Ficha 7

1.

$$(x+y)_n = \sum_{k=0}^{n} \binom{n}{k} (x)_k (y)_{n-k}$$

$$(x)_n = \frac{x!}{(x-n)!} = (x)(x-1)(x-2)\cdots(x-n+1)$$

2. $$\binom{1/2}{3} = \frac{\left(\frac{1}{2}\right)\cdot\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!} = \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6} = \frac{\frac{3}{8}}{6} = \frac{1}{16}$$

$$\binom{-2}{3} = \frac{(-2)(-3)(-4)}{3!} = \frac{6(-4)}{6} = -4$$

3.

$$\binom{x}{2} = \frac{x(x-1)}{2} = \frac{x^2-x}{2}$$

$$\binom{x}{2} = 28 \Leftrightarrow \frac{x^2-x}{2} = 28 \Leftrightarrow x^2 - x = 56 \Leftrightarrow x^2 - x - 56 = 0$$

$$\Leftrightarrow x = \frac{1 \pm \sqrt{1+224}}{2} \Leftrightarrow x = \frac{1 \pm 15}{2} \Leftrightarrow x = -7 \vee x = 8$$

4.

a) $$\binom{-n}{r} = \frac{(-n)(-n-1)\cdots(-n-r+1)}{r!} = \frac{(-1)^n (n)(n+1)\cdots(n+r-1)}{r!}$$

$$= \frac{(-1)^n (n+r-1)!}{r!\,(n+r-1-r)!} \underset{a=n+r-1}{=} \frac{(-1)^r (a)!}{r!\,(a-r)!} = (-1)^r \binom{a}{r} = (-1)^r \binom{n+r-1}{r}$$

b)

$$(1+x)^{-n} = \frac{1}{(1+x)^n} = \frac{1}{(1-(-x))^n} = \sum_{k=0}^{\infty} \binom{n+k-1}{k} (-x)^k$$

$$= \sum_{k=0}^{\infty} \underbrace{\binom{n+k-1}{k}(-1)^k}_{\text{pelo que foi provado em 4a)}} x^k = \sum_{k=0}^{\infty} \binom{-n}{k} x^k$$

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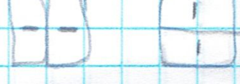
Os azulejos podem estar dispostos 

Ou seja para preencher uma área

2x1: 

2x2: 

2x3: adicionar o  a 2x2

ou  a 2x1, logo

~~Há~~ há ~~2~~ $P_1 + P_2$ maneiras

Para $2 \times n$, então, há: $P_{n-1} + P_{n-2}$

Ora ~~P~~ com $P_1 = 1$

$P_2 = 2$

Ora $F_n = F_{n-1} + F_{n+1}$

mas $F_1 = 1$

$F_2 = 1 = P_1$

$F_3 = 2 = P_2 \Rightarrow P_n = F_{n+1} = F_n + F_{n-1}$

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Para formar 1 numero par, pode-se somar 2 pares

$a + b =$, sendo $a = 2k, k \in \mathbb{N}$

$= 2k + 2i$ $\quad b = 2i, i \in \mathbb{N}$

$= 2(k+i)$

↳ par

Ou 2 impares

$a + b =$ $\quad a = 2k+1, k \in \mathbb{N}$

$= 2k+1+2i+1$ $\quad b = 2i+1, i \in \mathbb{N}$

$= 2k+2i+2$

$= 2(k+i+1)$ par

~~Há~~ logo, ~~como~~

$F_n = F_{n-1} + F_{n-2}$

F_n é par se e só se F_{n-1} e F_{n-2} forem ambos pares ou impares

Considerando os primeiros 5 elementos de F_n

1 1 2 3 5

Verifica-se que não pode haver 2 números pares seguidos, pois quando 1 par aparece, o seu antecedente será um impar, fazendo outro impar

Logo só há pares quando se somam 2 impares, ou seja cada 2 impares há 1 par, ~~pe~~ logo os pares serão F_{3n}

7 $\quad F_n = F_{n-1} + F_{n-2}$, $F_1 = 1$, $F_2 = 1$

$f(x) = \sum_{n=1}^{\infty} F_n x^n = x + x^2 + \sum_{n=3}^{\infty} F_{n-1} x^n + \sum_{n=3}^{\infty} F_{n-2} x^n$

$= x + x^2 + x \sum_{n=3}^{\infty} F_{n-1} x^{n-1} + x^2 \sum_{n=3}^{\infty} F_{n-2} x^{n-2}$

$= x + x^2 + x \sum_{n=2}^{\infty} (F_n x^n) + x^2 \sum_{n=1}^{\infty} F_n x^n$

$= x + x^2 + x \sum_{n=1}^{\infty} (F_n x^n - x) + x^2 f(x)$

$= x + x^2 - x^2 + x f(x) + x^2 f(x)$

$f(x) = \dfrac{x}{1 - x - x^2}$

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$F_n = F_{n-1} + F_{n-2}$

$F_3 = F_2 + F_1 = 2+1 = 3$

$\sum_{j=0}^{2} \binom{2-j}{j} = \binom{2-0}{0} + \binom{2-1}{1} + \binom{2-2}{2} = \binom{2}{0} + \binom{1}{1} + \binom{0}{2} = 1+1+0 = 2$

$F_4 = 3+2 = 5$

$\sum_{j=0}^{3} \binom{3-j}{j} = \binom{3-0}{0} + \binom{3-1}{1} + \binom{3-2}{2} + \binom{3-3}{3} = \binom{3}{0} + \binom{2}{1} + \binom{1}{2} + \binom{0}{3}$
$= 1 + 2 + 0 + 0$
$= 3$ X

Welp

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a)

$L_n = F_{n-1} + F_{n+1} = F_{n-3} + F_{n-2} + F_n + F_{n-1} = F_{n-3} + F_{n-1} + F_{n-2} + F_n$
$= L_{n-2} + L_{n-1}$

$L_0 = 2$
$F_1 = 1$
$F_2 = 1$

$\underbrace{L_0 + L_1}_{L} + L_2 + L_3 + \dots + \underbrace{L_n}_{L_{n+1}} = L_2 + L_2 + L_3 + \dots + L_n$

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Ciclos: $\begin{bmatrix}0\\0\end{bmatrix} = 1$ $\quad \begin{bmatrix}0\\k\end{bmatrix} = 1$ $\quad \sum_{k=0}^{n} \begin{bmatrix}n\\k\end{bmatrix} = n!$ $\quad \begin{bmatrix}n\\k\end{bmatrix} = \begin{bmatrix}n-1\\k-1\end{bmatrix} + (n-1)\begin{bmatrix}n-1\\k\end{bmatrix}$

$\begin{bmatrix}n-1\\2\end{bmatrix} = \begin{bmatrix}n-1\\1\end{bmatrix} + (n-1)\begin{bmatrix}n-2\\2\end{bmatrix}$