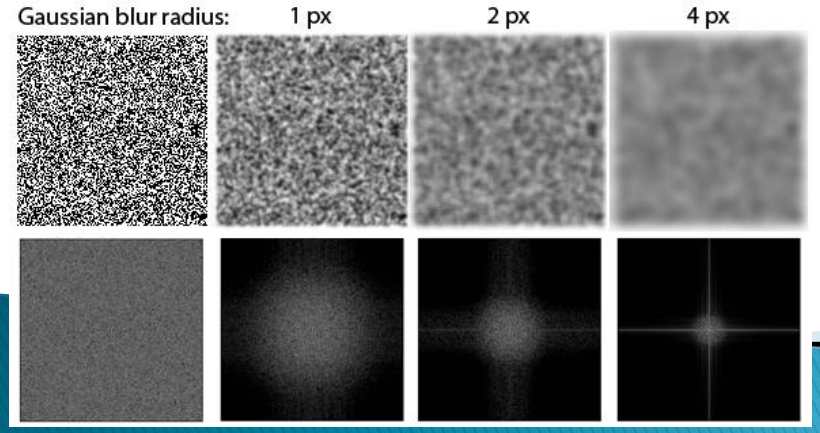
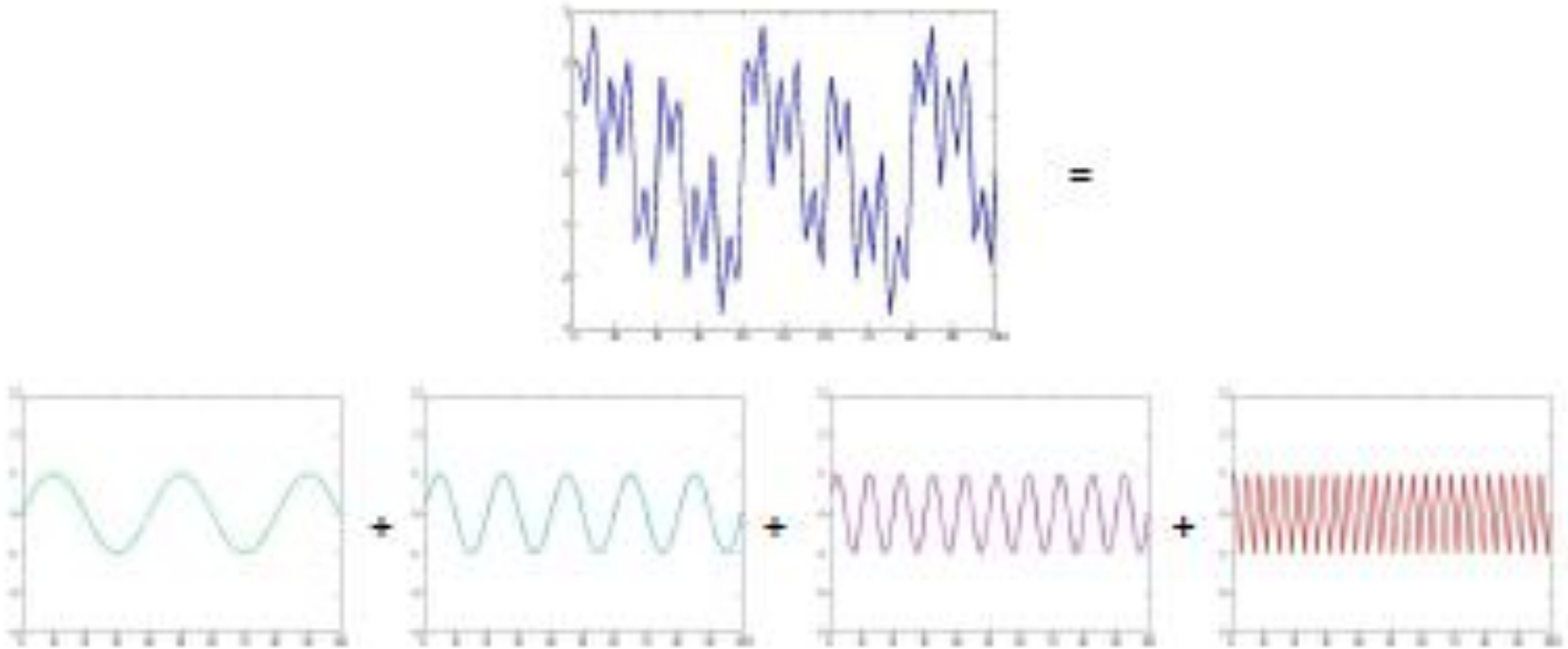
Processamento de Imagem na Frequência

A transformada de Fourier



A série de Fourier 1D

- Os sinais periódicos podem construir-se com base na soma de senos e cosenos com diferentes coeficientes



A transformada de Fourier 2D

- A transformada de Fourier decompõe uma imagem num conjunto de senos e cosenos de diferentes frequências
- Seja f(x,y) com x = 0,1,…,M-1 e y = 0,1,…,N-1 uma imagem de MxN
- A transformada directa de Fourier é dada por:

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y) e^{-i2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$com\ u = 0,1,\ldots,M-1\ e\ v = 0,1,\ldots,N-1$$

$$e^{-iwt} = \cos(wt) + i\sin(wt)$$

- A transformada inversa de Fourier é dada por:

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v) e^{i2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$com\ x = 0,1,\ldots,M-1\ e\ y = 0,1,\ldots,N-1$$

A transformada de Fourier

- A transformada de Fourier de uma função real é uma função complexa – **difícil de visualizar** – pelo que se representa normalmente como amplitude e fase.
- Uma curiosidade importante é que quase todas as imagens “naturais” têm:
  - uma amplitude muito idêntica.
  - uma fase que difere significativamente.
- O que acontecerá se se fizer a reconstrução de uma imagem com a amplitude de uma imagem e a fase de outra?

Componentes da Transformada de Fourier

- O valor de amplitude da transformada de Fourier (o espectro da imagem) é dado por:

$$|F(u,v)| = \sqrt{\left(R^2(u,v) + I^2(u,v)\right)}$$

- O valor da fase por:

$$\phi(u,v) = \tan^{-1}\left(\frac{I(u,v)}{R(u,v)}\right)$$

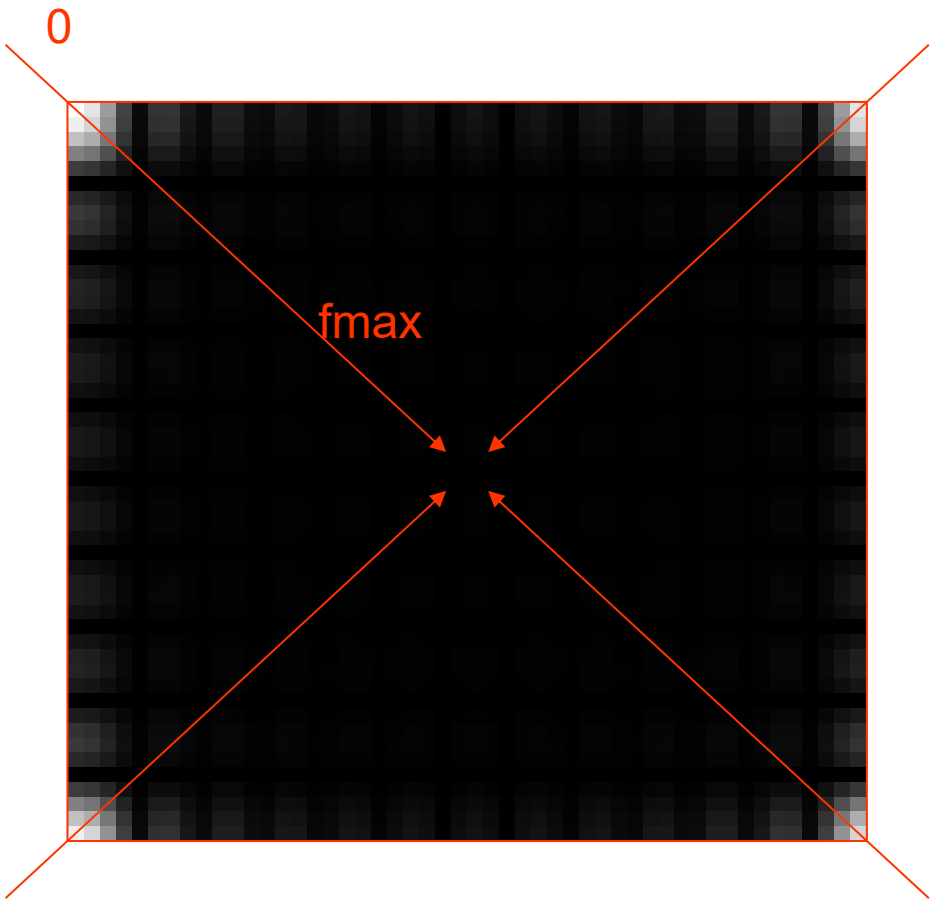
- Podemos então dizer:

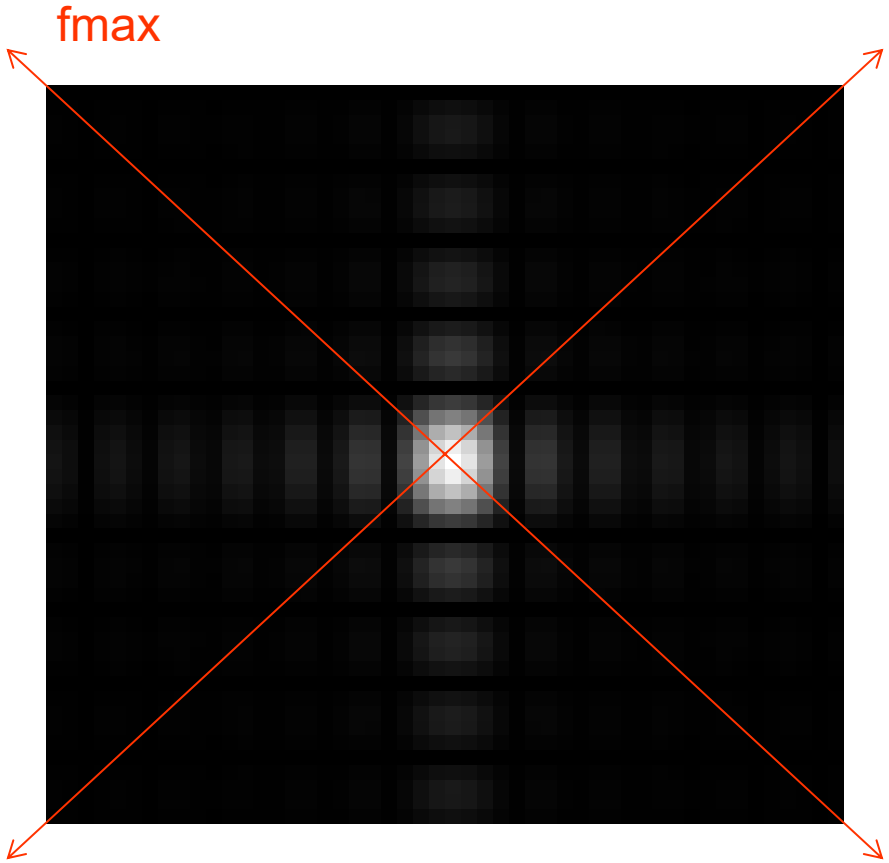
$$F(u,v) = |F(u,v)|e^{i\phi(u,v)}$$

- O espectro de potência é definido como o quadrado da amplitude:

