Optimization of Energy Systems using MILP and RC Modeling: A Real Case Study in Canada

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***Abstract*—In today's world, where technology is rapidly evolving and people are becoming more concerned about cost and comfort, the demand for electricity for heating, ventilation, and air conditioning (HVAC) systems is on the rise. This puts a strain on efficient power grid management. Demand response strategies have become a critical solution to address this issue. One method is to utilize the inherent inertia of HVAC systems to regulate energy consumption intelligently. This paper presents an optimization model specifically designed for the Varennes Library in Quebec, which is a Net-Zero building. The purpose of the model is to evaluate the energy adaptability of the building and explore the interplay between various systems, such as a building-integrated photovoltaic array, a geothermal heat pump, and a thermal storage-equipped radiant floor system. The approach used to develop the model is based on a mixed-integer linear programming problem, which incorporates a resistance-capacitance (RC) model that describes the building's thermal behavior. The model is validated through on-site measurements, which enable a rigorous quantitative analysis of the building's potential flexibility, particularly under different weather conditions. The study assesses proposed control strategies in contrast to baseline measures, providing valuable insights into efficient energy management within the context of the Varennes Library.**

***Index Terms*—Demand response, resistance-capacitance model, flexibility, energy management system, optimization.**

NOMENCLATURE

I. INTRODUCTION

Buildings are responsible for almost 40% of energy consumption in the United States[[1]. As such, it is crucial to address this issue to reduce greenhouse gas emissions and combat climate change, given that building energy usage accounts for a third of all greenhouse gas emissions [2]. It is also worth mentioning that energy consumption in buildings varies throughout the day, with a significant portion of electricity production capacity (20%) being reserved for peak demand that is only utilized 5% of the time [3]. Heating, Ventilation, and Air Conditioning (HVAC) systems consume a significant amount of energy in commercial and institutional buildings, up to 40% [4]. In this context, the appropriate application of control strategies in HVAC systems is a key factor in improving energy efficiency [5, 6].

Providing sufficient electricity to meet the increasing needs of society is a complicated issue that presents numerous challenges to modern power grids, impacting their dependability and safety. To address these challenges, experts have recognized the effectiveness of demand response (DR) [7], which offers a cost-efficient solution. DR strategically shifts power consumption from peak times to off-peak hours, lowering peak demand on the grid and improving its operational stability and resilience. When it comes to modeling complex systems, there are three approaches: white box, black box, and grey box [8]. Resistance-capacitance (RC) models are one type of grey-box model, which integrates physics-based insights with system identification methods for parameter estimation [9]. They rely more on data and exhibit greater adaptability than deterministic models [10]. Additionally, grey-box models retain essential characteristics of physical processes, enabling effortless integration of new dynamics, such as a new heating system, provided that the requisite physical parameters have been identified.

Various modeling techniques are employed in optimizing demand response, such as the ones utilized by Pedersen et al.[11] and Knudsen et al. [12]. These techniques involve using mixed-integer linear programming (MILP) with grey-box models representing systems to provide demand response profiles and set points in temperature-controlled apartments, with the model parameter being estimated from EnergyPlus models or real building data. Green and Garimella [13] also performed residential electricity cost optimization by addressing HVAC demand issues through a combination of black-box and grey-box models, utilizing a genetic algorithm. Additionally, a reductionist white-box approach has been frequently used in multiple studies to facilitate MILP-compatible single-node models [14, 15, 16].

