$$\nabla\cdot\left(\underline{u}\,(\underline{v}\cdot\underline{w})\right) = \partial_i\left(u_i\, v_j\, w_j\right) = (\partial_i u_i)\, v_j\, w_j + u_i\,\partial_i v_j\, w_j + u_i\,\partial_i w_j\, v_j =$$

$$= (\nabla\cdot\underline{u})(\underline{v}\cdot\underline{w}) + (\underline{u}\cdot\nabla)\underline{v}\cdot\underline{w} + (\underline{u}\cdot\nabla)\underline{w}\cdot\underline{v}$$

INTEGRATION + DIVERGENCE THEOREM:

$$\underbrace{\cancel{\int_{\partial\Omega}(\underline{u}\cdot\underline{n})(\underline{v}\cdot\underline{w})}}_{=0} = \left((\nabla\cdot\underline{u})\underline{v}, \underline{w}\right) + \left((\underline{u}\cdot\nabla)\underline{v}, \underline{w}\right) + \left((\underline{u}\cdot\nabla)\underline{w}, \underline{v}\right)$$

$$\Rightarrow \left((\underline{u}\cdot\nabla)\underline{v}, \underline{w}\right) = -\left((\underline{u}\cdot\nabla)\underline{w}, \underline{v}\right) - \left((\nabla\cdot\underline{u})\underline{v}, \underline{w}\right)$$

$$\Rightarrow \boxed{\tilde c(u,v,w) = \left((\underline{u}\cdot\nabla)\underline{v}, \underline{w}\right) + \frac{1}{2}\left((\nabla\cdot\underline{u})\underline{v}, \underline{w}\right)} =$$

$$= -\left((\underline{u}\cdot\nabla)\underline{w}, \underline{v}\right) - \frac{1}{2}\left((\nabla\cdot\underline{u})\underline{v}, \underline{w}\right) = -\tilde c(\underline{u}, \underline{w}, \underline{v})$$

SKEW SYMMETRIC FORM

Also equivalent to $\frac{1}{2}\left[\left((\underline{u}\cdot\nabla)\underline{v}, \underline{w}\right) - \left((\underline{u}\cdot\nabla)\underline{w}, \underline{v}\right)\right]$.