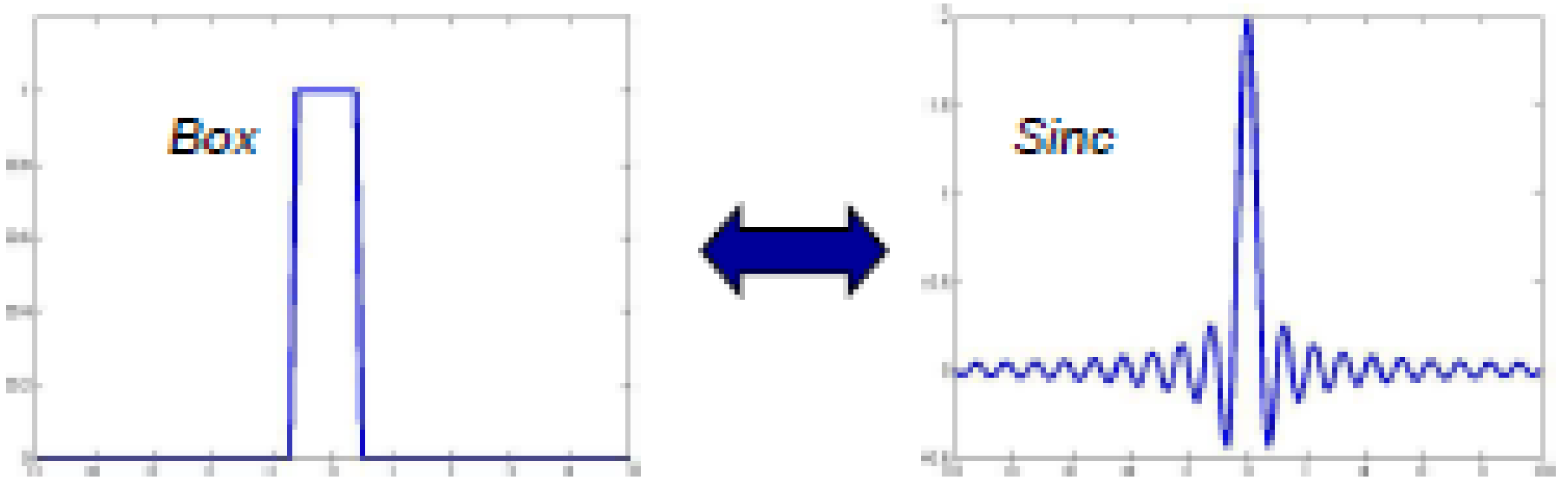
$$P(u,v) = |F(u,v)|^2 = R^2(u,v) + I^2(u,v)$$

