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Exercise Sheet 6

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Discussion Questions (Confidentiality and integrity).

(a) In Exercise Sheet 4, we discussed why IND-CPA security of symmetric encryption does not capture integrity. Why do the notions for UF-CMA security of MACs or INT-CTXT security of symmetric encryption not capture confidentiality?

(b) What is the difference between integrity of plaintexts (INT-PTXT) and integrity of ciphertexts (INT-CTXT) for authenticated encryption? Which of the two do you think is preferable?

Suggested focus. Attempt these problems
before class: Problem 2.
in class: Problems 1 and 4.
in your own time: Problem 3.

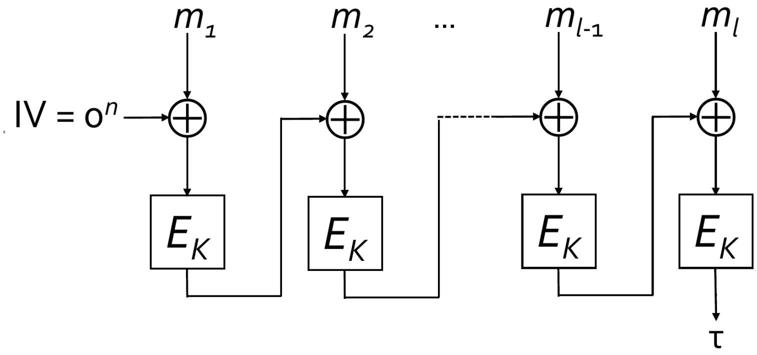
Suggested reading. Reading the following sections in the Boneh-Shoup book [1] might help with the problems on this exercise sheet: Sections 6.2, 6.4, 6.7–6.10 (message integrity), 9.1, and 9.4 (AE).

Problem 1 (CBC-MAC and insecure variants). The CBC mode of operation you saw in the lectures for encryption can also be used to construct a MAC scheme, called CBC-MAC. In this problem we will explore the basic, as well as several insecure, variants of CBC-MAC. In each case a slight change in the specification of CBC-MAC leads to a complete loss of security, meaning it becomes easy to forge tags. Most of these changes look innocent to non-experts, and could end up being implemented. (For further discussion of CBC-MAC and standardized variants, see, e.g., the Boneh-Shoup book [1] Sections 6.4 and 6.7–6.10.)

(a) Basic CBC-MAC. Let $\mathsf{E}\colon \{0,1\}^k \times \{0,1\}^n \to \{0,1\}^n$ be a block cipher. The basic CBC-MAC scheme $\mathcal{I}_{\mathsf{basic}} = (\mathrm{KGen}, \mathrm{Tag}, \mathrm{Vfy})$ is defined by KGen sampling a uniformly random key $\mathsf{K} \leftarrow_\$ \{0,1\}^k$ for E and the following Tag algorithm:

```
Tag(K, m)
$m_1 \| \ldots \| m_\ell \leftarrow m$   // s.t. $\forall i\colon |m_i| = n$
$\mathsf{IV} \leftarrow 0^n$ ; $c_0 \leftarrow \mathsf{IV}$
For $i = 1, \ldots, \ell$ do $c_i \leftarrow \mathsf{E}(\mathsf{K}, m_i \oplus c_{i-1})$
Return $c_\ell$
```

Here is a pictorial representation of Tag, which runs CBC-mode encryption with a zero IV, and outputs the final CBC block c_ℓ as the tag τ.



1) $\mathcal{I}_{\mathsf{basic}}$ is an example of a deterministic MAC scheme with a single tag per message. Write down the verification algorithm Vfy for $\mathcal{I}_{\mathsf{basic}}$.

```
Vfy(K, m, τ)
...
```

2) The basic CBC-MAC scheme is provably SUF-CMA secure (under the assumption that the used block cipher is PRP secure) if messages have the same *fixed* length, e.g., if the message space is $\mathcal{M} = \{0,1\}^{n \cdot z}$ for an arbitrary fixed $z \in \mathbb{N}$.

We now show that the scheme is *insecure* if arbitrary-length messages are allowed. Let $\mathcal{M}$ contain all messages whose length is a multiple of n, meaning $\mathcal{M} = (\{0,1\}^n)^*$. Define an adversary $\mathcal{A}_{\mathsf{basic}}$ that breaks the SUF-CMA security of $\mathcal{I}_{\mathsf{basic}}$ using only one tag query.

