

# Optimal control of a regional power microgrid network driven by wind and solar energy

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**Abstract** - In this paper, a model to support optimal decisions in a network of microgrids is formalized as an original discrete and centralized problem defined here as cooperative network of smart power grids problem. The control variables are the instantaneous flows of power in the network of microgrids, which can be obtained from the solution of a linear quadratic Gaussian (LQG) problem on a fixed time horizon. The state is represented by the energy stored in each microgrid. The goal is to minimize the variations of the energy stored in each storage device from a reference value, as well as to minimize the exchange of power between the microgrids. An application of the model is proposed taking into account wind and solar data in three sites in Liguria region (Castellari, Capo Vado and Savona). It is assumed that each microgrid is composed by a renewable hybrid energy system, a cluster of 100 households and a storage device.

**Keywords:** *renewable energy; optimal control; power microgrids; storage system; LQG control*

## I. INTRODUCTION

In accordance with fossil fuel depletion, increased air pollution, climate changes and the rapid economical and societal development, global concerns about renewable energy resources systems alternatives have grown significantly. In fact, renewable energy resources have been perceived serious attention in last decades due to their great potential in leading to a sustainable energy systems. The main problem of these energy resources is that their energy are intermittent, thus making the power generated fluctuant and uncertain. In addition, in terms of electric power, the fluctuating power of renewable energy might degrade power quality such as voltage and frequency [1]. Thus, technological and methodological approaches are needed to make power grids more flexible and agile. In this term, the microgrid is being analyzed as a feasible alternative to solve these issues. The concept of the microgrid involves an interconnection of distributed energy resources, electric loads, energy storage system and thus can be defined as a small electric power system [2]. The microgrid can operate in both configurations, connected to the main grid or operated in an autonomously manner. Through its configuration, the microgrid offers the continuously operation of the distributed

energy resources driven by renewable energy sources, and with the support of energy storage system, the microgrid has the ability to store energy locally in periods of low demand, and to acquire it back in period of peaks and high demand.

In addition, the microgrid has the possibility to use a cooperative exploitation of distributed generation renewable energy sources with others grid even if they are far from each others. This cooperation permits to exchange information on the predicted demand and production, as well as power in order to enhance the overall service provision. In this context, each microgrid can deliver power to the local load from its own power sources in an independently way. Alternatively, it can choose to get (or send) power either from other microgrid or from the main grid to supply its load when its local sources are insufficient or fail.

Many countries develop and use renewable energy as a key strategic direction in the process of constructing smart power grid. In fact, for instance, solar and wind renewable energy's power scale is growing, so the knowledge of the consumption of the generated output and simultaneously ensure system's security, stability and economy are new problems for developing smart grid [3, 4]. The management of operation mechanism of renewable energy sources (solar and wind) have significant role in power generation planning, specifically, including forecast of data, supporting the management associated with renewable energy such as demand, energy storage and others links [5].

In the literature, many authors have studied the technological and methodological development of microgrids systems. Li et al, [6] studied the stability of microgrid operation and discussed the control techniques of combining micro turbine and fuel cell, hydrogen tank, and electrolyzer system hybrid System to expand the microgrid system as ability for solving power quality issue of frequency fluctuation. Yu et al, [7] proposed a hybrid control architecture to balance the power shared among the multiple interfacing inverters and optimize the system-operating efficiency. Chung et al, [8] presented control schemes for coordination of multiple microgrid generators, especially with voltage source inverter type interface, for both grid-connected and autonomous modes. Zhu et al, [9] proposed a novel multi-step coordinated control

approach for microgrid management considering economic and environmental aspects.

Dagdougui et al., [10] have introduced a dynamic decision model for the real time control of hybrid renewable energy production systems, which can be particularly suitable for autonomous systems, such as islands or isolated villages. In their paper, the demand of energy is coupled with the demand of water and hydrogen, where the hydrogen and water reservoirs also work as storage energy systems.

Zhao et al, [11] have addressed the problem of how to split an operating grid into islands to best serve current loads with operating sources, thus in order to find out where to split the grid to best serve demand. Molderink et al, [12] presented a three-step control methodology to manage the cooperation between distributed generation, distributed storage, and demand-side load management.

