Happy Set problem

Luidi Simonetti

October 2023

1 Maximum Happy Set problem (MaxHS)

For an undirected graph $G = (V, E)$ and a subset $S \subseteq V$ of vertices, a vertex v is happy if v and all its neighbors are in S; otherwise unhappy. Given an undirected graph $G = (V, E)$ and an integer k, the goal of MaxHS is to find a subset $S \subseteq V$ of k vertices such that the number of happy vertices is maximized.

$$
\begin{aligned}
\max \quad & \sum_{i \in V} h_i && && (1)\\
\text{s.t.} \quad & \sum_{i \in V} y_i = k, && && (2)\\
& h_i \leq y_i, && \forall i \in V, && (3)\\
& h_i \leq y_j, && \forall i \in V, j \in N(i), && (4)\\
& h_i \geq \sum_{j \in N(i)} y_j - |N(i)| + y_i, && \forall i \in V, && (5)\\
& y \in \mathbb{B}^{|V|} && && (6)
\end{aligned}
$$