FFT Resultado



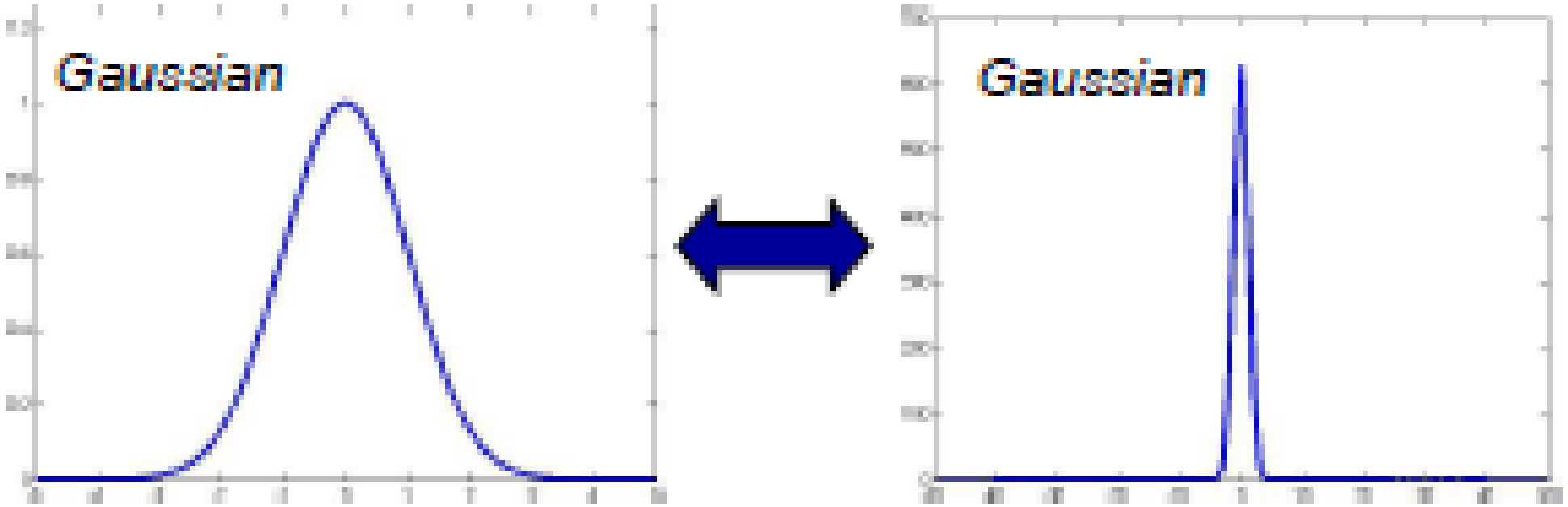


Algumas transformadas básicas



Time/Spatial
vs
Frequency

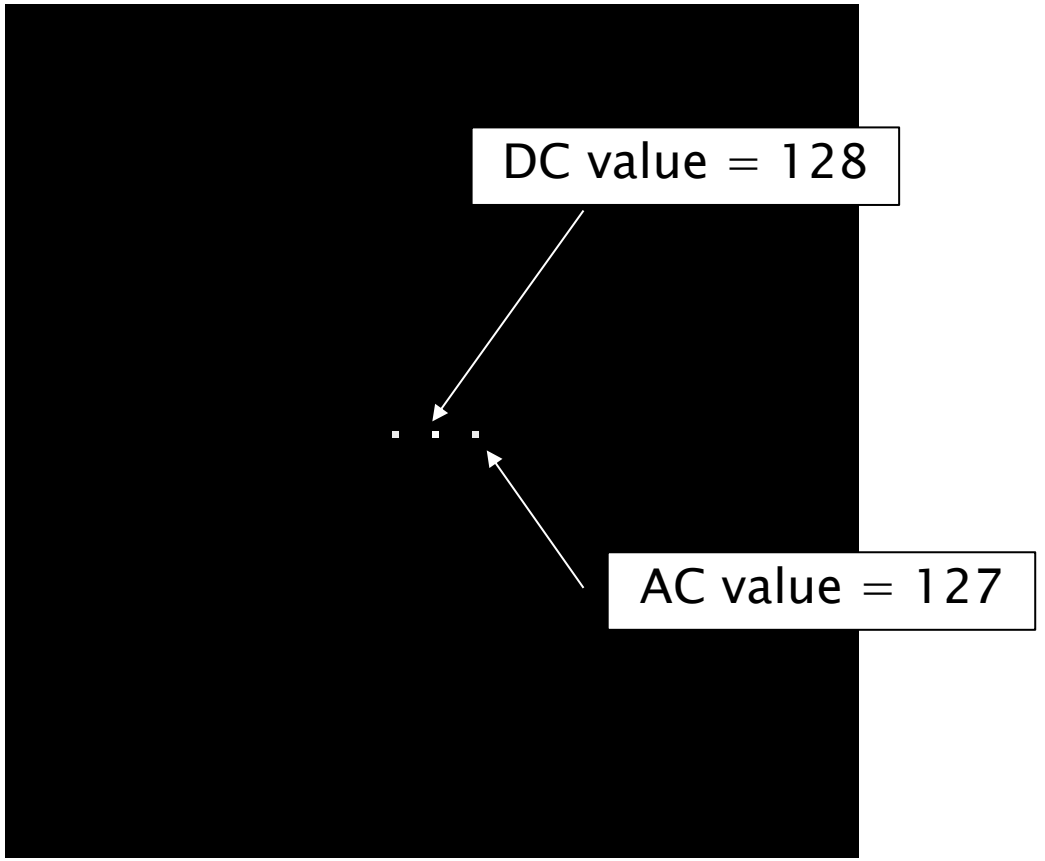
Algumas transformadas básicas



Time/Spatial
vs
Frequency

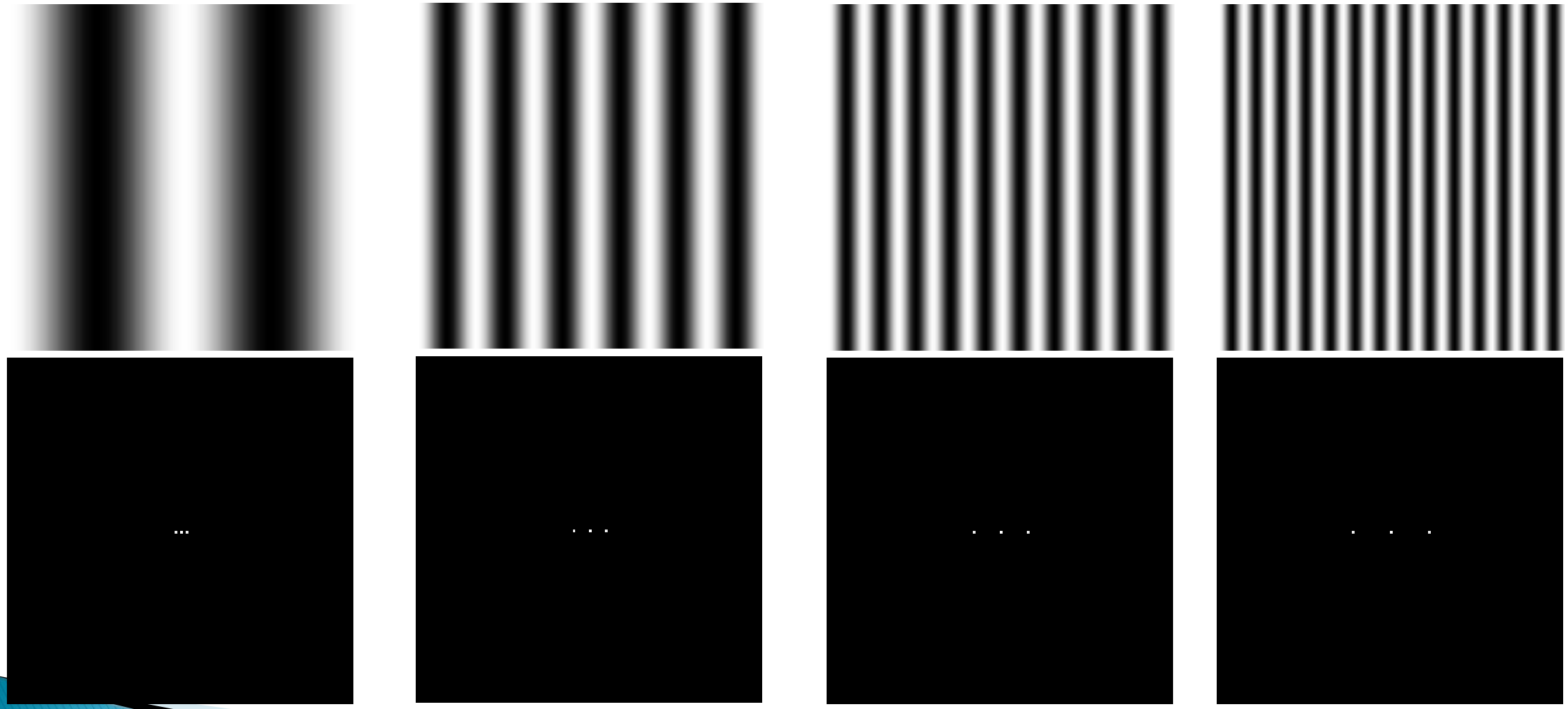
FFT – Exemplo

Espaço
img=Sin(wt)*127 + 128

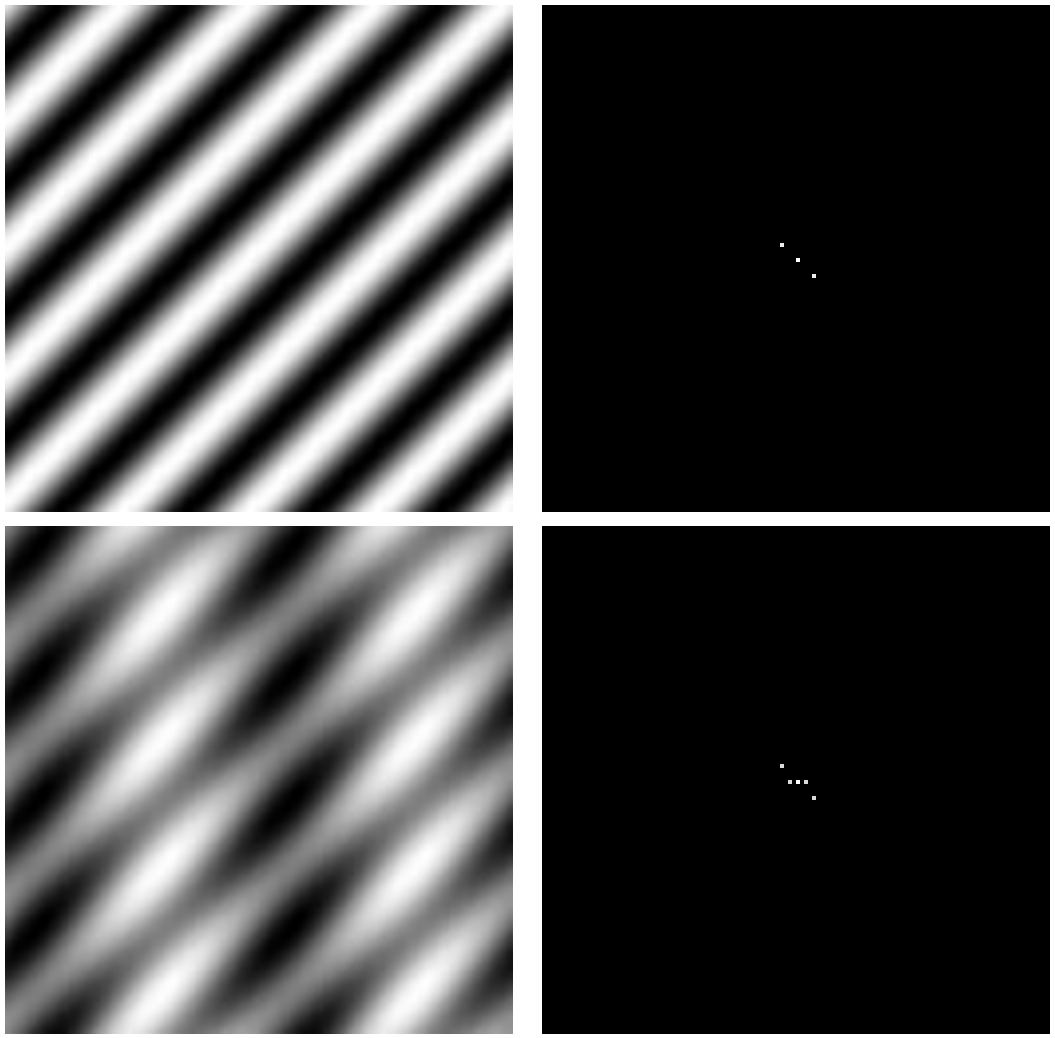


Frequência (magnitude)

