Aula 3: Funções hiperbólicas

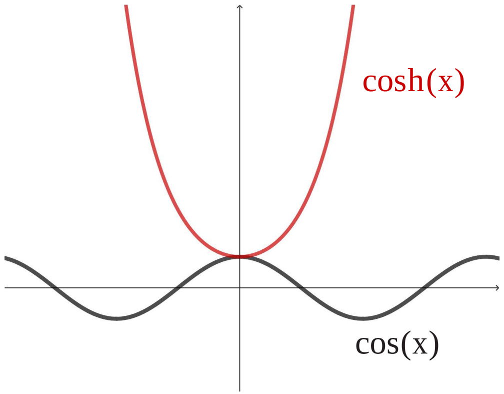
Fórmula de Euler: $\boxed{e^{ix} = \cos(x) + i\,\mathrm{sen}\,(x)}$ → Justificamos depois com a série de Taylor $(i^2 = -1)$

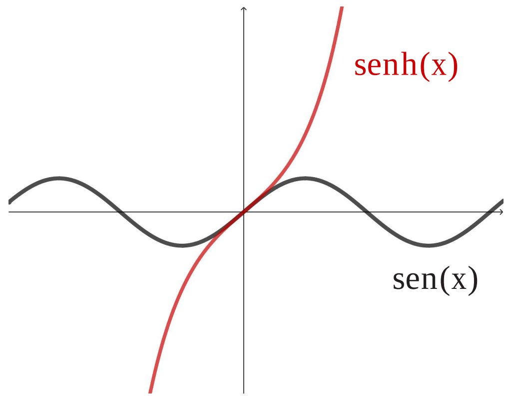
Daqui deduzimos:

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}, \quad \mathrm{sen}\,(x) = \frac{e^{ix} - e^{-ix}}{2i} \tag{1}$$

Eliminando a i nas formulas anteriores obtemos o coseno e o seno hiperbólicos:

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2} \quad , \quad \mathrm{senh}\,(x) = \frac{e^{x} - e^{-x}}{2} \tag{2}$$





$$\frac{e^{i\varphi}+e^{-i\varphi}}{2} = \frac{\cos(\varphi)+i\sin(\varphi)+\left(\cos(-\varphi)+i\sin(-\varphi)\right)}{2} =$$

$$= \frac{\cos(\varphi)+\cancel{i\sin(\varphi)}+\cos(\varphi)-\cancel{i\sin(\varphi)}}{2} = \cos(\varphi$$

Aula 3: Fórmulas hiperbólicas e trigonométricas

$\cosh(x) = \cos(ix)$

$\operatorname{senh}(x) = -i \operatorname{sen}(ix)$

$e^{ix} = \cos(x) + i \operatorname{sen}(x)$

$\cosh^2(x) - \operatorname{senh}^2(x) = 1$

$\operatorname{tgh}(x) := \frac{\operatorname{senh}(x)}{\cosh(x)} = \frac{e^{2x}-1}{e^{2x}+1}$

$\operatorname{cotgh}(x) := \frac{\cosh(x)}{\operatorname{senh}(x)} = \frac{e^{2x}+1}{e^{2x}-1}$

$\cosh(x+y) = \cosh(x)\cosh(y) + \operatorname{senh}(x)\operatorname{sen}(y)$

$\cosh(x-y) = \cosh(x)\cosh(y) - \operatorname{senh}(x)\operatorname{sen}(y)$

$\operatorname{senh}(x+y) = \operatorname{senh}(x)\cosh(y) + \cosh(x)\operatorname{senh}(y)$

$\operatorname{senh}(x-y) = \operatorname{senh}(x)\cosh(y) - \cosh(x)\operatorname{senh}(y)$