This paper outlines the problem of a cooperation of a network of microgrid, where each one exploits renewable energy sources as a source of power generation, and exchange real-time information on power demand and local energy production. The connections among microgrids might be used to exchange power and to decrease the variability of the load of the power exchanged with the connection to the main grid. The objective is to minimize the power exchanges among the grids, and to make each local storage system works around a proper optimal value. The problem is formalized as a model to support optimal decisions in a regional network of renewable power microgrids. The model has been solved for a network of three microgrids in a regional scale.

## II. OPTIMAL CONTROL OF THE ENERGY STORAGE IN A COOPERATIVE NETWORK OF SMART POWER GRIDS

The microgrids are modeled as a network of power generation, where each microgrid is composed by a renewable hybrid power generation, an energy storage system and a cluster of households. The grid has the possibility either to put or to get power from one or more connections to other grids, thus under the external and internal power flows adopted.

The local power production of the grid is RES, mainly solar and wind based, whose power production is supposed to be fully exploited. It is also supposed that the user demand can be fully satisfied. The main decisions are whether to store instantaneous exceeding energy production or to send it to some of the grid connections, or, alternatively, in case of lack of energy, whether it is convenient to acquire energy from some other grids or to use (if any) the energy stored in the local energy reservoir.

These decisional aspects - specifically under a collaborative framework which is of interest for example in a regional network of microgrids - and their optimal solution are described hereinafter. Under this modeling vision, under a terminology viewpoint, each grid will be referred hereinafter as a “smart power microgrid” (or simply as a “microgrid”), the connection to the local energy provider will be referred to as a connection to the “main grid”, and the overall set of microgrids connected among them and to the main grid will be referred to as the “network”.

Each smart power microgrid is supposed to be connected to a regional network of similar grids, and, at least for one microgrid, to one main grid. This network is modeled as a directed graph  $G = (V, E)$ , where  $V$  is the set of vertex with cardinality  $S$ , representing either microgrids or the main grid,

and  $E$  is the set of edges with cardinality  $W$ , representing the power connections existing among the vertexes. As a convention, the  $S$ -th node is associated to the main grid. It is worthwhile to underline that real world power grids are sparsely connected, and  $B$  will generally be associated to small-world networks [13].

The microgrids are supposed to be subject to the following discrete time state equation:

$$z_{t+1} = Az_t + (A - I)\hat{z} + (in_t - out_t + Bu_t)\Delta t \quad t = 0..T - 1$$

$$z_0 = z0$$

(1)

where

- $u_t \in R^W$  (kW), decisional variables, is the vector of power flows sent to (or received by, when the element of  $u_t$  is a negative value) other grids in time interval  $(t, t + 1]$ . Specifically, the generic  $j$ -th element is the directed flow of power in link  $j$  in time interval  $(t, t + 1]$ .
- $z_t \in R^{S-1}$  (kWh), state variables, is the vector of energy storage inventory of each microgrid at instant  $t$ . Specifically, the generic  $i$ -th element is the energy inventory in the storage of microgrid  $i$  at time instant  $t$ , with respect to an optimal working level  $\hat{z}$  for the storage technology present in that microgrid. In this respect, it is supposed that each element of  $z_t$  may assume both positive and negative values.
- $in_t \in R^{S-1}$  (kW) is the vector of stochastic processes of power flow in input to the microgrid in time interval  $(t, t + 1]$  as given by the renewable energy sources exploited in each microgrid.
- $out_t \in R^{S-1}$  (kW) is the vector of stochastic power demand processes of each microgrid in time interval  $(t, t + 1]$ .
- $0 \leq \alpha \leq 1$  is a parameter representing the efficiency of the device used to store energy.
- $A$  is a  $(S - 1) \times (S - 1)$  diagonal matrix describing, in each diagonal element  $\alpha_{ii}$ , the efficiency of the energy storage technology in the  $i$ -th grid. In this respect, it holds that  $0 \leq \alpha_{ii} \leq 1$ .
- $B$  is the  $(S - 1) \times W$  incidence matrix, representing the network topology, such that each element  $b_{i,j} = -1$  if there is an edge (that is a power connection) leaving the  $i$ -th microgrid, 1 if it enters the  $i$ -th microgrid and 0 otherwise.
- $\Delta t$  is the optimization time interval.