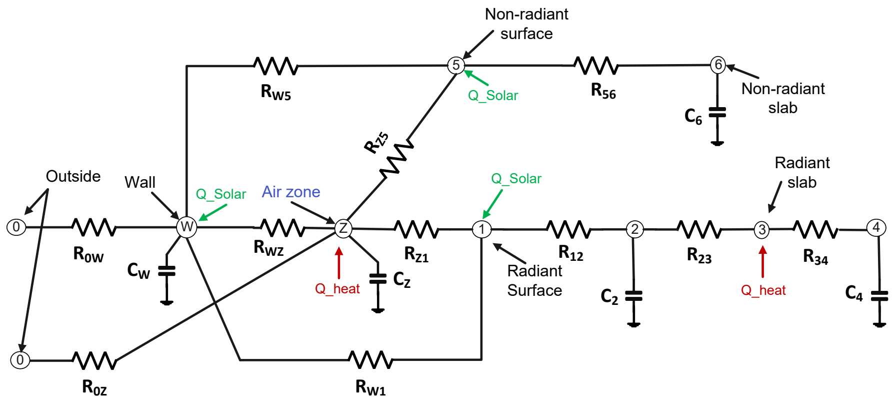

Figure 1. RC model of Varennes library.

C$_w$

II. MATHEMATICAL MODEL

When designing control-oriented models, it is essential to achieve a balance between detailed process requirements, precise modeling, and high time resolution while also minimizing inputs and maximizing ease of use, reliability, and computational efficiency. To meet these criteria, many turn to thermal RC models that rely on the heat balance equations. However, given their high computational demands, lower-order models are frequently recommended, with parameters identified through a range of system identification techniques.

A. RC Modeling

The grey-box model is a widely adopted approach for regulating building indoor temperatures, also known as the RC model. Its key feature is to model the various heat-transfer components (including walls and windows) as a set of resistances and capacitances, which enables accurate representation of heat transfer and storage more effectively. The zone mass is modeled as capacitors, representing thermal storage. The RC model developed in this paper is the lower-order representation of the 11th-order RC model presented in [17]. Figure 1 represents an 11R5C model consisting of five capacitors and eleven resistances.

The RC model predicts how the temperature changes in one area based on its connection with neighboring zones, how heat is exchanged among them, and the material properties involved. Such equations are commonly utilized in analyzing and forecasting temperature fluctuations in diverse settings, from building environments to broader environmental systems. The Zone temperature T_t^Z of the building depends on the previous time step T_{t-1}^Z as follows:

$$T_t^Z = T_{t-1}^Z + \left(\frac{U_{WZ}(T_t^W - T_t^Z)}{k_{ef} * C_Z} + \frac{U_{Z2}(T_t^Z - T_t^2)}{k_{ef} * C_Z} + \frac{U_{Z5}(T_t^Z - T_t^5)}{k_{ef} * C_Z} + \frac{U_{0Z}(T_t^0 - T_t^Z)}{k_{ef} * C_Z}\right) \cdot \Delta t \quad (1)$$

The equation (1) models the change in temperature (T_t^Z) of a zone at a given time (t). It considers the temperature variances between the zone and other linked zones (T_t^W, T_t^2, T_t^5, T_t^0), along with parameters indicating the transfer of heat between these zones (U_{WZ}, U_{Z2}, U_{Z5}, U_{0Z}), the heat storage capacity (C_Z), and the rate of temperature change (Δt).

$$T_t^W = T_{t-1}^W + \left(\frac{U_{0W}(T_t^0 - T_t^W)}{k_{ef} * C_W} + \frac{U_{W6}(T_t^W - T_t^6)}{k_{ef} * C_W} + \frac{U_{WZ}(T_t^W - T_t^Z)}{k_{ef} * C_W}\right) \cdot \Delta t \quad (2)$$

The equation (2) represents the temperature evolution in a wall (T_t^W) over time t, considering its previous temperature T_{t-1}^W, heat transfer between connected areas (T_t^0, T_t^6, T_t^Z), material properties (k_{ef}, C_W), and the time step Δt.