Hint: Call tag with an arbitrary single-block message as input.

```
Adversary $\mathcal{A}^{\mathsf{tag}}_{\mathsf{basic}}$
...



Return $(m, \tau)$
```

3) Now assume that an appropriate message space is used, meaning $\mathcal{M} = \{0,1\}^{n \cdot z}$ for an arbitrary fixed $z \in \mathbb{N}$. Show that the basic CBC-MAC becomes insecure if the Tag algorithm uses a random IV (as opposed to a fixed string 0^n). Let $\mathcal{I}_{\mathsf{random\text{-}iv}}$ be the resulting MAC scheme with the Tag algorithm defined below. Define an adversary $\mathcal{A}_{\mathsf{random\text{-}iv}}$ that breaks SUF-CMA security of $\mathcal{I}_{\mathsf{random\text{-}iv}}$, again using only one tag query.

```
Tag(K, m)
$m_1 \| \ldots \| m_\ell \leftarrow m$   // s.t. $\forall i\colon |m_i| = n$
$\mathsf{IV} \leftarrow_{\$} \{0,1\}^n$ ; $c_0 \leftarrow \mathsf{IV}$
For $i = 1, \ldots, \ell$ do $c_i \leftarrow \mathsf{E}(\mathsf{K}, m_i \oplus c_{i-1})$
Return $(\mathsf{IV}, c_\ell)$
```

```
Adversary $\mathcal{A}^{\mathsf{tag}}_{\mathsf{random\text{-}iv}}$
...




Return $(m, \tau)$
```

(b) CBC-MAC with prepended message length. In order to allow messages of variable length (in the number of blocks), the basic CBC-MAC can be tweaked to always prepend the message length (in the number of blocks) to the message that needs to be tagged. This results in a MAC scheme $\mathcal{I}_{\mathsf{prepend}}$ that is based on the following Tag algorithm, requiring that $\mathcal{M} = (\{0,1\}^n)^{\leq 2^n}$ contains full-block messages with at most $2^n - 1$ blocks total. This scheme is SUF-CMA. Here $\mathsf{bin}_n : \{0, \ldots, 2^n - 1\} \rightarrow \{0,1\}^n$ is a function that on input a non-negative integer $i \leq 2^n - 1$ returns the binary n-bit representation of i.

```
Tag(K, m)
$m_2 \| \ldots \| m_\ell \leftarrow m$   // s.t. $\forall i$: $|m_i| = n$
$m_1 \leftarrow \mathsf{bin}_n(\ell - 1)$   // prepend length of $m$
$\mathsf{IV} \leftarrow 0^n$ ; $c_0 \leftarrow \mathsf{IV}$
For $i = 1, \ldots, \ell$ do $c_i \leftarrow \mathsf{E}(\mathsf{K}, m_i \oplus c_{i-1})$
Return $c_\ell$
```

1) Informally motivate why the attack on the basic scheme from part (a) 2) doesn't work on $\mathcal{I}_{\mathsf{prepend}}$.

2) Now let $\mathcal{M} = \{0,1\}^*$, such that $\mathcal{I}_{\mathsf{prepend}}$ needs to be modified to pad the messages prior to processing them. Show that 0-padding leads to an attack against SUF-CMA. Let $\mathcal{I}_{\mathsf{zero\text{-}pad}}$ be a MAC scheme based on the Tag algorithm defined below. Define an adversary $\mathcal{A}_{\mathsf{zero\text{-}pad}}$ that breaks the SUF-CMA security of $\mathcal{I}_{\mathsf{zero\text{-}pad}}$, again using only one tag query.

```
Tag(K, m)
$m_2 \| \ldots \| m_\ell \leftarrow m$   // s.t. $\forall i < \ell$: $|m_i| = n, |m_\ell| \leq n$
$m_1 \leftarrow \mathsf{bin}_n(\ell - 1)$   // prepend block length of $m$
$m_\ell \leftarrow m_\ell \| 0^{n-|m_\ell|}$   // pad with 0s
$\mathsf{IV} \leftarrow 0^n$ ; $c_0 \leftarrow \mathsf{IV}$
For $i = 1, \ldots, \ell$ do $c_i \leftarrow \mathsf{E}(\mathsf{K}, m_i \oplus c_{i-1})$
Return $c_\ell$
```

```
Adversary $\mathcal{A}^{\mathsf{tag}}_{\mathsf{zero\text{-}pad}}$
...


Return $(m, \tau)$
```

(c) Output truncation. Another approach to making CBC-MAC secure for variable-length messages is to not output the full last block c_ℓ as tag, but to return a truncated version of it. Briefly and informally, how does this help to prevent the "chaining" attack we saw on the basic CBC-MAC scheme in part (a) 2)?