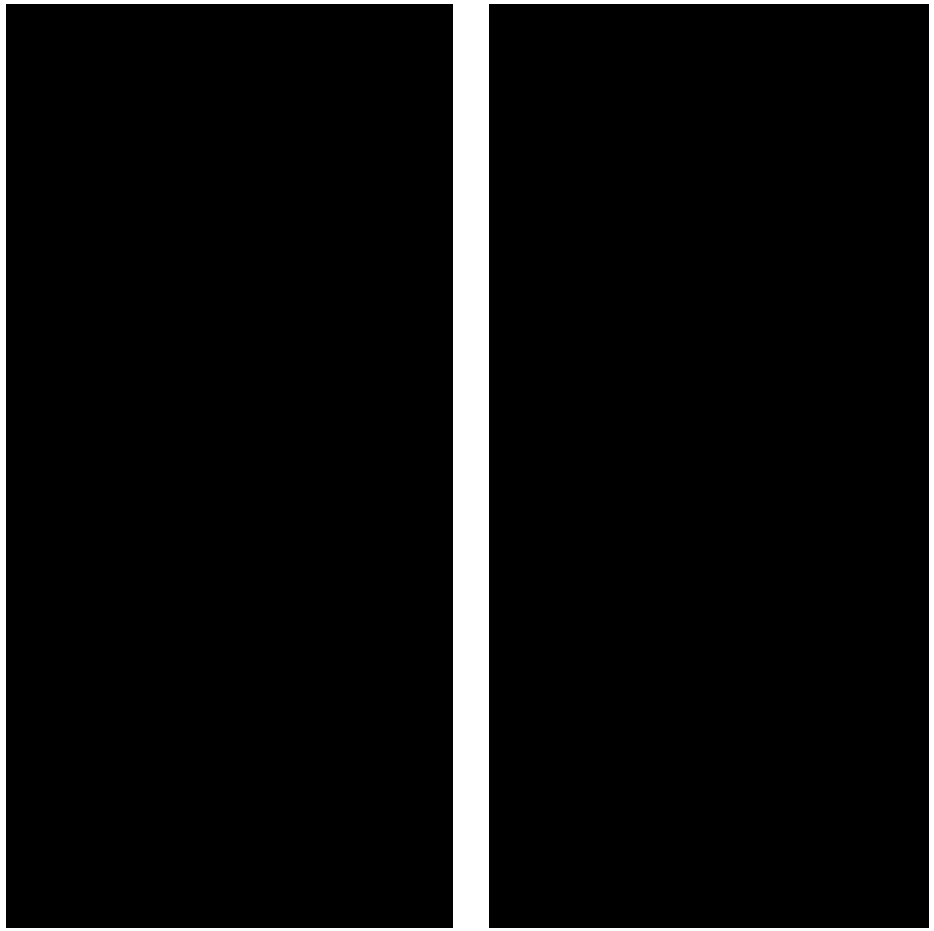
FFT – Exemplos



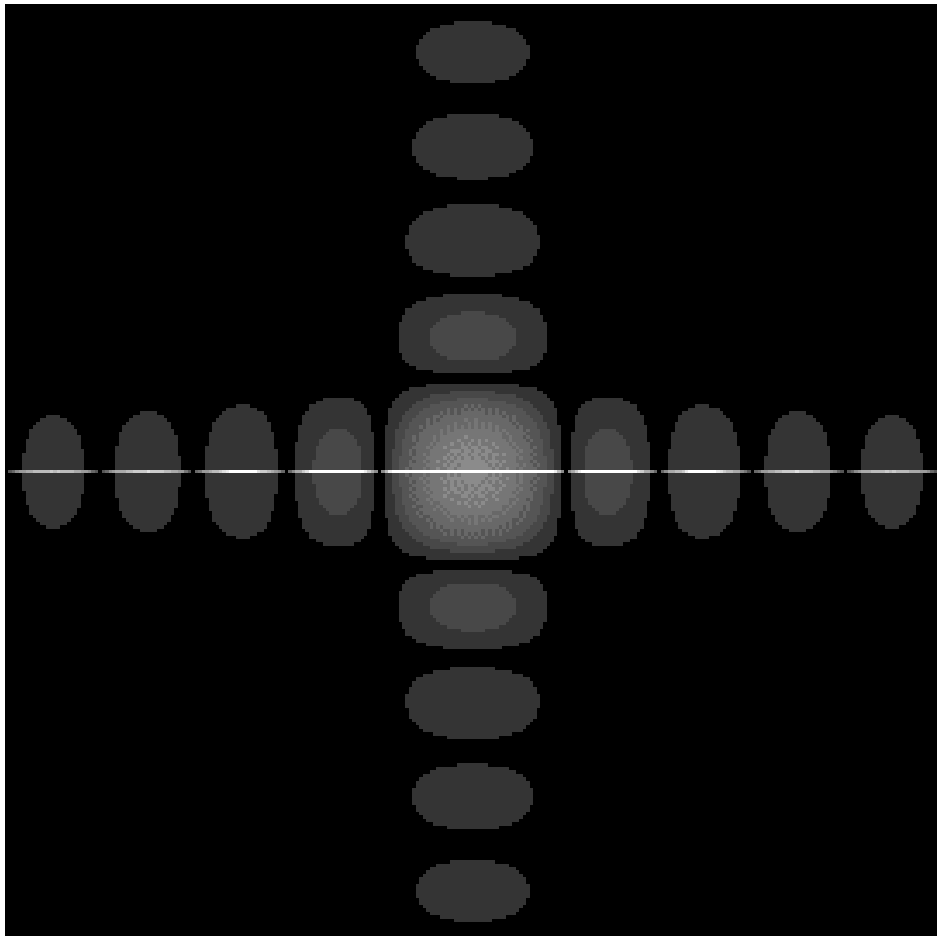
FFT – Exemplos



FFT – Exemplo



Espaço
BOX



Frequência (Magnitude)
Sync

FFT – Exemplo

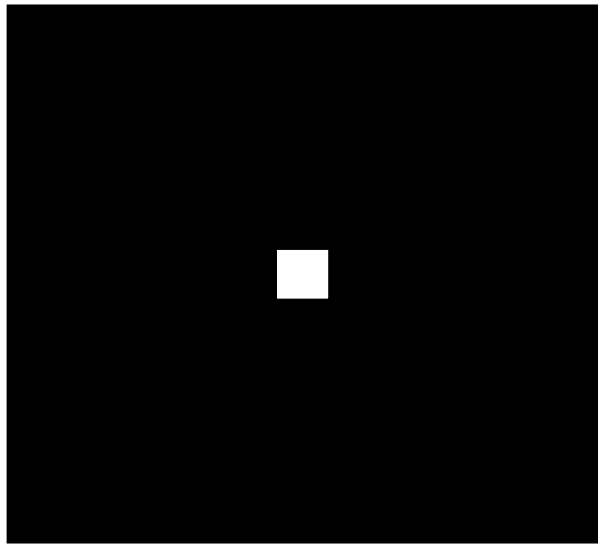
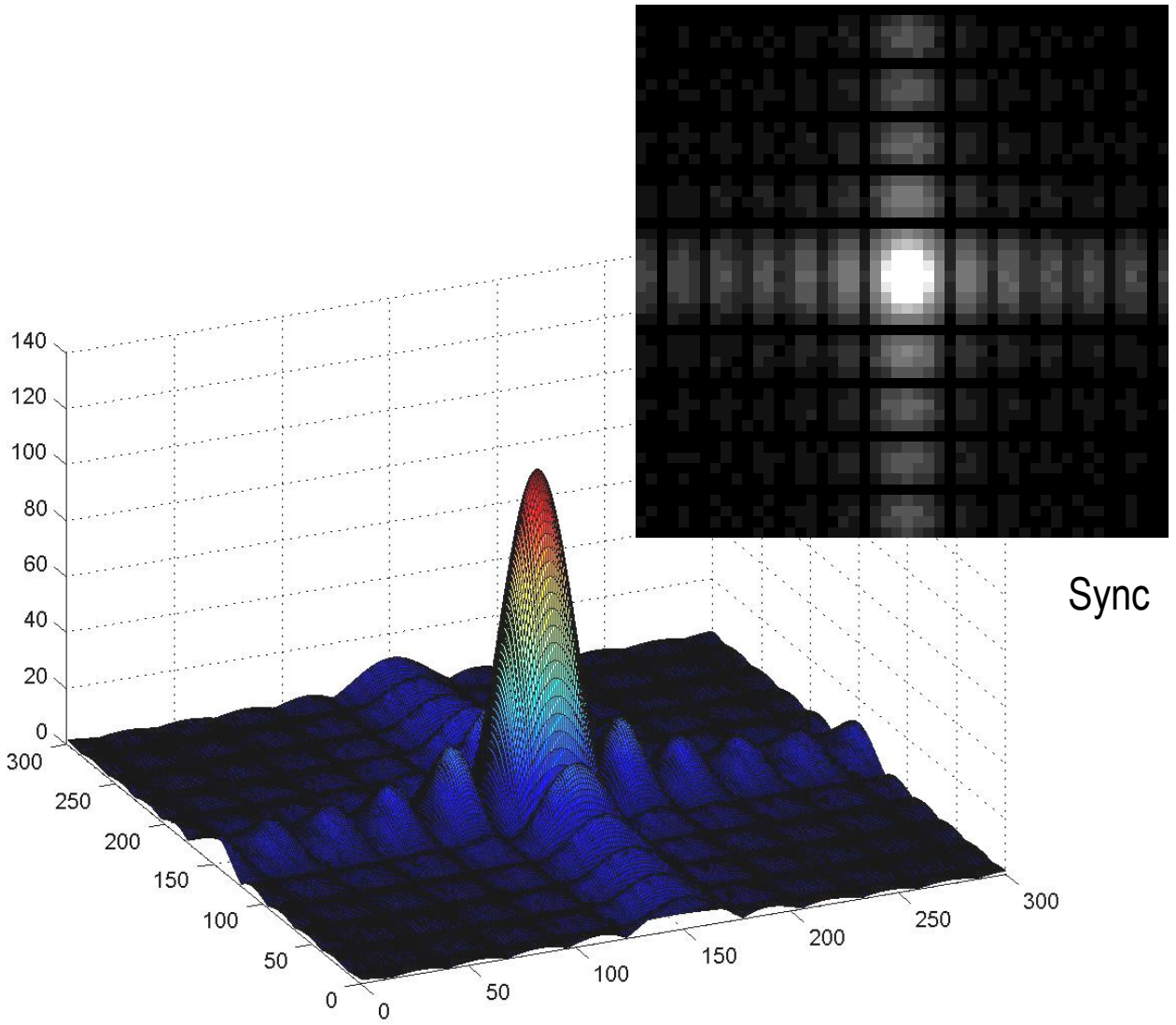


