

$$\begin{cases} -\varepsilon \Delta u + \underline{b} \cdot \nabla u = f & \text{in } \Omega \\ u = g & \text{on } \partial\Omega \end{cases}$$

$$u_{\text{ex}} = \sin(\pi x) \cos(\pi y)$$

$$\nabla u_{\text{ex}} = \begin{bmatrix} \pi \cos(\pi x) \cos(\pi y) \\ -\pi \sin(\pi x) \sin(\pi y) \end{bmatrix}$$

$$g := u_{\text{ex}}$$

$$\begin{aligned} \Delta u &= -\pi^2 \sin(\pi x) \cos(\pi y) - \pi^2 \sin(\pi x) \cos(\pi y) = \\ &= -2\pi^2 \sin(\pi x) \cos(\pi y) = -2\pi^2 u_{\text{ex}} \end{aligned}$$

$$\underline{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow \underline{b} \cdot \nabla u_{\text{ex}} = \pi \cos(\pi(x+y))$$

$$f := -\varepsilon \Delta u_{\text{ex}} + \underline{b} \cdot \nabla u_{\text{ex}} = 2\varepsilon \pi^2 u_{\text{ex}} + \pi \cos(\pi(x+y))$$

$$\begin{cases} -\varepsilon \Delta u + \underline{b} \cdot \nabla u = 2\varepsilon \pi^2 u_{\text{ex}} + \pi \cos(\pi(x+y)) & \text{in } \Omega \\ u = u_{\text{ex}} & \text{on } \partial\Omega \end{cases}$$