Multiplication Analysis

Report

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The main problem of the research is

To Estimate Complexity and Working Time Of The Grade School Multiplication Algorithm, Divide and Conquer Approach Algorithm and Caratsuba Multiplication Algorithm.

To speak precisely, finding the most optimal algorithm and the conditions of its optimality is the crucial part of this research.

The main reason of finding the most optimal algorithm is limitation of computing capabilities and necessity to minimalize computing time.

The multiplication has been chosen as the object of comparison due to its ubiquity in mathematical issues.

Theoretical Reference:

Grade School Multiplication Algorithm' s complexity is known to be equal O(n^2) in ideal case.

Divide and Conquer Approach Algorithm's complexity is equal O(n*log(n)) in ideal case.

Caratsuba Multiplication Algorithm's complexity is equal $O(n^{\log_2(3)})$ in ideal case.

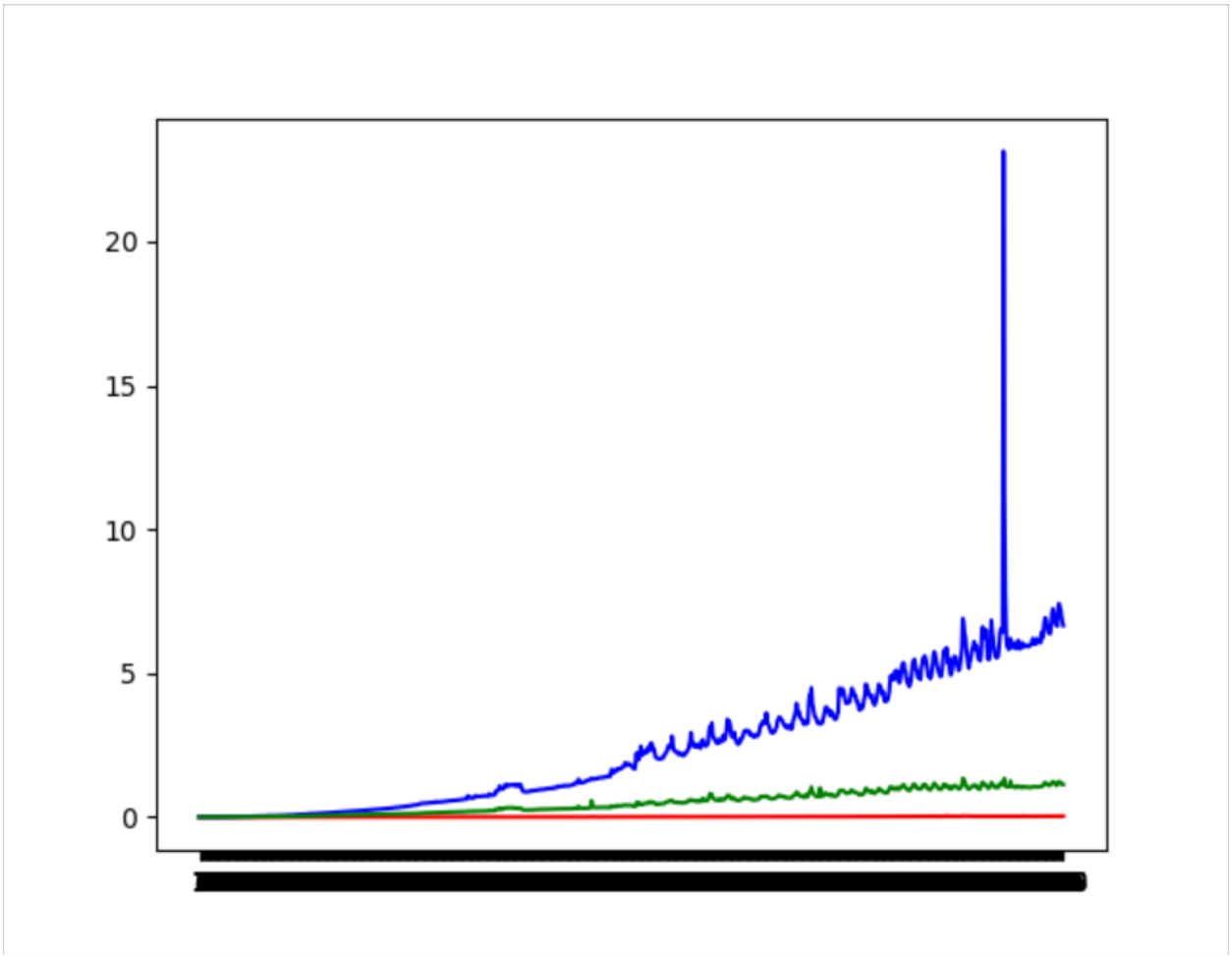
Research plan:

Aim: Estimate 3 algorithms described above and to build plots showing these algorithm's graphs. Compare results.

Plan:

1. Write 3 algorithms and find time of their calculation on numbers of length<=1000. Check time 3 times to make more objective result. Make a table of the results.
2. Build a plot describing calculation time and analyze it.

Analysis



Here’s presented graph which was made according to the plan.

Blue line describes **Divide and Conquer Approach Algorithm.**

Green lines describes **Caratsuba Multiplication Algorithm.**

Red line describes **Grade School Multiplication Algorithm.**

Axis x presents numbers from 1 to 1000.

Axis y presents average time which is spent on the algorithm.

So, it could be seen that

1. Grade School Multiplication is faster on smaller numbers, but there exists such n0 that other algorithms are working faster.
2. Such n0 can be described using ideal asymptotic*C, where is C is a constant factor.
3. Caratsuba Multiplication Algorithm asymptotic is close to its ideal one on such numbers.

4. Divide and Conquer algorithm is ineffective on smaller numbers.
5. Divide and Conquer algorithm is close to n*log(n) asymptotics.
6. Grade School Multiplication algorithm is too good on this graph, but it is not on higher numbers. (It is realized using nested loop = = 2, so its T(n)>=O(n^2).

Result

Graph lines are closed to ideal ones on small numbers, but it is not clear will it be same on higher numbers. Program requires further analysis on higher numbers to find more or less clear results.

Comment(I can't provide any numbers more than thousand due to some hardware problems)(500 number cycle was compiled more than 20 hours).

Used resourses:

-*Wikipedia*

- *Introduction to Algorithms by Thomas Cormen, Charles Leiserson*