Imagem original
Box



Sync

Amplitude da FFT resultante

Transformadas de Fourier

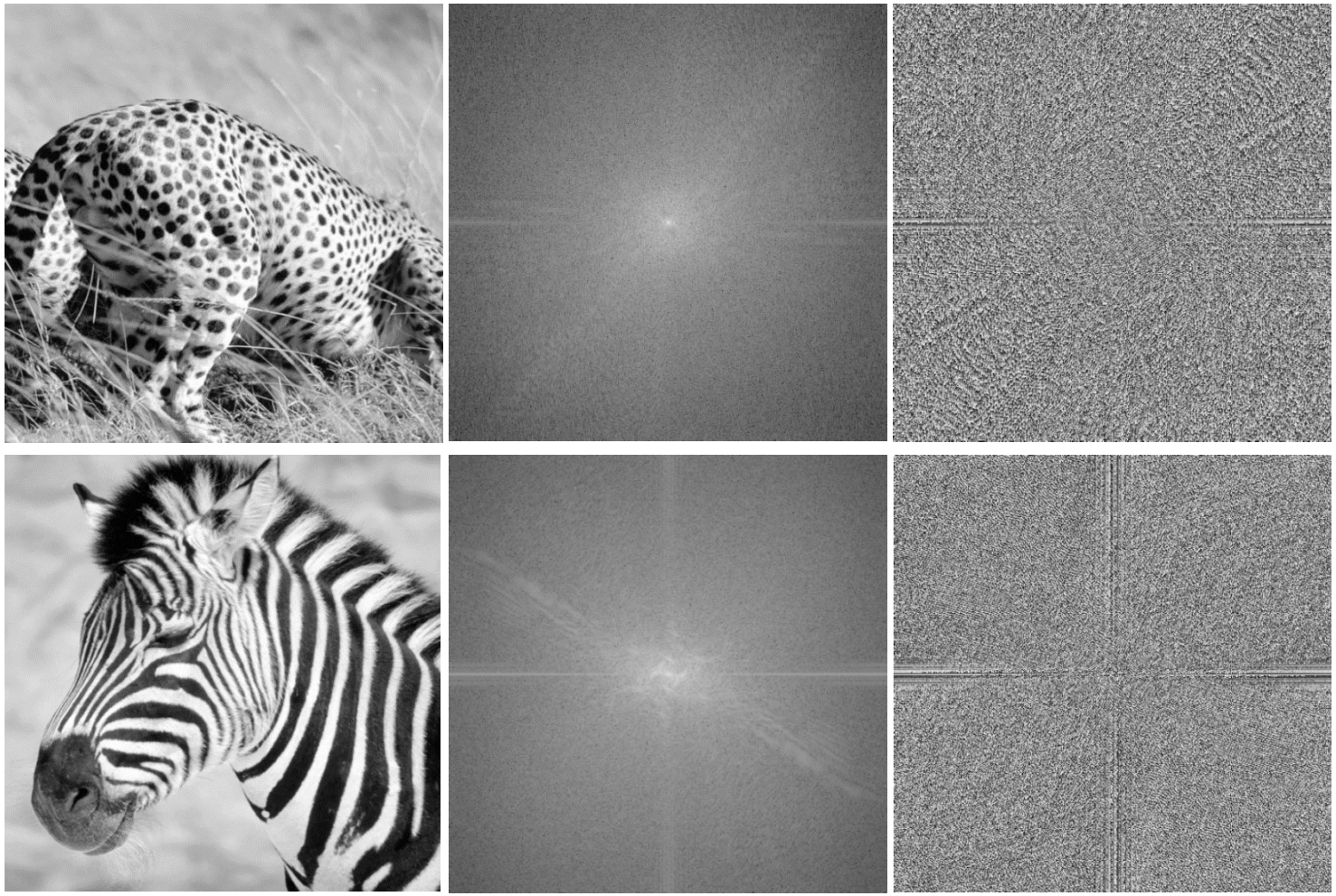
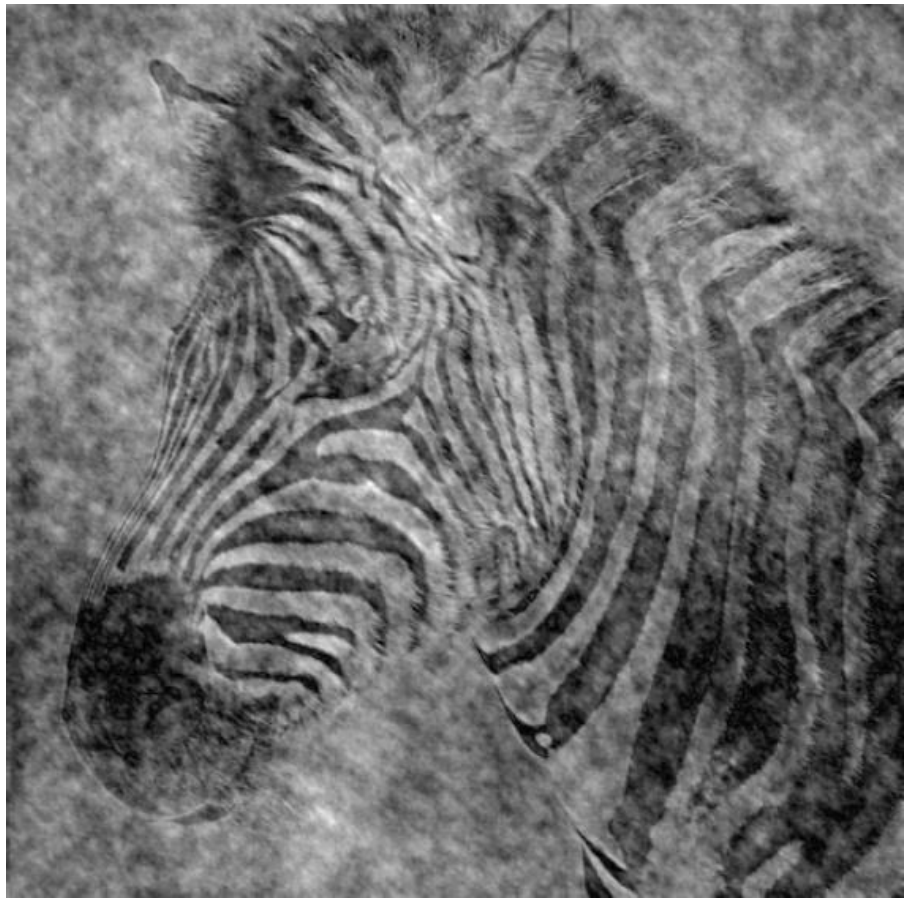
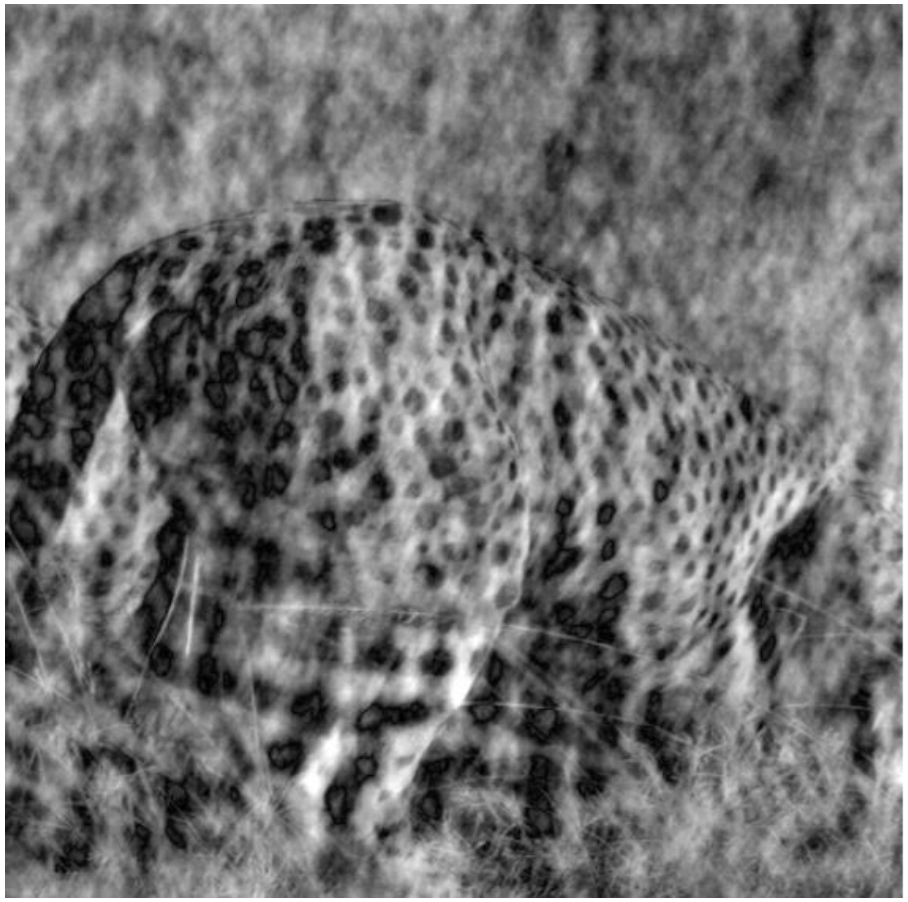


Imagem original | Amplitude | Fase

Comparação de importância amplitude/fase



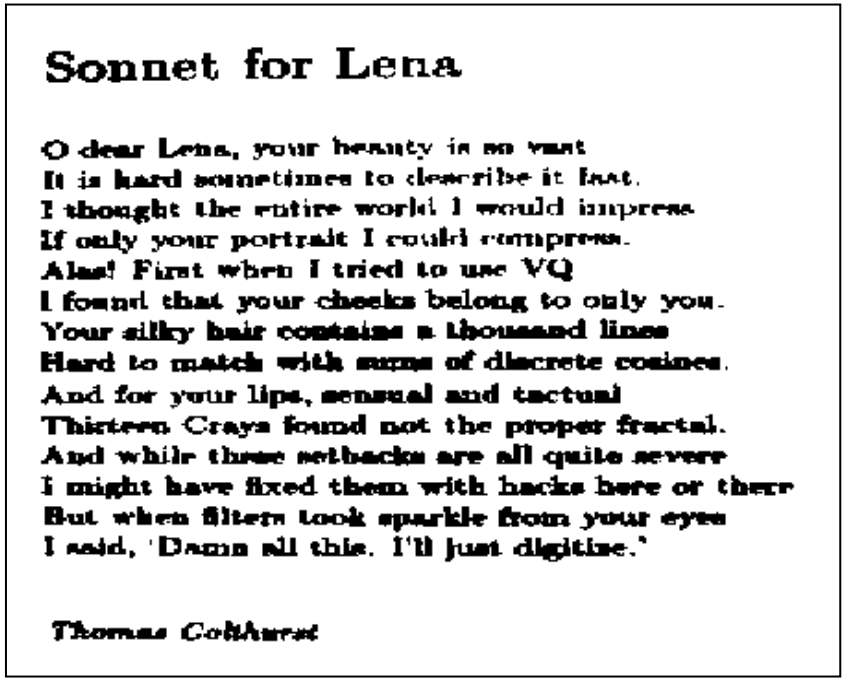
Amplitude da chita com a fase da zebra



Amplitude da zebra com a fase da chita

Exemplo de aplicação da FFT em imagem

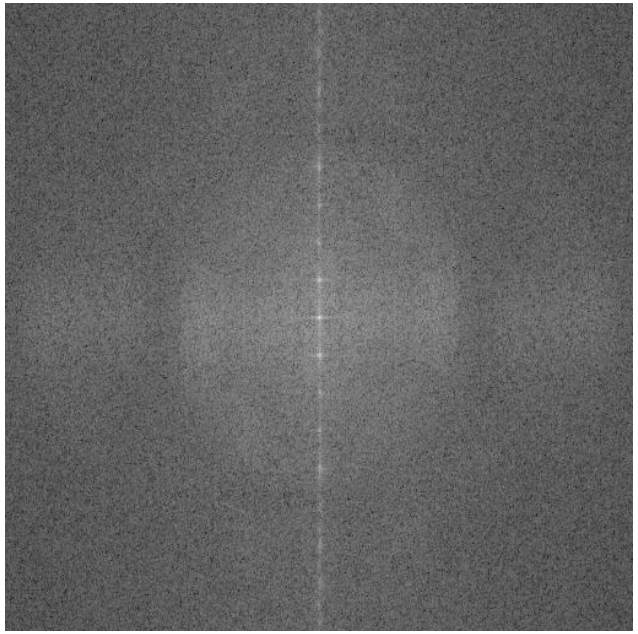
- Determinação da orientação do texto na imagem:



Sonnet for Lena

O dear Lena, your beauty is so vast
It is hard sometimes to describe it fast.
I thought the entire world I would impress
If only your portrait I could compress.
Alas! First when I tried to use VQ
I found that your cheeks belong to only you.
Your silky hair contains a thousand lines
Hard to match with sums of discrete cosines.
And for your lips, sensual and tactual
Thirteen Crays found not the proper fractal.
And while these setbacks are all quite severe
I might have fixed them with hacks here or there
But when filters took sparkle from your eyes
I said, 'Damn all this. I'll just digitize.'

