Instantaneous R for COVID-19 in Turkey: Estimation by Bayesian Statistical Inference

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Abstract

The instantaneous R in Turkey is estimated by Bayesian statistical inference that utilizes a 68-days-long dataset from the beginning of the COVID-19 outbreak in Turkey for monitoring the progression of the pandemic. As it is also globally adapted, enforced social distancing measures help to keep the instantaneous reproduction number below one. The low levels of instantaneous R are referred to as a basis for several countries to relax their country-wide restrictions, while hindsight involves a possible second wave of infections to follow in China, Germany, and South Korea. Thus, policy and decision-makers need to be vigilant regarding the pandemic's progress. It is not yet sure if it is possible to maintain the instantaneous reproduction number below one, even at the lack of societal measures.

Keywords

COVID-19, Turkey, epidemic models, Bayesian statistical inference, EpiEstim, coronavirus.

Özet

Türkiye'deki anlık bulaştırma katsayısı COVID-19 salgınının başlangıcından itibaren 68 günlük bir veri seti kullanılarak Bayesyen istatistiksel çıkarım ile tahmin edilmiştir. Salgının kontrol altında tutulabilmesi için anlık bulaştırma katsayısının cari seviyesinin sürekli bir biçimde tahmin edilmesinin önemi vurgulanmıştır. Model çıktılarıyla etkin bulaştırma katsayısı tahminleri sunulmuştur. Zaman ilerledikçe elde edilen model çıktıları karşılaştırıldığında, sosyal mesafe önlemlerinin anlık bulaştırma katsayısının birin altında tutulması yönünde olumlu etkisi gözlemlenmektedir. Bununla birlikte, önlemlerin gevşetilmesi sonrası Çin, Güney Kore ve Almanya gibi ülkelerde salgının ikinci dalgasının başlamış olabileceği de dikkate alındığında, anlık bulaştırma katsayısının kalıcı olarak birin altında tutulup tutulamayacağı belirsizliğini korumaktadır. Bu noktadan hareketle, politika yapıcılar ve karar vericilerin salgının sonraki aşamaları için tetikte olmaları gerekmektedir..

Anahtar kelimeler

COVID-19, Turkiye, epidemik modeller, Bayesyen istatistiksel çıkarım, EpiEstim, koronavirüs.

As of May 16, 2020, it has been 68 days since the reporting of the first COVID-19 case in Turkey on March 11, 2020. During this period, the total number of confirmed cases reached 148,067, according to figures reported by the Ministry of Health - Turkey.

In our previous study[2], where we employed the SIR model to predict the progress of the COVID-19 pandemic, it was emphasized how imperative it is to forecast the pandemic's progression in the coming future to devise an appropriate policy response. Besides predicting the future progress of the pandemic, an equally maybe more critical policy question concerns the timing for easing and eventually lifting limitations such as curfews and closure of schools and businesses. If the restrictions are relaxed and/or lifted prematurely, there might be a substantial risk of rebound. On the other hand, as long as such movement restrictions and social isolation principles remain intact, economic hardship for millions of people is exacerbated.

Estimating the instantaneous reproduction number may help us answer the second policy question regarding the timing for easing and eventually lifting limitations. WHO suggests that the value for reproduction number should be equal to or less than 1.0 to alleviate the measures imposed by governments without further potential distress on their healthcare systems. When the effective reproduction number is larger than 1.0, the exponential growth of the outbreak poses distress risk to the healthcare system.

There are variants of the reproduction number, such as the basic reproduction number, the effective reproduction number, the case reproduction number, and the instantaneous reproduction number.

The instantaneous reproduction number, R_t, at time t can be estimated as in Equation (1).[3]

$$R_t = \frac{E(I_t)}{\sum_{s=1}^{t} I_{t-s} w_s} \qquad (1)$$

In equation (1), I_t stands for the number of new infections generated at time step t, whereas w_s is the probability distribution of the infectivity profile which is dependent on time since infection of the case, s, but independent of calendar time, t. Hence, an individual will be most infectious at time s when w_s is the largest. w_s is typically related to individual biological factors such as symptom severity.