The equations (3)-(8) presented here describe how temperatures change over time in multiple nodes. The system has eight nodes, each representing a temperature variable from $T1$ to $T6$. The equations take into account various factors, such as the thermal properties of the material, the heat transfer coefficients between zones, and the temperatures of adjacent zones in the previous time step. For example, the equation for $T1$ at time t considers the temperature differences between $T1$ and its neighboring zones ($T2$, Z, and W) and their respective conductance values. Similarly, the other equations explain the temperature changes in their respective nodes based on neighboring zones and their thermal characteristics. These equations are crucial for studying heat transfer and thermal behavior in complex structures or environments.

$$T_t^1 = T_{t-1}^1 + (U_{Z1}(T_t^Z - T_t^1) + U_{12}(T_t^1 - T_t^2) + U_{W1}(T_t^W - T_t^1)) \cdot \Delta t \quad (3)$$

$$T_t^2 = T_{t-1}^2 + \left(\frac{U_{12}(T_t^1 - T_t^2) + U_{23}(T_t^2 - T_t^3)}{k_{ef} * C_2}\right) \cdot \Delta t \quad (4)$$

$$T_t^3 = T_{t-1}^3 + (U_{23}(T_t^2 - T_t^3) + U_{34}(T_t^3 - T_t^4)) \cdot \Delta t \quad (5)$$

$$T_t^4 = T_{t-1}^4 + \left(\frac{U_{34}(T_t^3 - T_t^4)}{k_{ef} * C_4}\right) \cdot \Delta t \quad (6)$$

$$T_t^5 = T_{t-1}^5 + (U_{W5}(T_t^W - T_t^5) + U_{Z5}(T_t^Z - T_t^5) + U_{56}(T_t^5 - T_t^6)) \cdot \Delta t \quad (7)$$

$$T_t^6 = T_{t-1}^6 + (\frac{U_{56}(T_t^5 - T_t^6)}{k_{ef} * C_6}) \cdot \Delta t \quad (8)$$

III. Building Energy Management System

A. Buying and Selling power from Hydro Quebec

Equation (9) ensures that the net power consumption of the appliances is equal to the power bought from the grid, as shown below:

$$P_t^{Dem,G} = P_t^{BL} + P_t^{HP} - P_t^{PV,H}, \qquad \forall t \in \mathcal{T} \quad (9)$$

In (9), the different terms represent the power demand of the base load, the power demand of the heat pump, and the power supply to the house from the PV. The building energy management system (BEMS) can also sell power to Hydro Quebec and reduce the electricity cost according to the Feed-in Tariff (FiT).

$$P_t^{Expt,G} = P_t^{PV,G} \qquad \forall t \in \mathcal{T} \quad (10)$$

In (10), the right-hand side is PV supply surplus power to the grid.

B. Heat Pump Operational Constraint

$$P_t^{HP} = \frac{Q_t^{HP,G}}{COP^{HP}} \qquad \forall t \in \mathcal{T} \quad (11)$$

Equation (11) represents the operational constraint of a heat pump system at time t. It calculates the electrical power consumption P_t^{HP} of the heat pump as the ratio of the thermal energy output $Q_t^{HP,G}$ and the Coefficient of Performance (COP^{HP}).

$$Q_{min,t}^{HP} \leq Q_t^{HP} \leq Q_{max,t}^{HP}, \quad \forall t \in \mathcal{T} \quad (12)$$

In equation (12) sets bounds on the thermal energy output Q_t^{HP} of the heat pump at time t. It ensures that the thermal output remains within the specified minimum ($Q_{min,t}^{HP}$) and maximum ($Q_{max,t}^{HP}$) limits.

These equations are integral in modeling and governing the behavior of a heat pump system, ensuring that it operates within specified efficiency parameters and thermal output limits across different time intervals.

C. PV Operational Constraint

The BEMS distributed power generated by the PV to the grid and building is described below in the form of the following equation:

$$P_t^{PV} = P_t^{PV,G} + P_t^{PV,H}, \quad \forall t \in \mathcal{T} \quad (13)$$

D. Temperature Constraint

$$T_{min,t}^{air} \leq T_t^{air} \leq T_{max,t}^{air}, \quad \forall t \in \mathcal{T} \quad (14)$$

The equation (14) enforces constraints on the air temperature T_t^{air} at time t. It ensures that the air temperature remains within the specified minimum ($T_{min,t}^{air}$) and maximum ($T_{max,t}^{air}$) limits for all time periods t in the set $\mathcal{T}$. These constraints are vital for maintaining comfortable and safe indoor environmental conditions.