Under the hypothesis that  $in_t$  and  $out_t$  can be forecasted on a given interval  $[0, T]$ , with a certain degree of uncertainty, both of them can be split into their deterministic (respectively  $in_t^d$  and  $out_t^d$ ) and stochastic (respectively  $\omega_t^{in}$  and  $\omega_t^{out}$ ) vector components as described in equation (2) and (3):

$$in_t = in_t^d + \omega_t^{in} \quad (2)$$

$$out_t = out_t^d + \omega_t^{out} \quad (3)$$

Let  $e_t$  be the vector of energy balance in each microgrid given by

$$e_t = in_t - out_t \quad (4)$$

Under a simplifying hypothesis, let  $e(t)$  be a vector whose elements are represented by a Gaussian white noise, where:

$$E\{e_t\} = \eta_t = in_t^d - out_t^d \quad (5)$$

So,  $\mu_t$  is a completely known function on the given interval  $[0, T]$ , while  $\omega_t$ , defined in (6), is a zero-mean normal distribution vector with variance  $n$ , not correlated with  $\mu_t$ .

$$\omega_t = e_t - \eta_t \quad (6)$$

Under the assumption quoted above, and, to simplify the notation, assuming  $\Delta t = 1h$ , equation (1<sup>i</sup>) represents the state equation of the energy storage of the microgrid network:

$$\begin{aligned} z_{t+1} &= Az_t + Bu_t + \mu_t + \omega_t \quad t = 0..T-1 \\ z_0 &= z0 \\ \mu_t &= \eta_t + (A - I)\hat{z} \end{aligned} \quad (1^i)$$

where here:

- $\mu_t \in R^{S-1}$  is a vector whose  $i$ -th element is the deterministic energy balance of microgrid  $i$  in time interval  $(t, t+1]$ , as resulting from the difference of the predictions of RES power supply and user demand.
- $\omega_t \in R^{S-1}$  is a vector whose  $i$ -th element is the stochastic error in the prediction of the deterministic energy balance  $\mu_t$  of microgrid  $i$  in time interval  $(t, t+1]$ , and modeled as a zero-mean normal distribution vector with variance  $n$ , not correlated with  $\mu_t$  and  $u_t$ .

Supposing that there is a perfect knowledge of the state of each local storage, and under a cooperative strategy among the grids, whose aim is to maintain an optimal level of energy in the distributed local storage systems, as well as to achieve a low flow of power among the grids, the following objective is formulated.

$$\min J(z, u) = E \left\{ \sum_{t=0}^{T-1} c(z_t, u_t) + z_T' M_T z_T \right\} \quad (7)$$

$$c(z_t, u_t) = z_t' M z_t + u_t' N u_t \quad (8)$$

where

- $M$  is a  $(S-1) \times (S-1)$  matrix, related to the cost of an exceeding/lacking quantity of energy stored in each energy storage device. This matrix is supposed to be  $M > 0$ , and constant for each instant  $t \neq T$ .
- $M_T, M_T > 0$ , has the same definition of  $M$ , but it is only defined for instant  $t = T$ .

- $N$  is a  $W \times W$  matrix,  $N > 0$ , related to the cost of the power sent on each edge of the network, whose elements are constant for each time interval  $t$ .

The problem is so completely defined by the cost function (7) and by equation (8), subject to the state system (1<sup>i</sup>).

The problem is a linear quadratic Gaussian (LQG) problem, that is a “non-standard” LQG due to the presence of the known input  $\mu_t$  in the state equation.