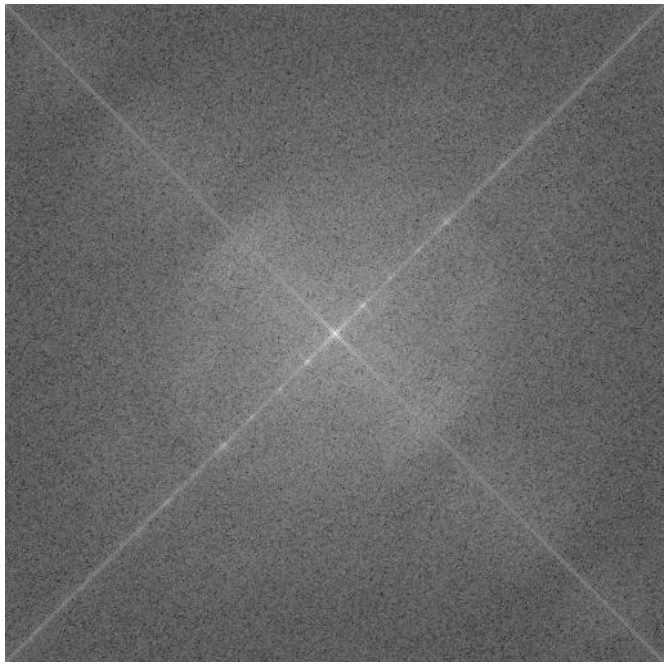
Thomas Colthurst



Amplitude

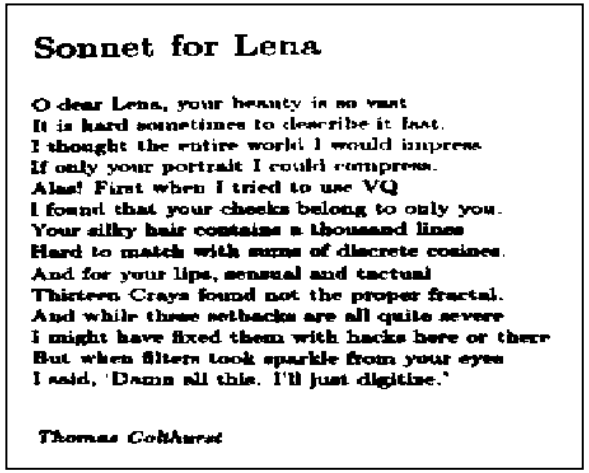
Exemplo de aplicação da FFT em imagem

- Partindo agora de uma imagem semelhante, mas rodada:



Os pontos de maior amplitude permitem determinar a orientação principal do texto

Comparação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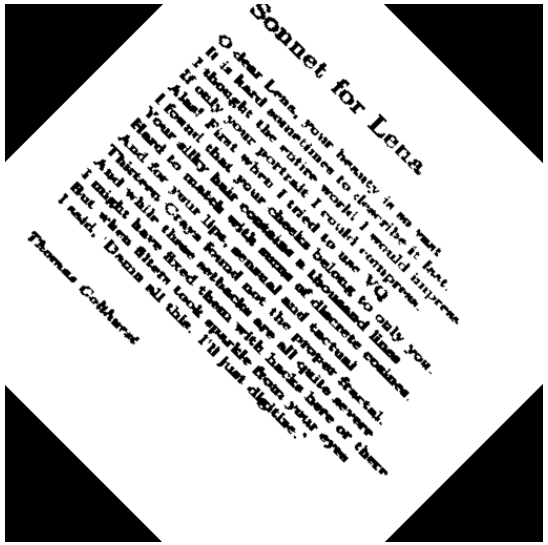
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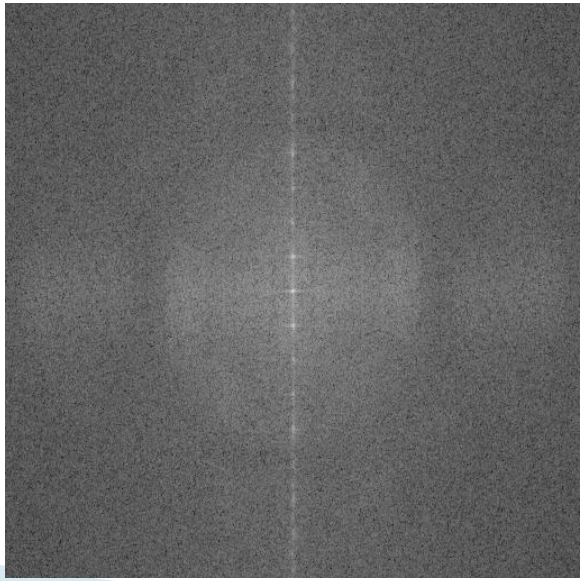
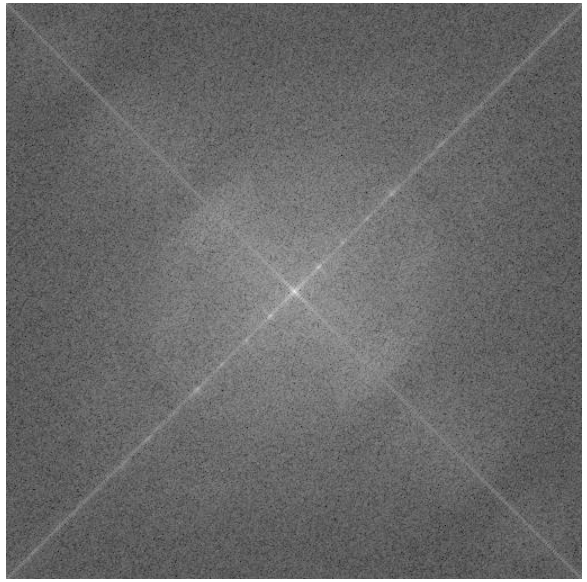
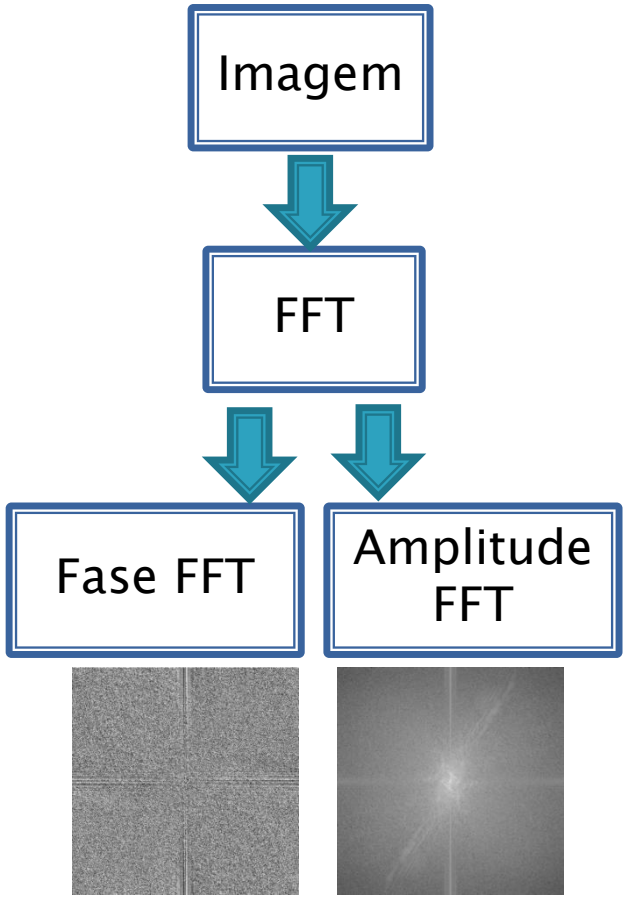
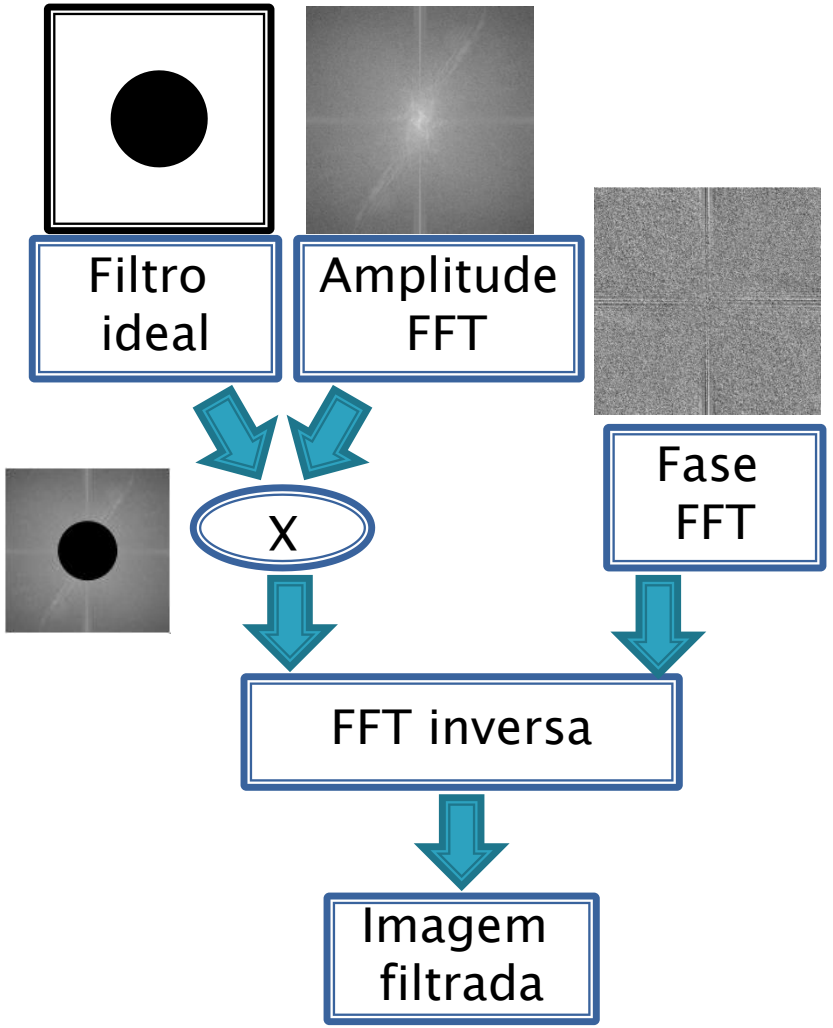


