 0 & 0 \\ 0 & 216.211 & 0 \\ 0 & 0 & 234.055 \end{bmatrix} (\mathrm{kg.m}^2), \tag{2}$$

where $\boldsymbol{\omega} \in \mathbb{R}^3$ is the angular velocity, $\boldsymbol{\tau} \in \mathbb{R}^3$ is the applied torque, $\mathbf{J} \in \mathbb{R}^{3\times 3}$ is the inertia matrix, and $\mathbf{S}(\mathbf{a})$ is a skew-symmetric matrix such that $\mathbf{S}(\mathbf{a})\mathbf{b} = \mathbf{a} \times \mathbf{b}$ for all $\mathbf{a}, \mathbf{b} \in \mathbb{R}^3$.

2 Tasks

Please note that in this stage, you are allowed to use any controller strategy or any software that you wish. Implement the following tasks:

1. (10 marks) The first task consists in designing an attitude controller that receives the state of the spacecraft and the desired orientation in Euler angles that should be named `attController` and outputs the actuation. Solutions will be validated in terms by using the cost function:

   $$\mathrm{cost}(\mathbf{u}) = 10\|\mathbf{u}_{\mathrm{avg}}\|^2 + 100T$$

   where T is the total time for the maneuver until the error of the angles is all below 0.1 degrees and the vector $\mathbf{u}_{\mathrm{avg}}$ corresponds to the average actuation over the entire simulation. After 20 minutes, the simulations will be halted. This evaluation will determine the assignment of 3 out of the 10 points for this question. For computing times above the sampling period of 0.2 s, that actuation value will only be applied to the system in the correspondent future sample.

2. (10 marks) Repeat the previous question but where the new function `constrainedAttController` receives an extra argument as a vector such that the angle of any of the axis with that vector should be below 10 degrees. Assume all the previous definitions and restrictions for the question.