**Theorem 1:** For the problem, defined by (1<sup>i</sup>), (7) and (8) the optimal control is:

$$u_t^* = K_t(z_t - z_t^{d2}) + K_t^g g_{t+1} \quad (9)$$

- $K_t$  is a  $W \times (S-1)$  matrix given by

$$K_t = -(N + B'P_{t+1}B)^{-1}(B'P_{t+1}A) \quad (10)$$

where  $P_{t+1}$  is a  $(S-1) \times (S-1)$  matrix given by the discrete time algebraic Riccati equations (DARE):

$$\begin{aligned} P_t &= M + A'P_{t+1}(I + BN^{-1}B'P_{t+1})^{-1}A \\ P_T &= M_T \end{aligned} \quad (11)$$

- $K_t^g$  is a  $W \times (S-1)$  matrix given by

$$K_t^g = (N + B'P_{t+1}B)^{-1}B' \quad (12)$$

- the vector  $z_t^{d2}$  is given by

$$\begin{aligned} z_{t+1}^{d2} &= Az_t^{d2} + \mu_t \quad t = 0..T-1 \\ z_0^{d2} &= z0 \end{aligned} \quad (13)$$

- and the vector  $g_t$  is given by

$$\begin{aligned} g_t &= (A' - A'P_{t+1}(I + BN^{-1}B'P_{t+1})^{-1}BN^{-1}B') \\ g_T &= M_T z_T^{d2} \end{aligned} \quad (14)$$

The demonstration of the optimal control is shown in Minciardi and Sacile [14].

### III. A CASE STUDY: A NETWORK OF THREE MICROGRIDS

A case study follows illustrating one possible application of the decisional model quoted in the previous section. Three microgrids each one composed by 100 households (Castellari (G1), Capo Vado (G2) and Savona (G3)) can produce power for their local user community using a mix renewable hybrid energy system. Their current power balance between demand and production is often subject to intermittences mainly due to the uncertainties of wind and solar resources as well as to a user demand significantly varying according to the hours of the day. As noted above, in order to overcome these issues, a local

energy storage is available within each microgrid (hydrogen storage system, batteries, water reservoir,..etc), whose size is adequate to the microgrid needs. It should be noted that the microgrids are connected and the third one is connected to the main grid, with which it may exchange power. Fig. 1 shows the existing connections between microgrids. The decision maker of both grids would like to evaluate whether a cooperative behavior, that is exchanging power between the three microgrids can help to improve the working conditions of the storage technology, and to reduce the overall input/output load to the main grid.

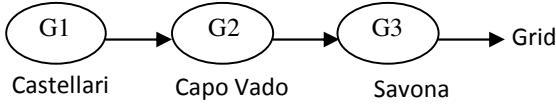


Figure 1. Directed connections planned among the microgrids.

The network to be studied is composed by  $S=4$  vertexes, where the 4-th vertex is associated to the main grid. In the network there are  $W=3$  links, including the new connection whose performance has to be evaluated.

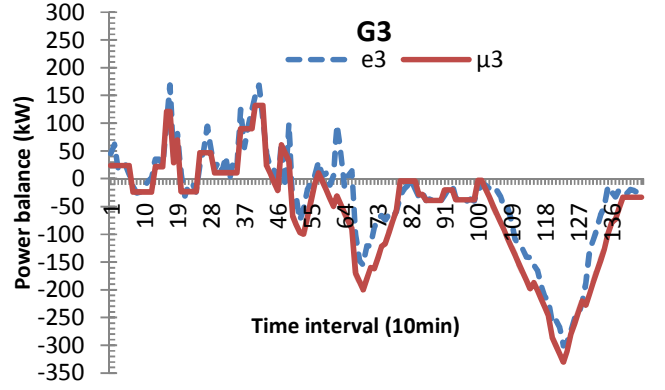
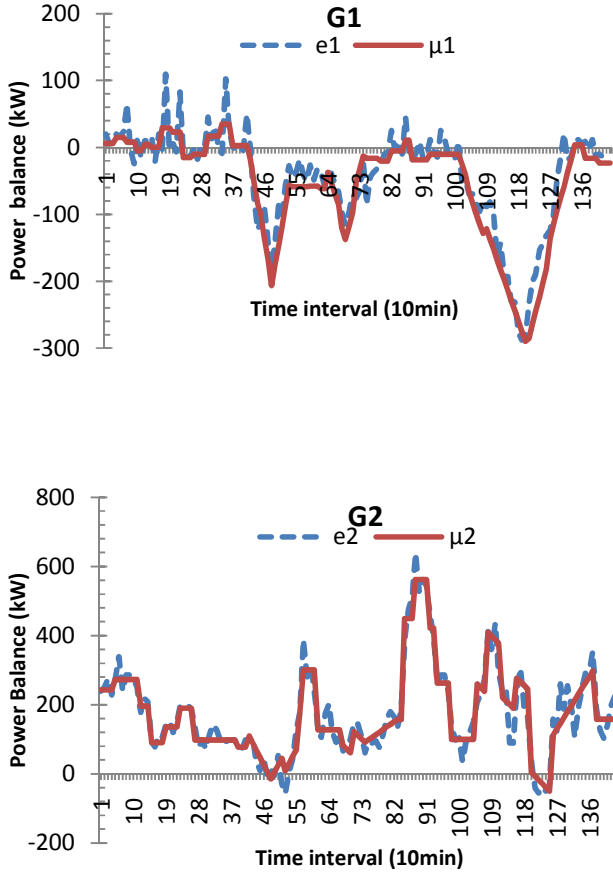


