Decision tree for basic logic functions

Machine Learning A.A. 2021/2022

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1 Truth table

P	Q	W	AND	OR	XOR
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	0	1	0
1	0	0	0	1	1
1	0	1	0	1	0
1	1	0	0	1	0
1	1	1	1	1	1

2 AND

Step 1

$A = \{P, Q, W\}$ $S = [1+, 7-]$

Since A isn't empty and samples in S aren't all of the same class, i have to select the optimal attribute A and partition S according to the values that the attribute a can take.

$E(S) = -\frac{1}{8}log_2\frac{1}{8} - \frac{3}{8}log_2\frac{3}{8} = 0.543$
Let's calculate the entropy for each attribute:

- $S_{P0} = [0+, 4-]$ $E(S_{P0}) = 0$

  $S_{P1} = [1+, 3-]$ $E(S_{P1}) = -\frac{1}{4}log_2\frac{1}{4} - \frac{3}{4}log_2\frac{3}{4} = 0.811$
- $S_{Q0} = [0+, 4-]$ $E(S_{Q0}) = 0$

  $S_{Q1} = [1+, 3-]$ $E(S_{Q1}) = -\frac{1}{4}log_2\frac{1}{4} - \frac{3}{4}log_2\frac{3}{4} = 0.811$
- $S_{W0} = [0+, 4-]$ $E(S_{W0}) = 0$

  $S_{W1} = [1+, 3-]$ $E(S_{W1}) = -\frac{1}{4}log_2\frac{1}{4} - \frac{3}{4}log_2\frac{3}{4} = 0.811$

Now let's calculate their information gain:

- $G(S, P) = 0.543 - \frac{4}{8} * 0 - \frac{4}{8} * 0.811 = 0.137$
- $G(S, Q) = 0.543 - \frac{4}{8} * 0 - \frac{4}{8} * 0.811 = 0.137$
- $G(S, W) = 0.543 - \frac{4}{8} * 0 - \frac{4}{8} * 0.811 = 0.137$

All information gains are equal and so i don't have an optimal attribute to select but i can pick one without distinguishing.

I create the root node with the attribute P:

Step 2