

Sigmoid derivative

Machine Learning Course A.A. 2022/2023

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October 26, 2022

1 Exercise

Given the sigmoid function defined as follows:

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

I need to prove:

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)(1 - \sigma(x))$$

Proof.

$$\begin{aligned} \frac{\partial \sigma(x)}{\partial x} &= \frac{e^{-x}}{(1 + e^{-x})^2} = \\ &= \frac{e^{-x}}{1 + e^{-x}} \cdot \frac{1}{1 + e^{-x}} = \\ &= \frac{e^{-x} + 1 - 1}{1 + e^{-x}} \cdot \frac{1}{1 + e^{-x}} = \\ &= \frac{1}{1 + e^{-x}} \cdot \left(\frac{1 + e^{-x}}{1 + e^{-x}} - \frac{1}{1 + e^{-x}} \right) = \\ &= \sigma(x)(1 - \sigma(x)) \end{aligned}$$

□