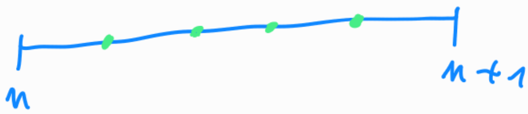
RUNGE-KUTTA METHODS

$$\begin{cases} \dfrac{dy}{dt} = g(t, y) \\ y^0 = \bar{y} \end{cases}$$



$$\longrightarrow y^{n+1} = y^n + K_n \cdot \sum_{i=1}^{s} b_i \, g\left(t_n + c_i K_n, \, y_n + \sum_{j=1}^{s} a_{ij} k_j\right)$$

$\underline{\underline{a}}, \underline{b}, \underline{c}$ BUTCHER TABLEAU

$$k_j = f(\tilde{t}_j, \tilde{y}_j^n)$$

RUNGE-KUTTA FOR TIME-DEPENDENT PDEs WITH FEM

$$\frac{\partial u}{\partial t} = \Delta u + f$$

$$\int_\Omega \frac{\partial u}{\partial t} \varphi_i = \underbrace{\int_\Omega \Delta u \cdot \varphi_i}_{-\int_\Omega \nabla u \cdot \nabla \varphi_i} + \int_\Omega f \varphi_i$$

$$u = \sum_{j=1}^{N} U_j(t) \varphi_j(x) \longrightarrow \frac{\partial u}{\partial t} = \sum_{j=1}^{N} \frac{dU_j}{dt} \varphi_j(x)$$

$$\sum_j \frac{dU_j}{dt} \underbrace{\int_\Omega \varphi_i \varphi_j}_{M_{ij}} = -\sum_j U_j \underbrace{\int_\Omega \nabla \varphi_i \cdot \nabla \varphi_j}_{S_{ij}} + \int_\Omega f \varphi_i$$

$$M \frac{d\underline{U}}{dt} = -S\underline{U} + \underline{F} \rightarrow \boxed{\frac{d\underline{U}}{dt} = M^{-1}(-S\underline{U} + \underline{F})}$$

$\rightarrow$ R-K with $g(t, \underline{y}) = M^{-1}(-S\underline{y} + \underline{F})$

1. ASSEMBLE M (AND INVERT IT) $\rightarrow$ Using UMFPACK Sparse Direct Solver
2. IMPLEMENT FUNCTION g