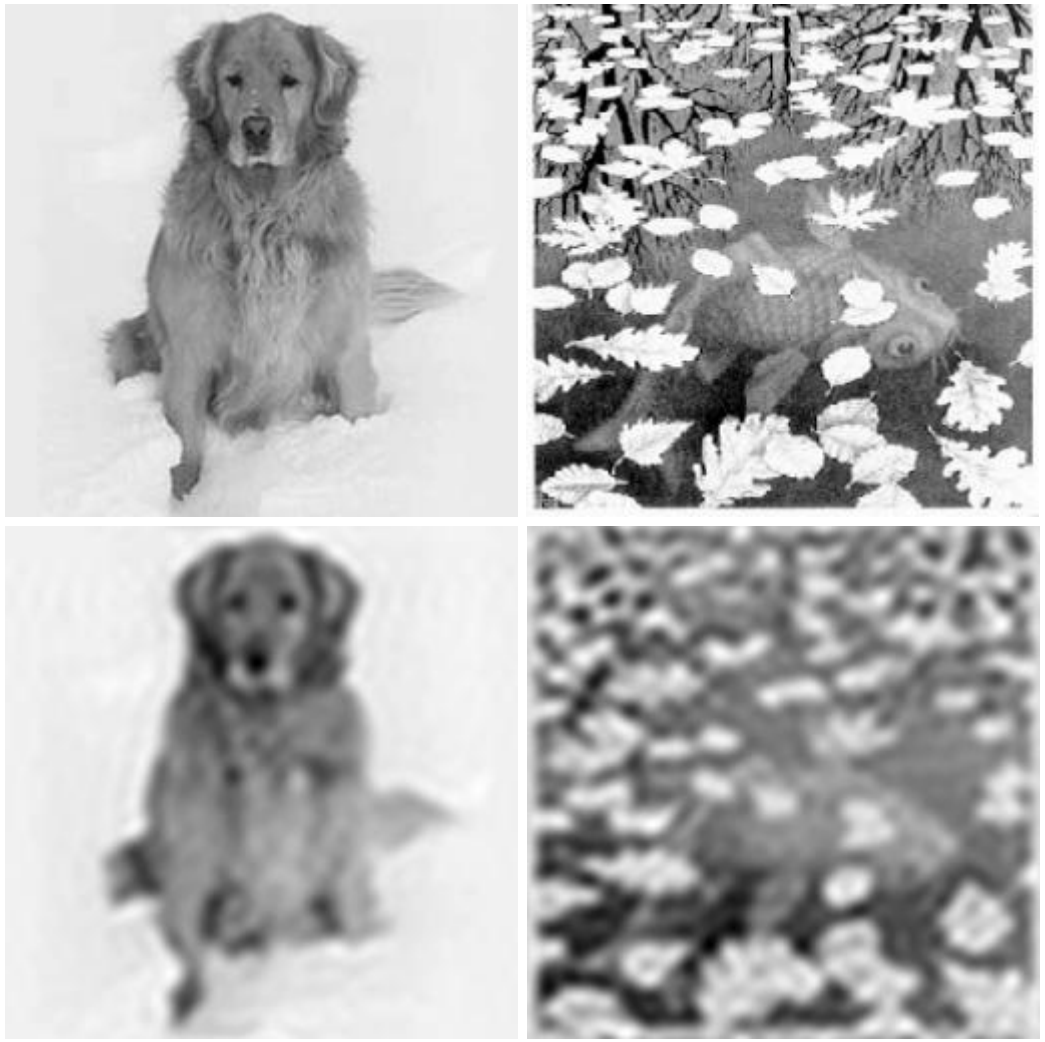


Filtragem na frequência



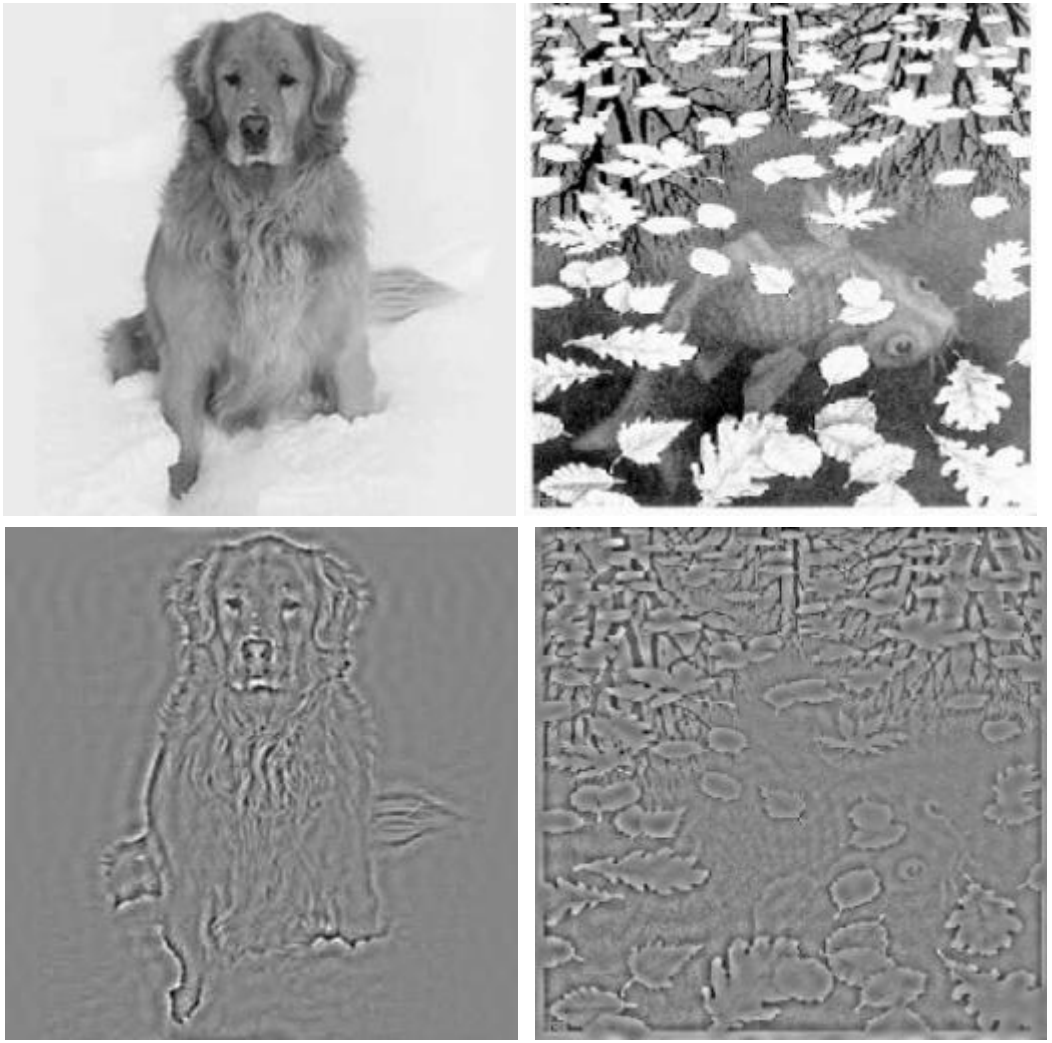


Filtragem passa-baixos



http://www.cs.unc.edu/~parente/igv/hw5/parente_hw5.html

Filtragem passa-altos



O efeito de ringing deve-se ao filtro ser ideal:
A FFT de uma função degrau é um Sinc

http://www.cs.unc.edu/~parente/igv/hw5/parente_hw5.html

Filtragem na frequência – Passa Baixo

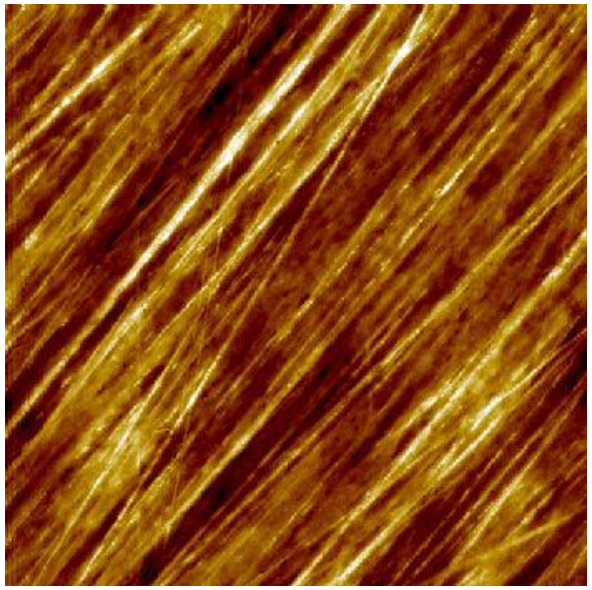
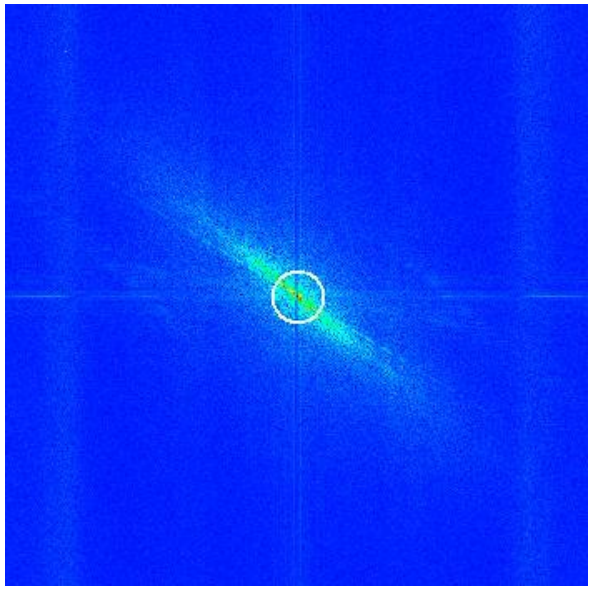


Imagem original



FFT com frequência de corte assinalada

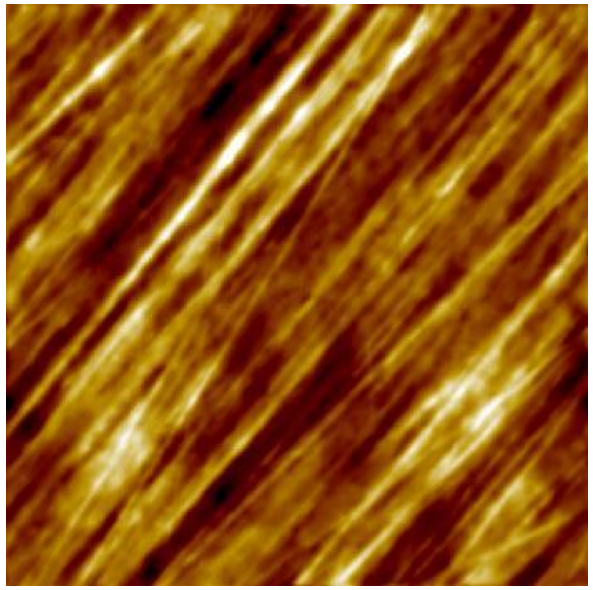


Imagem resultante

FFT para filtragem selectiva (1)

