$$\begin{cases} -\Delta u = f & \text{in } \Omega = (0,1)^2 \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

$$V \ni \varphi \qquad \int_\Omega -\varphi \Delta u = \int_\Omega f \varphi$$

$$\int_\Omega \nabla u \cdot \nabla \varphi - \cancel{\int_{\partial\Omega} \varphi \nabla u \cdot \underline{n}} = \int_\Omega f \varphi \qquad \forall \varphi \in V$$

CHOOSE $\varphi = 0$ on $\partial\Omega$

$V_h \subset V$

$\uparrow$

FIN. DIM.

$$\int_{\Omega_h} \nabla u_h \cdot \nabla \varphi_h = \int_{\Omega_h} f \varphi_h \qquad \underline{\underline{\forall \varphi_h \in V_h}}$$

$$V_h = \text{span}\{\varphi_1, \dots, \varphi_N\}$$

$$\int_{\Omega_h} \nabla u_h \cdot \nabla \varphi_i = \int_{\Omega_h} f \varphi_i \qquad \forall i = 1, \dots, N$$

$$u_h \in V_h \Rightarrow u_h = \sum_{j=1}^N U_j \varphi_j$$

$$\sum_{j=1}^N U_j \underbrace{\int_{\Omega_h} \nabla \varphi_j \cdot \nabla \varphi_i}_{A_{ij}} = \underbrace{\int_{\Omega_h} f \varphi_i}_{F_i} \qquad \forall i = 1, \dots, N$$

$$A\,\underline{U} = \underline{F} \qquad \Omega_h = \bigcup_{K=1}^{N_{el}} K$$

$$A_{ij} = \int_{\Omega_h} \nabla\varphi_i \cdot \nabla\varphi_j = \sum_K \int_K \nabla\varphi_i \cdot \nabla\varphi_j \, dx$$

$$\underset{\substack{\uparrow \\ \text{Mapping}}}{=} \sum_K \int_{\hat{K}} \nabla\hat{\varphi}_i \cdot \nabla\hat{\varphi}_j \, |\det J_K| \, d\hat{x} \approx$$

$$\approx \sum_K \underbrace{\sum_{q=1}^{N_q} \nabla\hat{\varphi}_i(\hat{x}_q) \cdot \nabla\hat{\varphi}_j(\hat{x}_q) \underbrace{|\det J_K(\hat{x}_q)|\, w_q}_{J_x W(\hat{x}_q)}}_{A_{ij}^K \text{ (local matrix)}}$$