$\cosh(2x) = \cosh^2(x) + \operatorname{senh}^2(x)$

$\operatorname{senh}(2x) = 2\operatorname{senh}(x)\cosh(x)$

$(\cosh(x))' = \operatorname{senh}(x)$

$(\operatorname{senh}(x))' = \cosh(x)$

$\cos(x) = \cosh(ix)$

$\operatorname{sen}(x) = -i \operatorname{senh}(ix)$

$e^{x} = e^{i(-ix)} = \cosh(x) + i \operatorname{senh}(x)$

$\cos^2(x) + \operatorname{sen}^2(x) = 1$

$\operatorname{tg} x = \frac{\operatorname{sen}(x)}{\cos(x)}$

$\operatorname{cotg} x = \frac{\cos(x)}{\operatorname{sen}(x)}$

$\cos(x+y) = \cos(x)\cos(y) - \operatorname{sen}(x)\operatorname{sen}(y)$

$\cos(x-y) = \cos(x)\cos(y) + \operatorname{sen}(x)\operatorname{sen}(y)$

$\operatorname{sen}(x+y) = \operatorname{sen}(x+y) = \operatorname{sen}(x)\cos(y) + \cos(x)\operatorname{sen}(y)$

$\operatorname{sen}(x-y) = \operatorname{sen}(x-y) = \operatorname{sen}(x)\cos(y) - \cos(x)\operatorname{sen}(y)$

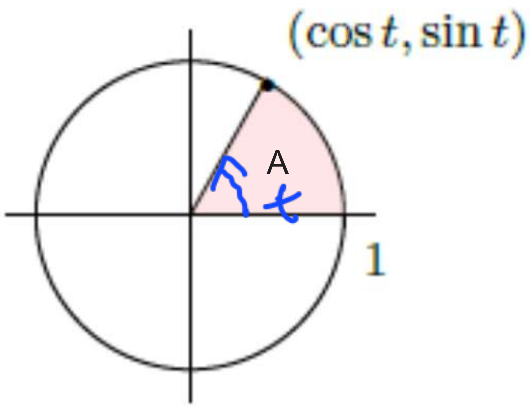
$\cos(2x) = \cos^2(x)\operatorname{sen}^2(x)$