Problem 2 (Do not use MACs as hash functions). A MAC is a keyed primitive, whereas a hash function is not keyed. While it may be tempting to build hash functions from secure MACs by picking an arbitrary MAC key and using it as a constant, the example below shows that this approach does not guarantee that the resulting hash function will be collision resistant.

The MAC scheme $\mathcal{I}_{\mathsf{prepend}} = (\mathrm{KGen}, \mathrm{Tag}, \mathrm{Vfy})$ from Problem 1 (b) is SUF-CMA. Let $\mathsf{K} \in \{0,1\}^n$ be an arbitrary fixed key for $\mathcal{I}_{\mathsf{prepend}}$. Define a hash function $\mathsf{H}(x) := \mathrm{Tag}(\mathsf{K}, x)$ for all $x \in (\{0,1\}^n)^*$.

Construct an adversary $\mathcal{A}$ that breaks the collision-resistance of H regardless of which key K is used.
Hint: The hash function H is *not* a keyed primitive, so K is a part of the public description of H and known to $\mathcal{A}$.

Problem 3 (MAC combiner). To obtain cryptographic constructions that are more resilient to break-downs of individual primitives, one sometimes combines two different constructions $\mathcal{I}_1, \mathcal{I}_2$ of the same primitive (e.g., MAC) to build a new construction $\mathcal{I}$ in such a way that $\mathcal{I}$ remains secure as long as one of $\mathcal{I}_1, \mathcal{I}_2$ is secure. This is called a "combiner" construction.

In this problem, we define such a construction for a MAC scheme $\mathcal{I}$ from deterministic MAC schemes $\mathcal{I}_1 = (\mathrm{KGen}_1, \mathrm{Tag}_1, \mathrm{Vfy}_1)$ and $\mathcal{I}_2 = (\mathrm{KGen}_2, \mathrm{Tag}_2, \mathrm{Vfy}_2)$. Your assignment is to prove that the construction remains SUF-CMA secure as long as one of the underlying MAC schemes is SUF-CMA secure. For your proof, you should use the SUF-CMA security model with no verification queries. Assume $\mathrm{Vfy}_1, \mathrm{Vfy}_2$ are built using $\mathrm{Tag}_1, \mathrm{Tag}_2$ as follows:

```
$\underline{\mathrm{Vfy}_i(\mathsf{K}, m, \tau)}$
$\tau' \leftarrow \mathrm{Tag}_i(\mathsf{K}, m)$
If $\tau = \tau'$ then return 1 else return 0
```

Let $\mathcal{I} = (\mathrm{KGen}, \mathrm{Tag}, \mathrm{Vfy})$ be a deterministic MAC scheme such that

$$\begin{aligned}
\mathrm{KGen}() &:= (\mathrm{KGen}_1(), \mathrm{KGen}_2())\\
\mathrm{Tag}((k_1, k_2), m) &:= (\mathrm{Tag}_1(k_1, m), \mathrm{Tag}_2(k_2, m))\\
\mathrm{Vfy}((k_1, k_2), m, (\tau_1, \tau_2)) &:= \text{If } \mathrm{Vfy}_1(k_1, m, \tau_1) \wedge \mathrm{Vfy}_2(k_2, m, \tau_2) \text{ then return 1 else return 0.}
\end{aligned}$$

Problem 4 (Secure authenticated encryption). Let $\mathrm{SE} = (\mathrm{KGen}, \mathrm{Enc}, \mathrm{Dec})$ be a symmetric encryption scheme that provides authenticated encryption (IND-CPA + INT-CTXT). Let SE be defined for key space $\mathcal{K}$, message space $\mathcal{M}$ and ciphertext space $\mathcal{C}$. Which of the following symmetric encryption schemes provide authenticated encryption? For those that do, give a short proof. For those that do not, present an attack that breaks either the IND-CPA or the INT-CTXT security of the new scheme.

You can assume that $\mathcal{M}$ and $\mathcal{C}$ are efficiently samplable, and that $|\mathcal{M}| > 1$ and $|\mathcal{C}| > 1$ (this means that you can sample distinct elements from either set, if necessary). Each of your adversaries should make at most 1 query to the encryption oracle enc (regardless of whether it attacks IND-CPA or INT-CTXT). Each of the new schemes we construct inherits the key generation algorithm KGen from the initial scheme.

