

Problem 4: Denoising altimetric measurements

The file `izoard.txt` contains altitude measurements taken at 200-meter intervals along the renowned 20-kilometer horizontal distance of the road ascent to the *Col de l'Izoard* in the French Alps. We are interested in recovering the continuous altitude profile as a smooth function of the horizontal distance, taking into account that the altitude measurements are subject to noise.

- a) Perform a penalized smoothing of the altitude measurements using a basis of cubic B-splines with breaks at each horizontal distance point, using a smoothing parameter $\lambda = 10^{-1}$. Report the number of splines used and the generalized cross validation (GCV) error.
- b) What is the approximate dimension of the space in which the fitted curve lives? Provide a plot of the fitted curve and its first derivative.
- c) Report the value of λ minimizing the GCV error and the GCV error corresponding to that λ , using a sequence of values in $[10^{-1}, 10^3]$, with a step of 0.5, for the grid search. Fit again the smoothed curve with this λ value.
- d) What is the slope¹ at the steepest part of the ascent?

Upload your results here: <https://forms.office.com/e/1GJZEvfKcR>

¹Tangent of the inclination angle