Effective control measures undertaken at time t are expected to result in a sudden decrease in R_t, whereas the other reproduction number variants tend to respond rather slowly. Therefore, evaluating the efficiency of control measures is more effective when estimates of R_t are used.[4]

Cori et al. (2013) developed a generic and robust tool, EpiEstim (implemented in Microsoft Excel and R), for estimating R_t. Assuming a gamma prior distribution for R_t, Bayesian statistical inference leads to an analytical expression for the posterior distribution of R_t. Since the resulting

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[2] Ozdinc, M., Senel, K., Ozturkcan, S., & Akgul, A., "Predicting the Progress of COVID-19: The Case for Turkey," Turkiye Klinikleri Journal of Medical Sciences, in press. 10.5336/medsci.2020-75741

[3] Cori, A., Ferguson, N. M., Fraser, C., & Cauchemez, S. (2013). A new framework and software to estimate time-varying reproduction numbers during epidemics. *American journal of epidemiology*, *178*(9), 1505-1512.

[4] Ibid.

R_t estimates are usually not robust when the time step is small, they calculate estimates over longer time windows, under the assumption that the instantaneous reproduction number is constant over that time window. At each time step t, they calculate the reproduction number over a time window of size τ ending at time t. These estimates, denoted $R_{t,\tau}$, yield the average transmissibility over the time window of length τ ending at time t. The posterior mean and standard deviation of $R_{t,\tau}$ are given in Equations (2) and (3), respectively.[5]

$$E\left(R_{t,\tau} \mid I_0,\ I_1,\ \ldots,\ I_t,\ w\right) = \frac{a + \sum_{s=t-\tau+1}^{t} I_s}{\frac{1}{b} + \sum_{s=t-\tau+1}^{t} \sum_{r=1}^{s} I_{s-r} w_r} \tag{2}$$

$$\sigma\left(R_{t,\tau} \mid I_0,\ I_1,\ \ldots,\ I_t,\ w\right) = \frac{\sqrt{a + \sum_{s=t-\tau+1}^{t} I_s}}{\frac{1}{b} + \sum_{s=t-\tau+1}^{t} \sum_{r=1}^{s} I_{s-r} w_r} \tag{3}$$

In Equations (2) and (3), a and b are the shape and scale parameters of the gamma prior distribution for R_t, respectively.

In order to employ this method for the Turkish COVID-19 data, we need distribution parameters of the serial interval for COVID-19. Serial interval is defined as the time between onset of systems of a case and onset of symptoms of his/her secondary cases. We obtained the distribution parameters from literature.[6] Hence, we assume a gamma distribution with shape parameter of 2.39 and rate parameter 0.48 for serial interval, which correspond to a mean of 4.98 days and a standard deviation of 3.22 days. Serial interval distribution is depicted in Figure 1.

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[5] Ibid.

[6] Supplement to: Zhang J, Litvinova M, Wang W, et al. Evolving epidemiology and transmission dynamics of coronavirus disease 2019 outside Hubei province, China: a descriptive and modelling study. *Lancet Infect Dis* 2020; published online April 2. https://doi.org/10.1016/S1473-3099(20)30230-9.