(a) Let $\mathrm{SE}_1 = (\mathrm{KGen}, \mathrm{Enc}_1, \mathrm{Dec}_1)$, where algorithms Enc_1 and Dec_1 are defined as follows:

```
$\underline{\mathrm{Enc}_1(k, m)}$                     $\underline{\mathrm{Dec}_1(k, c)}$
$c_0 \leftarrow\!\$\ \mathrm{Enc}(k, m)$          $(c_0, c_1) \leftarrow c$
$c_1 \leftarrow\!\$\ \mathrm{Enc}(k, m)$          $m_0 \leftarrow \mathrm{Dec}(k, c_0)$
Return $(c_0, c_1)$                $m_1 \leftarrow \mathrm{Dec}(k, c_1)$
                                   If $m_0 = m_1$ then return $m_0$
                                   Else return $\perp$
```

Hint: Note that $\mathrm{Enc}(k, \cdot)$ is randomized; running it twice on the same input will result in different outputs with high probability.

(b) Let $\mathrm{SE}_2 = (\mathrm{KGen}, \mathrm{Enc}_2, \mathrm{Dec}_2)$, where algorithms Enc_2 and Dec_2 are defined as follows:

```
$\underline{\mathrm{Enc}_2(k, m)}$
$c \leftarrow\!\$\ \mathrm{Enc}(k, m)$
Return $(c, c)$
```

```
$\underline{\mathrm{Dec}_2(k, c)}$
$(c_0, c_1) \leftarrow c$
$m_0 \leftarrow \mathrm{Dec}(k, c_0)$
If $c_0 = c_1$ then return $m_0$
Else return $\perp$
```

(c) Let $\mathrm{SE}_3 = (\mathrm{KGen}, \mathrm{Enc}_3, \mathrm{Dec}_3)$, where algorithms Enc_3 and Dec_3 are defined as follows:

```
$\underline{\mathrm{Enc}_3(k, m)}$
$c \leftarrow\!\$\ \mathrm{Enc}(k, m)$
Return $(c, c)$
```

```
$\underline{\mathrm{Dec}_3(k, c)}$
$(c_0, c_1) \leftarrow c$
$m_0 \leftarrow \mathrm{Dec}(k, c_0)$
Return $m_0$
```

(d) Let $\mathsf{H}_1 \colon \mathcal{M} \to \{0,1\}^n$ be a collision-resistant hash function for any $n \in \mathbb{N}$. Let $\mathrm{SE}_4 = (\mathrm{KGen}, \mathrm{Enc}_4, \mathrm{Dec}_4)$, where algorithms Enc_4 and Dec_4 are defined as follows:

```
$\underline{\mathrm{Enc}_4(k, m)}$
$c \leftarrow\!\$\ \mathrm{Enc}(k, m)$
$h \leftarrow \mathsf{H}_1(m)$
Return $(c, h)$
```

```
$\underline{\mathrm{Dec}_4(k, (c, h))}$
$m \leftarrow \mathrm{Dec}(k, c)$
$h' \leftarrow \mathsf{H}_1(m)$
If $h' = h$ then return $m$
Else return $\perp$
```

(e) Let $\mathsf{H}_2 \colon \mathcal{C} \to \{0,1\}^n$ be a collision-resistant hash function for any $n \in \mathbb{N}$. Let $\mathrm{SE}_5 = (\mathrm{KGen}, \mathrm{Enc}_5, \mathrm{Dec}_5)$, where algorithms Enc_5 and Dec_5 are defined as follows:

```
$\underline{\mathrm{Enc}_5(k, m)}$
$c \leftarrow\!\$\ \mathrm{Enc}(k, m)$
$h \leftarrow \mathsf{H}_2(c)$
Return $(c, h)$
```

```
$\underline{\mathrm{Dec}_5(k, (c, h))}$
$m \leftarrow \mathrm{Dec}(k, c)$
$h' \leftarrow \mathsf{H}_2(c)$
If $h' = h$ then return $m$
Else return $\perp$
```

(f) Now answer part (e) again, assuming only that SE is IND-CPA secure (but *not* AE secure).

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References

[1] D. Boneh and V. Shoup. *A Graduate Course in Applied Cryptography.* Online, version 0.6 edition, Jan. 2023.