Figure 2 (a, b, c). The prediction of  $\mu_t^i$ , and the a posteriori true energy balance  $e_t^i$  for the microgrid  $i = 1, \dots, 3$  (in top down order)

This evaluation is done on a period of one day (1440 minutes). For the three microgrids, the predictions of the local power balance (that is  $\mu_t$ ) are available. For this case study, the prediction of  $\mu_t^i$ , and the a posteriori true power balance  $e_t^i$  for the microgrid  $i = 1, \dots, 3$  are plotted in Fig.2 (a, b, c). The true power balance is supposed to result from the sum of the predicted energy balance and an additional Gaussian white noise  $\omega_t^i$ .

Here the three microgrids and the main grid are supposed to be cooperative and to be connected according to the network shown in Fig.1, including the dotted connection. The related  $3 \times 3$  incidence matrix  $B$  is as follows:

$$B = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \quad (15)$$

The  $3 \times 3$  matrix  $A$  is as follows:

$$A = \begin{bmatrix} 0.85 & 0 & 0 \\ 0 & 0.85 & 0 \\ 0 & 0 & 0.85 \end{bmatrix} \quad (16)$$

The following  $3 \times 3$   $N$  matrix is defined:

$$N = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 100 \end{bmatrix} \quad (17)$$

Finally, both microgrids have the same inventory pay-off. So, the following  $3 \times 3$   $M$  and  $3 \times 3$   $M_T$  matrices are defined:

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (18)$$

$$M_T = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 100 \end{bmatrix} \quad (19)$$

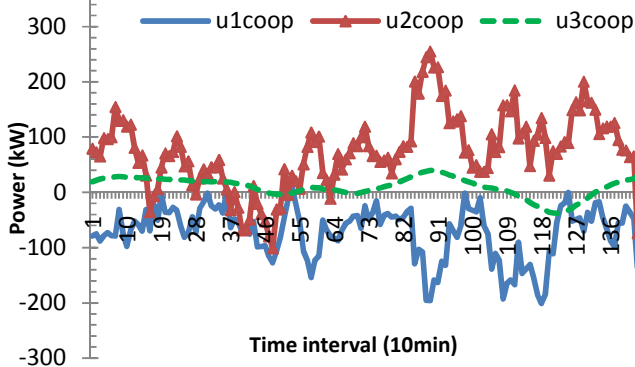


Figure 3. Trends of the optimal values for the  $u_t^{*,i}$   $i=1,2,3$  elements of the control variables under a cooperative strategy (u1coop, u2coop and u3coop in the legend)

Fig. 3 shows the trends of  $u_t^{*,i}$   $i=1,2,3$  while Fig.4 shows the trends of  $z_t^*$ , supposing again an initial value of  $z_0 = [2 \ 4 \ 3]$ .

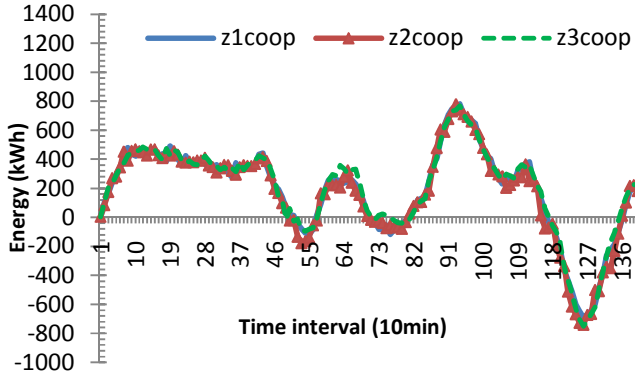


Figure 4. Trends of the optimal values for the state variables  $z^*$  under a cooperative strategy (respectively for microgrids 1,2,3 referred to in the legend as z1coop, z2coop and z3coop)

