CS319: Scientific Computing (with C++)

CS319 Lab 5: Numerical Integration I

Week 7 (22+23 Feb, 2024)

Goal:

- Implement Simpson's Rule;
- Use a Jupyter Notebook to verify its convergence.

You don't have to submit your work this week: we'll develop the idea some more next week to include a Newton-type method for this problem.

Review

1. Review your notes from Weeks 4, 6 and 7 on Numerical Integration.
2. Download and run `04CompareRules.cpp` from Week 6. That has implementations of both the Trapezium and Simpson's Rule. It tests them for $f(x) = e^x$ and for values of N that the user enters.
3. Download and run `00CheckConvergence.cpp` from Week 7. It computes the error for the Trapezium rule applied to this problem for $N = 8, 16, 32, \ldots, 512$.
4. Upload the `CS319-Week07.ipynb` notebook from Week 7 to Jupyter. Pay particular attention to the final section ("Analysing the Quadrature Data") which shows to to verify that, if the error for the Trapezium Rule is

$$E_n \approx CN^{-q},$$

   Then, for this problem, $C \approx 0.13$ and $q \approx 2.0$.

Simpson's Rule

1. Adapt `00CheckConvergence.cpp` from Week 7 so that Simpson's Rule is also implemented (see `Quad2()` from `04CompareRules.cpp`.
2. Use it to compute estimates of the error for Simpson's Rule for various values of N.
3. We suppose again that

$$E_n \approx CN^{-q},$$

   Adapt the Jupyter Notebook to find C and q for Simpson's Rule for this method.

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Boole's Rule

1. George Boole (from Cork) developed a quadrature rule. Read about it at https://en.wikipedia.org/wiki/Boole's_rule
2. Adapt your C++ code so that it also implements the Composite Boole Rule. Note that, in the Wikipedia page, N is used to represent the number of points, whereas we use it to represent the number of intervals. Therefore, in our notation, the formula works for any N that is a multiple of 4.
3. Suppose that Boole's Rule's error as roughly

   $$E_n \approx CN^{-q},$$

   Adapt the Jupyter Notebook to find C and q for Boole's Rule.
4. **Tip:** Boole's method should be more accurate than Simpson's. However, that means it can only be used for relatively small values of N, before round-off error becomes a problem.

Next week...

In next week's lab we'll extend some of these methods to 2D problems. In that setting we'll be interested, not just in the accuracy of the method, bu also the **efficiency** too.

we may also consider the application of Monte Carlo methods to these problems.