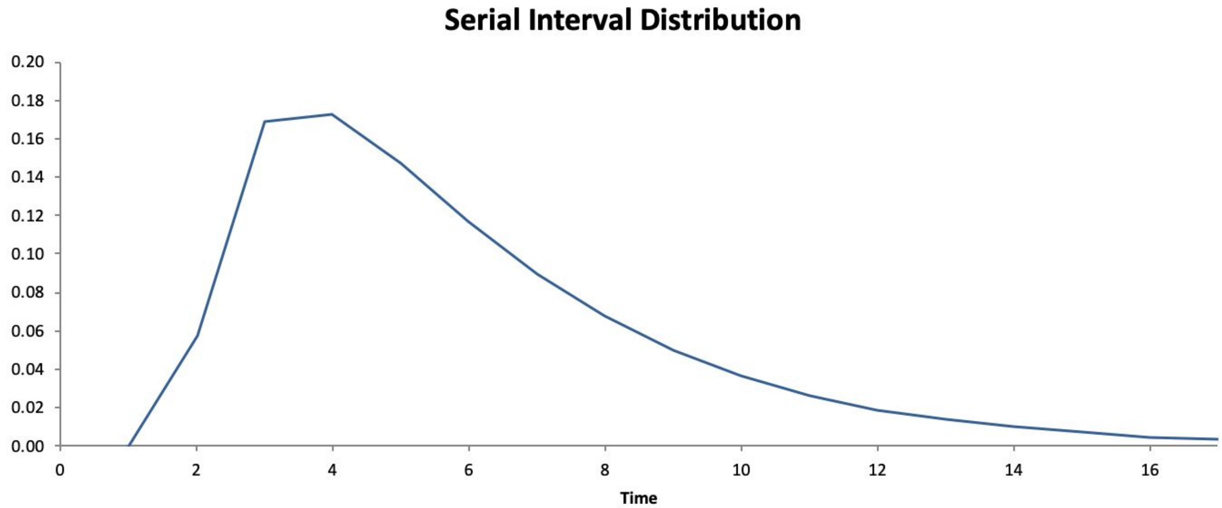

Figure 1. Serial Interval Distribution for COVID-19 - Γ(2.39 days, 0.48 days)

Accordingly, the length of time steps, τ, is chosen as 4 days since the method requires that τ should be less than the mean serial interval (4.98 days).

The resulting $R_{t,4}$ estimates are depicted in Figure 2 and tabulated in Appendix 1.

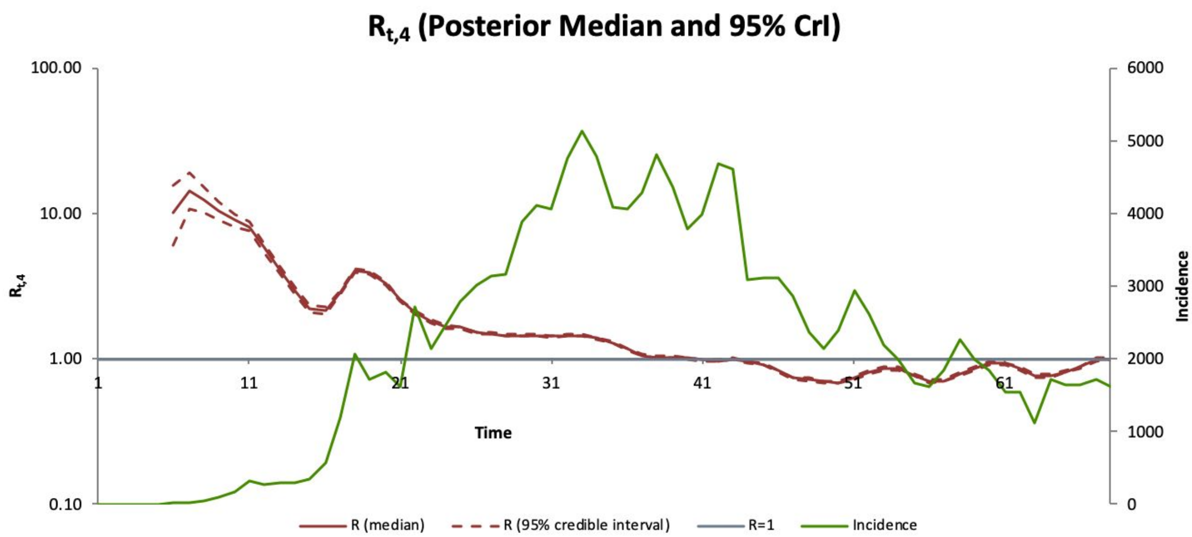

Figure 2. Instantaneous R Averaged Over 4 Days (Posterior Median and 95% Credible Interval)

The median $R_{t,4}$ has declined to the critical threshold level of 1 on Day 40 (April 18, 2020). Since then, it seemingly has plateaued and oscillated between 0.69 and 1.00. As of May 16, 2020, the median $R_{t,4}$ is estimated at 1.00 whereas the 95% credible interval is [0.98, 1.02].