$\operatorname{sen}(2x) = 2\operatorname{sen}(x)\cos(x)$

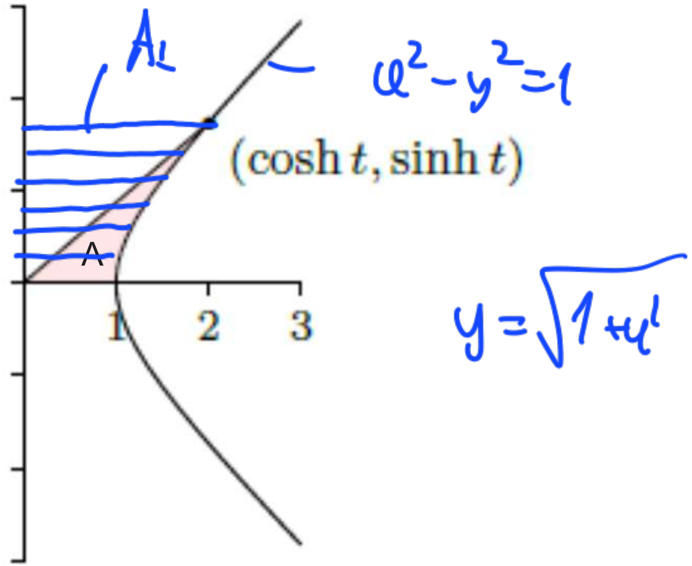
$(\cos(x))' = -\operatorname{sen}(x)$

$(\operatorname{sen}(x))' = \cos(x)$

Aula 3: Semelhanças



circunferência unitária



hipérbole unitária

$$A = \pi \times 1^2 \times \frac{t}{2\pi}$$

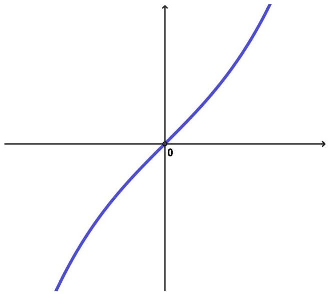
$$= \frac{t}{2}$$

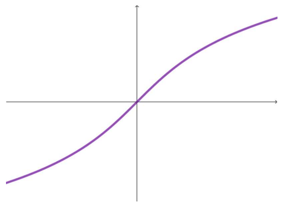
$$A = \frac{t}{2}$$

$$A_1 = \int_0^{\sinh t} \sqrt{1+u^2}\, du$$

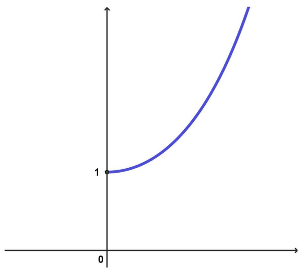
Aula 3: Funções hiperbólicas inversas

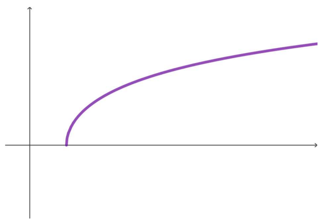
pf: parte de f ao subdominio (f restrita ao subdominio)

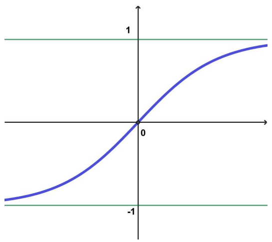
senh(x) 

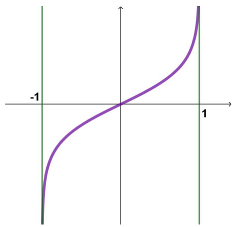
$\xrightarrow{\text{inversa}}$ $\text{senh}^{-1}(x) = \ln(x + \sqrt{x^2+1})$ 

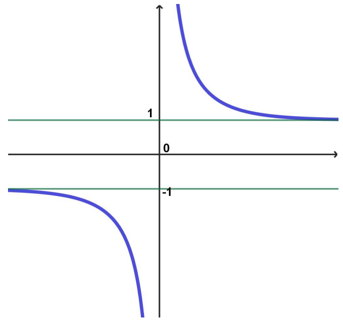
pcosh(x)

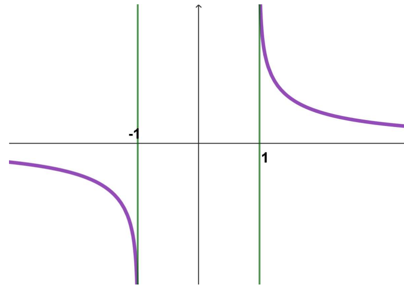
$x \geqslant 0$ 

$\xrightarrow{\text{inversa}}$ $\text{pcosh}^{-1}(x) = \ln(x + \sqrt{x^2-1})$ 

tgh(x) 

$\xrightarrow{\text{inversa}}$ $\text{tgh}^{-1}(x) = \frac{1}{2}\ln(\frac{1+x}{1-x})$ 

cotgh(x) 

$\xrightarrow{\text{inversa}}$ $\text{cotgh}^{-1}(x) = \frac{1}{2}\ln(\frac{x+1}{x-1})$ 

Inversa de $\sinh u = \dfrac{e^u - e^{-u}}{2}$, $D = CD = \mathbb{R}$.

$$y = \frac{e^u - e^{-u}}{2} \iff 2y = e^u - e^{-u} \quad \text{mult. por } e^u$$

$$2ye^u = e^{2u} - e^0 \iff$$

$$(e^u)^2 - 2ye^u - 1 = 0, \quad e^u = z$$

$$z^2 - 2yz - 1 = 0 \iff z = \frac{2y \pm \sqrt{4y^2+4}}{2}$$

$$= y \cancel{\pm}\, 2\sqrt{y^2+1}$$

$$z = y + \sqrt{y^2+1}$$

$$e^u = y + \sqrt{y^2+1}$$

$$u = \ln\left(y + \sqrt{y^2+1} \right)$$