E. Objective function

The primary goal of a BEMS is to either reduce the operating costs of the building or energy consumption (maximize self-consumption) or maximize export. These objectives can be mathematically modeled through the application of optimization equations as follows:

1) Cost minimization:

$$Obj_1 = \sum_{t \in T} (P_t^{Dem,G} \cdot \lambda_t^{TOU} - P_t^{Expt,G} \cdot \lambda_t^{FiT})\tau \quad (15)$$

2) Maximizing self-consumption of the building:

$$Obj_2 = \sum_{t \in T} (P_t^{BL} + P_t^{HP} - P_t^{PV,H})\tau \quad (16)$$

3) Maximizing Exported energy:

$$Obj_3 = \sum_{t \in T} (-P_t^{Expt,G})\tau \quad (17)$$

The objective of minimizing a household's energy costs is represented by Equation (15). The first component $P_{h_k,t}^{Dem,G}$ reflects the prosumer purchased power from the grid, with the cost represented flex-rate of Hydro Quebec pricing λ_t^{TOU} from the grid [18]. The second term in (15) represents the power generated by each household's DERs that are sold back to the grid and is indicated by $P_{h_k,t}^{Expt,G}$, with the revenue represented by the selling price λ_t^{FiT}. Equation (16) presents the energy demand minimization or maximizing the self-consumption, which utilizes all the PV power to the building ($P_t^{PV,H}$). The objective function (17) maximizes the power exported to the grid.

IV. Problem Formulation

In our study, we introduce an integrated formulation that merges the RC model, a standard methodology for characterizing building thermal dynamics, within the context of a MILP optimization model known as the BEMS. This integration aims to harness the strengths of both approaches. The RC model provides a detailed representation of the building's thermal behavior, offering insight into how heat propagates through the building. Our focus controllable variable, the heat pump output, serves as the pivotal link between the RC model and the optimization problem. It acts as the modulating factor, allowing the optimization model to influence the building's thermal dynamics by regulating the power consumption of the heat pump.

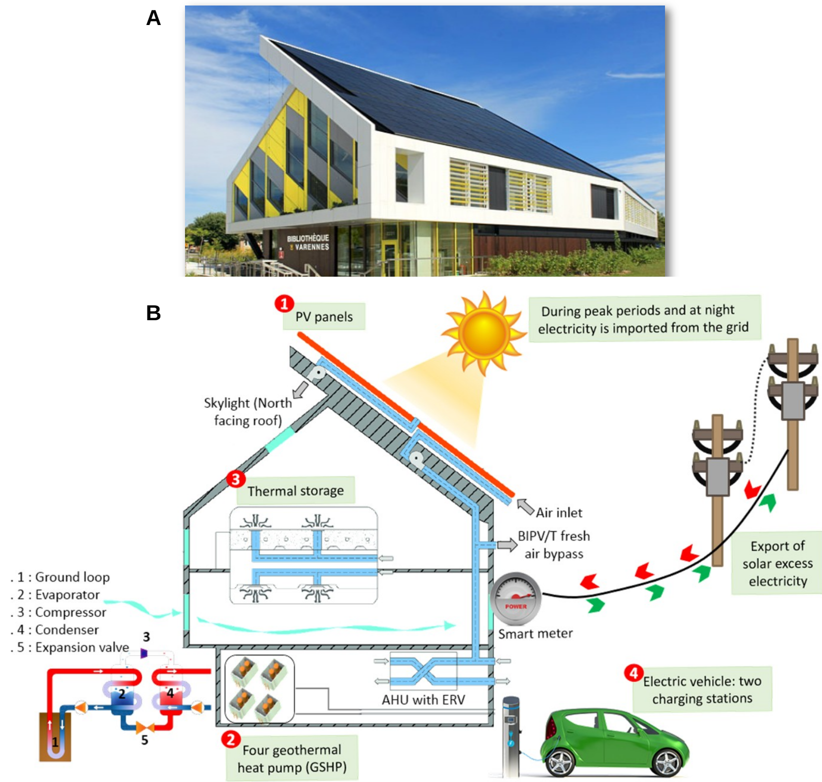

Figure 2. Front view (A) and plan view (B) of the case study building [19].