Although we observe that the lockdown measures have been quite effective in containing the pandemic, it is still uncertain if the instantaneous reproduction number can be decisively kept under the critical threshold. It should also be noted that the second wave of the pandemic might have already started in countries such as China, South Korea, and Germany.[7, 8] Therefore, we suggest that policy and decision makers should be extremely vigilant before easing or lifting the precautionary measures.

[7] https://www.aa.com.tr/en/asia-pacific/new-cases-in-china-skorea-spark-fears-of-2nd-wave/1836458 Access: May 16, 2020.

[8] https://www.euronews.com/2020/04/28/coronavirus-germany-s-covid-19-infection-rate-rises-after-lockdown-lifted Access: May 16, 2020.

Appendix 1.

Estimates of the instantaneous reproduction number R

[redacted]

Time periods		Posterior R moments		Main R Quantiles						
Start	End	Mean	Std	0.025 quantile	0.05 quantile	0.25 quantile	Median	0.75 quantile	0.95 quantile	0.975 quantile
3	6	10.17	2.40	6.03	6.57	8.47	9.98	11.67	14.40	15.38
4	7	14.50	2.11	10.65	11.20	13.02	14.39	15.86	18.14	18.93
5	8	12.37	1.28	10.00	10.35	11.49	12.33	13.21	14.55	15.00
6	9	10.45	0.77	9.00	9.22	9.92	10.43	10.96	11.74	12.01
7	10	8.95	0.48	8.03	8.17	8.62	8.94	9.27	9.76	9.92
8	11	8.12	0.33	7.50	7.59	7.90	8.12	8.34	8.66	8.77
9	12	5.78	0.20	5.40	5.46	5.65	5.78	5.92	6.11	6.18
10	13	4.07	0.13	3.82	3.86	3.98	4.07	4.15	4.28	4.32
11	14	2.96	0.09	2.79	2.82	2.90	2.96	3.01	3.10	3.13
12	15	2.19	0.06	2.07	2.09	2.15	2.19	2.24	2.30	2.32
13	16	2.14	0.06	2.03	2.05	2.10	2.14	2.17	2.23	2.25
14	17	2.86	0.06	2.75	2.76	2.82	2.86	2.90	2.96	2.97
15	18	4.02	0.06	3.90	3.92	3.98	4.02	4.06	4.12	4.14
16	19	3.90	0.05	3.80	3.82	3.87	3.90	3.94	3.99	4.01
17	20	3.31	0.04	3.23	3.24	3.28	3.31	3.33	3.37	3.39
18	21	2.51	0.03	2.45	2.46	2.49	2.51	2.53	2.56	2.57
19	22	2.08	0.02	2.04	2.04	2.07	2.08	2.10	2.12	2.13
20	23	1.79	0.02	1.75	1.76	1.78	1.79	1.81	1.83	1.83
21	24	1.65	0.02	1.61	1.62	1.63	1.65	1.66	1.68	1.68
22	25	1.64	0.02	1.61	1.61	1.63	1.64	1.65	1.67	1.67
23	26	1.51	0.01	1.48	1.49	1.50	1.51	1.52	1.53	1.54
24	27	1.49	0.01	1.46	1.47	1.48	1.49	1.50	1.51	1.52
25	28	1.44	0.01	1.42	1.42	1.43	1.44	1.45	1.46	1.47
26	29	1.45	0.01	1.42	1.43	1.44	1.45	1.45	1.47	1.47
27	30	1.45	0.01	1.42	1.43	1.44	1.45	1.45	1.47	1.47
28	31	1.42	0.01	1.40	1.40	1.41	1.42	1.43	1.44	1.44

Estimates of the instantaneous reproduction number R

[redacted]