#### IV. CONCLUSION

The problem described in this study can be considered as a support for decision makers in order to control and optimize the exchange of power in a network of microgrids and grids. The model developed allows determining the optimal flows of power exchanged in real-time according to a linear closed-loop control function. An interesting development of the proposed approach will be devoted to adopt a distributed control

strategy, thus avoiding the delay that may happen in the transmission of power flow and allowing decisions to be taken locally.

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#### REFERENCES

- [1] S. Bando, Y. Sasaki, H. Asano, and S. Tagami, "Balancing control method of a microgrid with intermittent renewable energy generators and small battery storage," IEEE Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century, 2008.
- [2] A. Etxeberria, I. Vechiu, J.M. Vinassa, H. Camblong, "Hybrid Energy Storage Systems for Renewable Energy Sources Integration in Microgrids: A Review," IPEC, Conference Proceedings. 2010.
- [3] Iris Krampitz, "Renewables: making the grid work," Renewable Energy Focus, vol. 9, Issue 6, Supplement 1, pp. 20-21, November-December 2008.
- [4] P.M. Jansson and R.A. Michelfelder, "Integrating Renewables into the U.S. Grid: Is It Sustainable?," The Electricity Journal, vol. 21, Issue 6, pp. 9-21, July 2008.
- [5] Z. M. Huang, "Research of the problems of renewable energy orderly combined to the grid in smart grid," Power and Energy Engineering Conference (APPEEC), Asia-Pacific, IEEE conferences, 2010.
- [6] X. Li, Yu-Jin Song, Soo-Bin Han, "Study on Power Quality Control in Multiple Renewable Energy Hybrid MicroGrid System Power Tech", 2007 IEEE Lausanne, 2007, pp 2000 - 2005 DOI: 10.1109/PCT.2007.4538625.
- [7] X. Yu, A.M. Khambadkone, H. Wang, and STS Terence. "Control of Parallel-Connected Power Converters for Low-Voltage Microgrid—Part I: A Hybrid Control Architecture," IEEE Transaction on Power Electronics, Vol. 25, No. 12, December 2010.
- [8] Il-Yop Chung, W. Liu, DA. Cartes, and K. Schoder, "Control Parameter Optimization for a Microgrid System Using Particle Swarm Optimization," IEEE International Conference on Sustainable Energy Technologies. DOI: 10.1109/ICSET.2008.4747124. 2008, P: 837 – 842
- [9] D. Zhu, R. Yang, G. Hug-Glanzmann, "Managing Microgrids with Intermittent Resources: A Two-Layer Multi-Step Optimal Control Approach," North American Power Symposium (NAPS), 2010.
- [10] H. Dagdougui, R. Minciardi, A. Ouammi, M. Robba, R. Sacile, "A dynamic decision model for the real time control of hybrid renewable energy production systems", IEEE Systems Journal, vol. 4, NO. 3, September 2010.
- [11] Zhao, Q., Sun, K., Zheng, D.-Z., Ma, J., and Lu, Q, "A study of system splitting strategies for island operation of power system: a two-phase method based on OBDDS," IEEE Trans. on Power Systems 18:1556–1565, 2003.
- [12] A. Molderink, V. Bakker, M.G. C. Bosman, J.L. Hurink, G.J. M. Smit, "Management and Control of Domestic Smart Grid Technology," IEEE Transactions on Smart Grid, vol. 1, no. 2, pp.109-118, September 2010.
- [13] Z. Wang, A. Scaglione, and R. J. Thomas, "Generating Statistically Correct Random Topologies for Testing Smart Grid Communication and Control Networks", IEEE Tr. Smart Grid, Vol. 1, No. 1, 28-39, 2010.
- [14] R. Minciardi and R. Sacile, "Distributed energy storage optimal control in a cooperative network of smart power grids", IEEE Systems Journal. 2011, submitted.