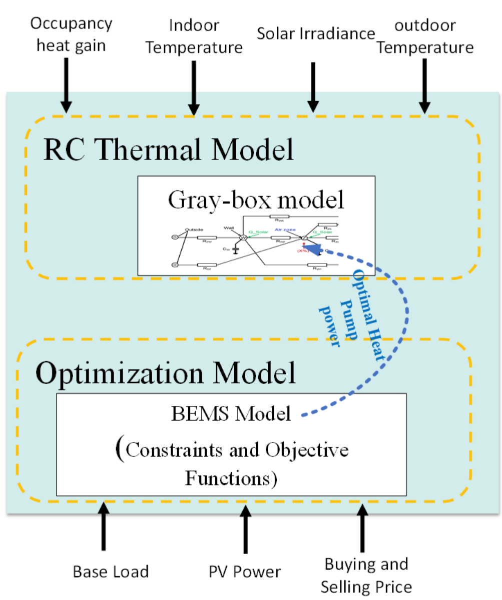

Figure 3. Unified framework of MILP with RC model.

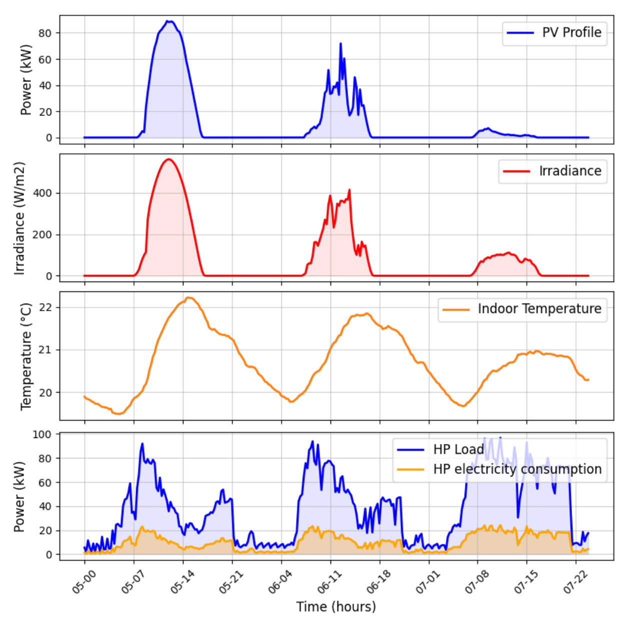

Figure 4. Measurement data of typical cloudy and sunny days at Varennes library.

V. RESULT AND DISCUSSIONS

A. CASE STUDY

The incorporation of flexible operation and control of the thermal and electrical system within buildings has the potential to offer valuable flexibility services to the grid. With buildings being the largest energy consumers worldwide, there is an increasing focus on enhancing energy flexibility within the

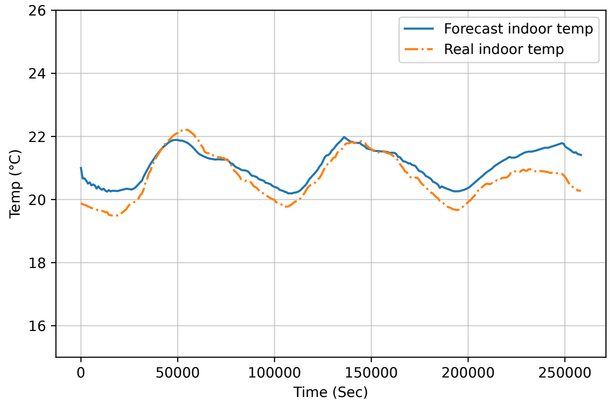

Figure 5. Indoor Temperature.