Time periods		Posterior R moments		Main R Quantiles						
Start	End	Mean	Std	0.025 quantile	0.05 quantile	0.25 quantile	Median	0.75 quantile	0.95 quantile	0.975 quantile
29	32	1.45	0.01	1.43	1.43	1.44	1.45	1.46	1.47	1.47
30	33	1.44	0.01	1.42	1.42	1.43	1.44	1.45	1.46	1.46
31	34	1.38	0.01	1.36	1.36	1.37	1.38	1.39	1.40	1.40
32	35	1.28	0.01	1.26	1.27	1.28	1.28	1.29	1.30	1.30
33	36	1.16	0.01	1.14	1.14	1.15	1.16	1.16	1.17	1.17
34	37	1.05	0.01	1.04	1.04	1.05	1.05	1.06	1.06	1.07
35	38	1.02	0.01	1.01	1.01	1.02	1.02	1.03	1.04	1.04
36	39	1.03	0.01	1.01	1.01	1.02	1.03	1.03	1.04	1.04
37	40	1.00	0.01	0.99	0.99	1.00	1.00	1.01	1.02	1.02
38	41	0.98	0.01	0.97	0.97	0.98	0.98	0.99	0.99	1.00
39	42	0.97	0.01	0.96	0.96	0.97	0.97	0.98	0.99	0.99
40	43	0.99	0.01	0.98	0.98	0.99	0.99	1.00	1.00	1.01
41	44	0.95	0.01	0.94	0.94	0.95	0.95	0.96	0.97	0.97
42	45	0.91	0.01	0.89	0.90	0.90	0.91	0.91	0.92	0.92
43	46	0.83	0.01	0.81	0.81	0.82	0.83	0.83	0.84	0.84
44	47	0.74	0.01	0.73	0.73	0.73	0.74	0.74	0.75	0.75
45	48	0.72	0.01	0.71	0.71	0.72	0.72	0.73	0.73	0.73
46	49	0.70	0.01	0.68	0.68	0.69	0.70	0.70	0.71	0.71
47	50	0.69	0.01	0.67	0.68	0.68	0.69	0.69	0.70	0.70
48	51	0.74	0.01	0.72	0.73	0.73	0.74	0.74	0.75	0.75
49	52	0.81	0.01	0.79	0.80	0.80	0.81	0.81	0.82	0.83
50	53	0.86	0.01	0.84	0.85	0.85	0.86	0.87	0.87	0.88
51	54	0.86	0.01	0.84	0.84	0.85	0.86	0.87	0.87	0.88
52	55	0.77	0.01	0.76	0.76	0.77	0.77	0.78	0.79	0.79
53	56	0.71	0.01	0.69	0.69	0.70	0.71	0.71	0.72	0.72
54	57	0.71	0.01	0.69	0.69	0.70	0.71	0.71	0.72	0.72

Estimates of the instantaneous reproduction number R

Time periods		Posterior R moments		Main R Quantiles						
Start	End	Mean	Std	0.025 quantile	0.05 quantile	0.25 quantile	Median	0.75 quantile	0.95 quantile	0.975 quantile
55	58	0.78	0.01	0.76	0.76	0.77	0.78	0.79	0.79	0.80
56	59	0.86	0.01	0.84	0.84	0.85	0.86	0.87	0.88	0.88
57	60	0.93	0.01	0.91	0.91	0.92	0.93	0.94	0.95	0.95
58	61	0.92	0.01	0.90	0.90	0.91	0.92	0.93	0.94	0.94
59	62	0.85	0.01	0.83	0.83	0.84	0.85	0.86	0.87	0.87
60	63	0.76	0.01	0.74	0.74	0.75	0.76	0.77	0.78	0.78
61	64	0.77	0.01	0.75	0.75	0.76	0.77	0.78	0.79	0.79
62	65	0.82	0.01	0.80	0.80	0.81	0.82	0.83	0.84	0.84
63	66	0.87	0.01	0.85	0.85	0.86	0.87	0.88	0.89	0.89
64	67	0.99	0.01	0.97	0.97	0.98	0.99	1.00	1.01	1.01
65	68	1.00	0.01	0.98	0.98	0.99	1.00	1.01	1.02	1.02