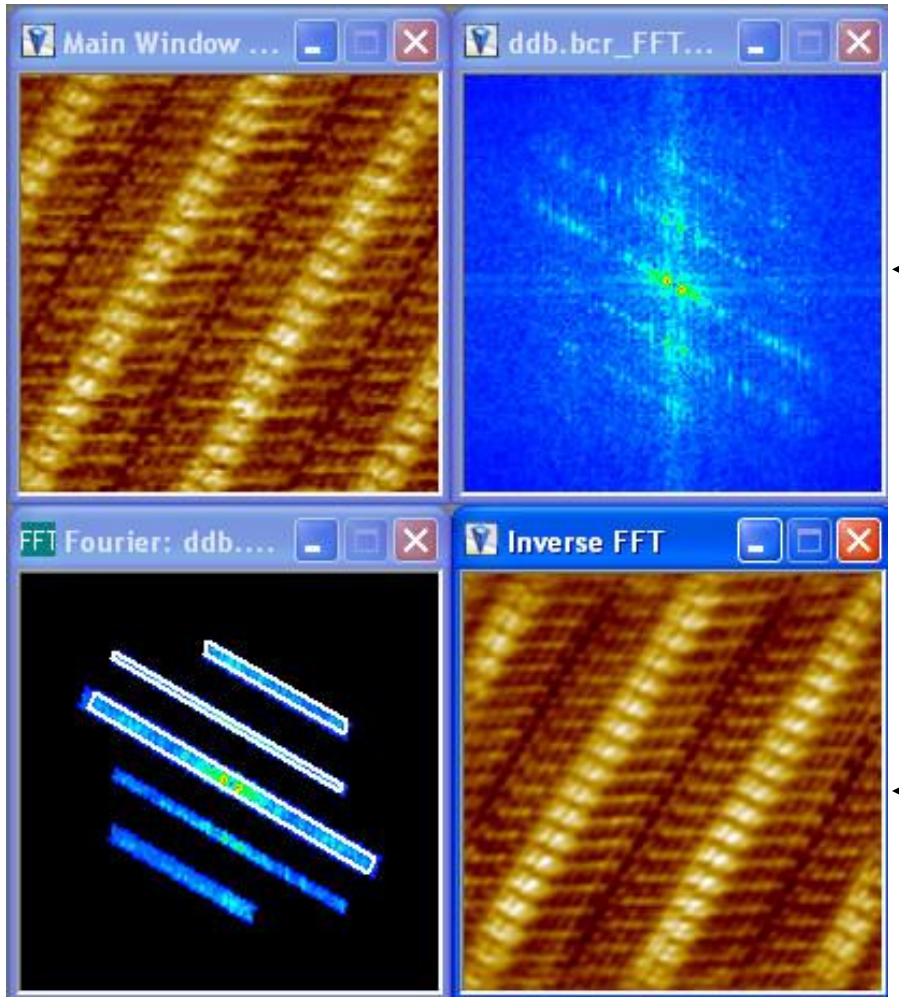


Imagem original

FFT (amplitude)

Selecção dos valores

Imagem resultante

http://www.imagemet.com/WebHelp/spip.htm#selective_fourier_filtering.htm

FFT para filtragem selectiva (2)

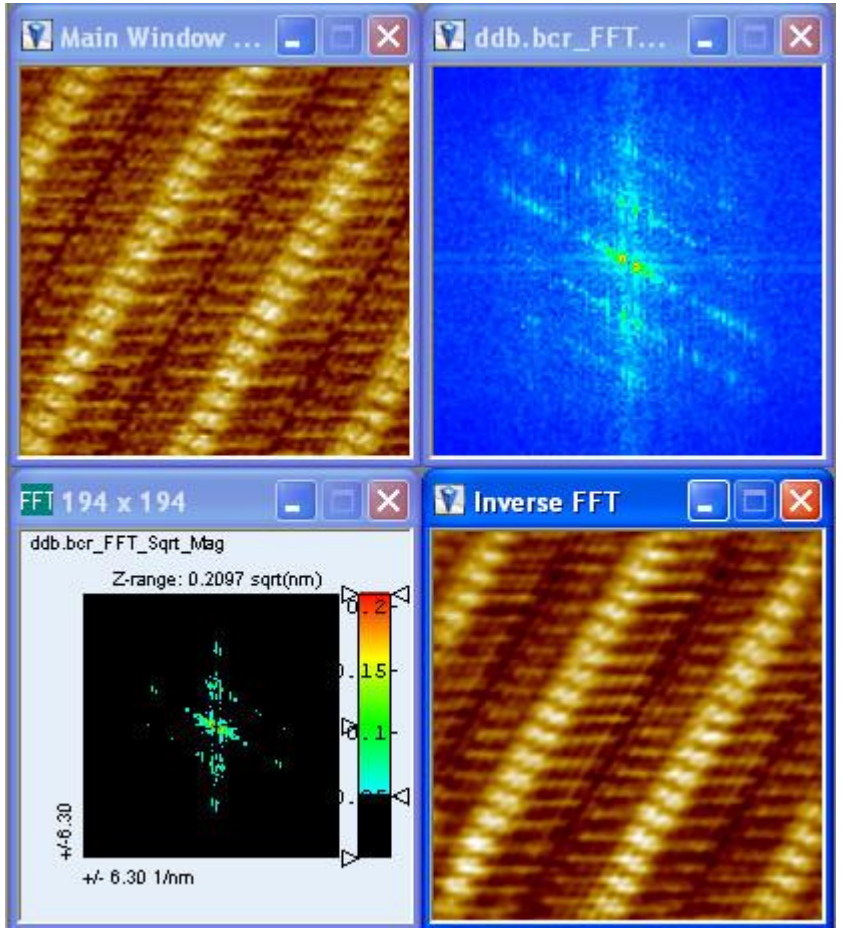
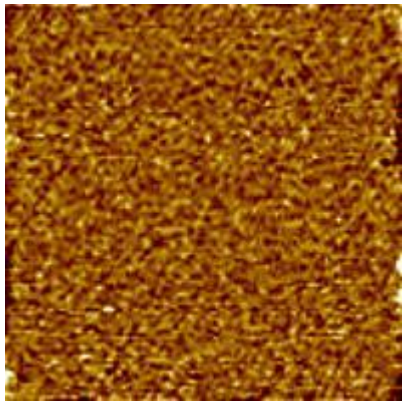


Imagem original →

← FFT (amplitude)

Selecção dos valores com amplitude dentro dos limites →

Imagem resultante



Diferença entre as imagens

http://www.imagemet.com/WebHelp/spip.htm#selective_fourier_filtering.htm

FFT para filtragem selectiva (3)

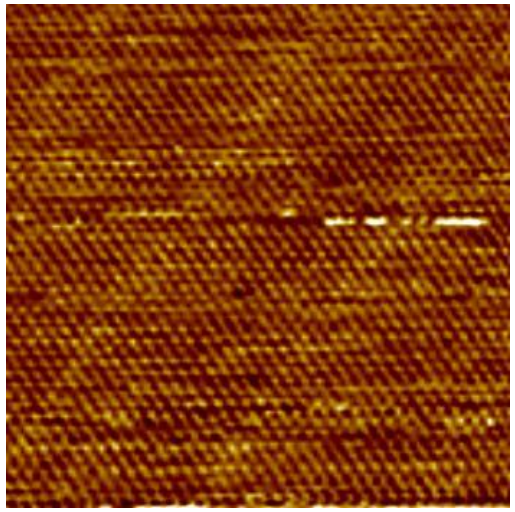
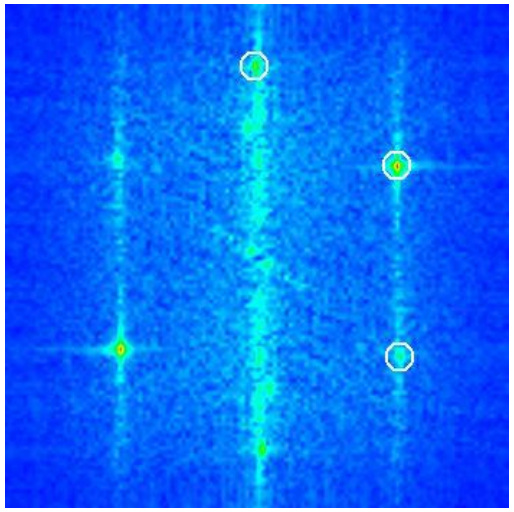
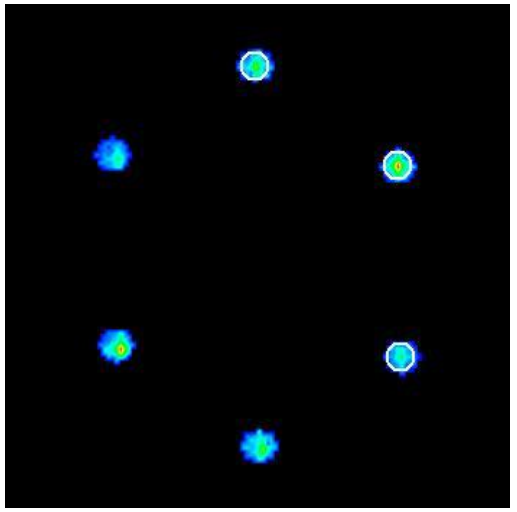


Imagem original



FFT



FFT seleccionada

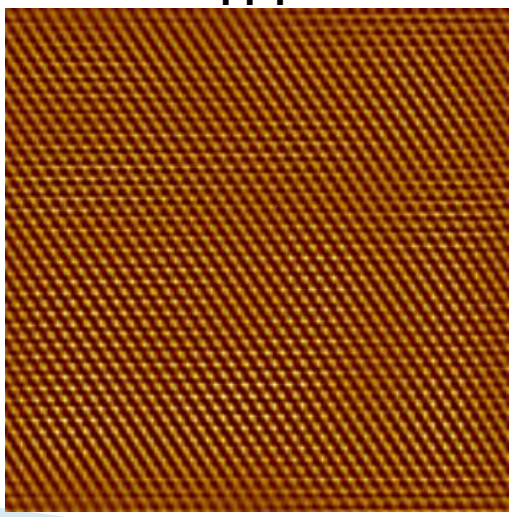


Imagem resultante

FFT para focagem

FFT – Aplicações

- Análise de frequência da imagem (ex: focagem)
- Filtragem passa baixo, passa alto, selectiva
- Acelerar a operação de convulsão entre duas imagens
  - A convulsão corresponde a avaliar cada pixel N_2xN_2 vezes
  - Convulsão corresponde à multiplicação na frequência:

$$\boldsymbol{I1}(\boldsymbol{X},\boldsymbol{Y}) \cdot \boldsymbol{I2}(\boldsymbol{X},\boldsymbol{Y}) \;\rightarrow \boldsymbol{I1}(\boldsymbol{u},\boldsymbol{v}) \times \boldsymbol{I2}(\boldsymbol{u},\boldsymbol{v})$$

- Compressão de imagem
- Reconstrução de imagem