building sector. These buildings are designed to be energy-efficient and require less energy import from the grid than traditional buildings due to their integrated generation and storage systems. Additionally, they can offer a significant source of energy flexibility. Most zero-energy buildings feature energy storage, on-site renewable generation systems, efficient HVAC systems, and building automation systems that enable advanced control logic, data collection, and remote control. The Varennes Library is a prime example of such a building, being the first institutional Net-Zero Energy Building in Canada when it opened in 2015 [20]. The library incorporates most of these technologies, making it an excellent model for non-residential buildings with similar performance.

The developed formulation takes advantage of data collected from the various smart meters and sensors available within the building. This data encompasses information about outdoor temperature T_{out}, global horizontal solar radiation, heat pump load, and indoor temperature at 15-minute intervals. Indoor temperatures vary widely because they include nodes with different temperature set points. These measured data were recorded for three years, but only the three-day sunny, partly sunny, and cloudy days of the 5th, 6th, and 7th of February 2018 are considered in this study, as illustrated in figure 4.

B. RC Model

Using the proposed 11R5C model presented in this paper, the forecast indoor temperature is compared with the reference real indoor temperature as shown in figure 5. From figure 5 it can be seen that the proposed RC model follows the real temperature of the indoor and slab temperature within acceptable tolerances. The absolute error between them is lower than 5%.

C. BEMS Model

In this work, we compare the three objectives, which are minimizing operating cost, maximizing self-consumption, and export power maximization, to provide different profiles for the bidding market. From figure 6 to 8, the red curve represents objective 1 (cost minimization), the blue curve represents objective 2 (energy demand minimization), and the green curves

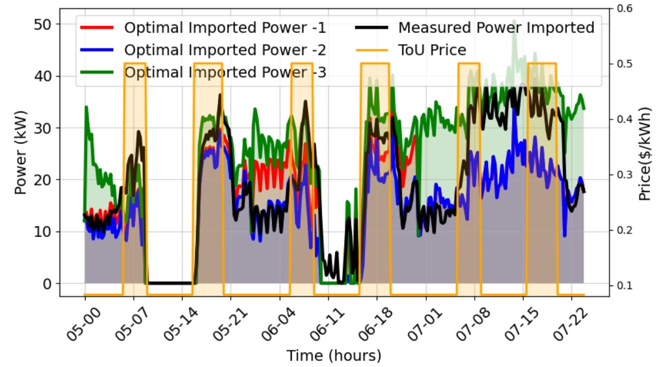

Figure 6. Power imported from the grid using objectives 1,2 and 3..

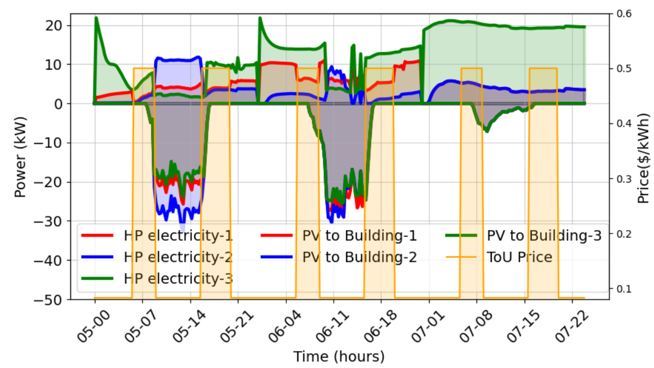

Figure 7. Controllable load profile using objectives 1,2 and 3.

present the third objective (maximizing revenue). Figure 6 presents the power profiles of the imported power from the grid using three objectives, with measured consumption serving as the baseline. From these profiles, it can be seen that power consumption is reduced when the peak price occurs using objectives 1 and 2, while the power imported from the grid is higher using the objective of maximizing power export.

From figure 7, it can be seen that the heat pump, which is a shiftable load, reduces the power consumption during high price intervals using the cost minimization objective. Figure 7 also shows that using maximizing local energy consumption, the PV supply to the building is high, and heat pump power is high when there is local production. When there is no solar, the heat pump is kept as much as possible to reduce power consumption.

Figure 8 shows the optimal indoor temperature of the building compared to the measured indoor temperature for those days, which is used as a non-optimized baseline. Cost minimization results in a reduction of the indoor temperature at peak price time intervals. Similarly, in energy minimization, the indoor temperature increases when solar power is there, and another interval is kept at a lower bound.

As in this case study, the heat pump is a flexible load, which reduces morning and evening peak power using cost and energy minimization. However, the exported power maximization increases the peak power as shown in table I.

In the table I, "*Cost Min*" shows cost minimization strategies

Table I
FLEXIBILITY OF IMPORTED POWER FOR THREE DAYS (SUNNY, PARTLY SUNNY, AND CLOUDY DAY)

Objective Function	Days	Peak reduction (%)	Peak reduction (%)
Cost Min	Sunny Day (5th)	43	19
	Partly Sunny day (6th)	37	14
	Cloudy day (7th)	26	29
Energy Min	Sunny Day (5th)	41	21
	Partly Sunny day (6th)	37	18.5
	Cloudy day (7th)	26.4	36
Export Max	Sunny Day (5th)	20.5	-9.37
	Partly Sunny day (6th)	-37.5	-20.5
	Cloudy day (7th)	-17	-20.9

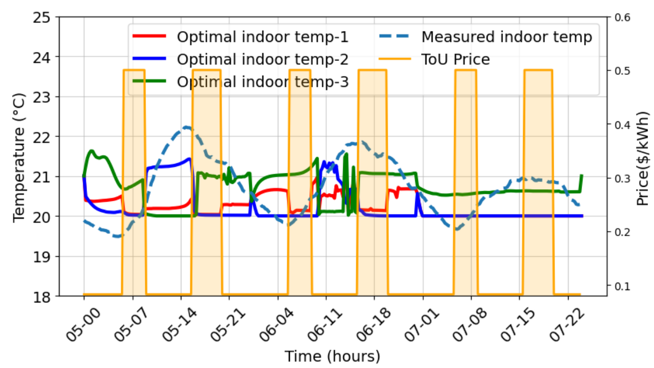

Figure 8. Indoor temperature profile using objectives 1,2 and 3.

for different weather scenarios. "*Energy Min*" presents findings for minimizing energy consumption. "*Export Max*" provides insights on maximizing power export under these weather conditions. Corresponding percentages of peak power reduction are documented for each case. Exported power maximization increases the power consumption instead of reduction by -11.7%, -28.5%, -12.5%, -17%, and -20.9%, as shown in table I. These red highlights indicate an increase in power consumption.

This table shows how the different optimization goals reduce peak power consumption in varying weather conditions.

VI. CONCLUSION

This study proposes a new way to assess energy flexibility in net-zero energy institutional buildings that use renewable energy sources, heat pump systems, and thermal storage. A new method has been presented to measure energy flexibility in high-performance buildings that utilize renewable energy technologies and heat pump systems. To achieve this, a combined MILP with an RC model was developed. The model predicts indoor air temperatures to optimize the operation of geothermal heat pumps and slab thermal storage. The results were used to evaluate the energy flexibility of the building by minimizing peak load and energy consumption during critical peak periods of the grid. The proposed strategies reduce the heat pump power and thus reduce the temperature in peak hours. Optimized profile outputs for a sunny day showed a significant reduction in peak load (by 43%, 41%, and 19%, 14% during morning and evening peak on sunny days using cost and energy minimization objectives), confirming the methodology's capability and potential to develop flexibility.

The next steps involve integrating a true simulation of the building to understand the true ability of the optimization. Following that, a test on the real building and the systems also would validate the formulation proposed. Finally, other future work should test the impact of the market and the impact of the bidding on the optimization. These are the most important three things to study as part of future work.

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