

HPIPM reference guide

Gianluca Frison

May 11, 2020

Contents

1	Introduction	2
2	Dense QP	3
3	OCP QP	4
3.1	QP dimensions structure	4
3.1.1	Create structure	4
3.1.2	Populate structure	4
3.2	QP structure	5
3.2.1	Create structure	5
3.2.2	Populate structure	5

Chapter 1

Introduction

HPIPM, which stands for High-Performance Interior Point Method, is a library providing a collection of quadratic programs (QP) and routines to manage them. Aim of the library is to provide both stand-alone IPM solvers for the QPs and the building blocks for more complex optimization algorithms.

At the moment, three QPs types are provided: dense QPs, optimal control problem (OCP) QPs, and tree-structured OCP QPs. These QPs are defined using C structures. HPIPM provides routines to manage the QPs, and to convert between them.

HPIPM is written entirely in C, and it builds on top of BLASFEO [1], that provides high-performance implementations of basic linear algebra (LA) routines, optimized for matrices of moderate size (as common in embedded optimization).

Chapter 2

Dense QP

The dense QP is a QP in the form

$$\begin{aligned} \min_{v,s} \quad & \frac{1}{2} \begin{bmatrix} v \\ 1 \end{bmatrix}^T \begin{bmatrix} H & g \\ g^T & 0 \end{bmatrix} \begin{bmatrix} v \\ 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} s^l \\ s^u \\ 1 \end{bmatrix}^T \begin{bmatrix} Z^l & 0 & z^l \\ 0 & Z^u & z^u \\ (z^l)^T & (z^u)^T & 0 \end{bmatrix} \begin{bmatrix} s^l \\ s^u \\ 1 \end{bmatrix} \\ \text{s.t.} \quad & Av = b, \\ & \begin{bmatrix} \underline{v} \\ \underline{d} \end{bmatrix} \leq \begin{bmatrix} J_{b,v} \\ C \end{bmatrix} v + \begin{bmatrix} J_{s,v} \\ J_{s,g} \end{bmatrix} s^l, \\ & \begin{bmatrix} J_{b,v} \\ C \end{bmatrix} v - \begin{bmatrix} J_{s,v} \\ J_{s,g} \end{bmatrix} s^u \leq \begin{bmatrix} \bar{v} \\ \bar{d} \end{bmatrix}, \\ & s^l \geq \underline{s}^l, \\ & s^u \geq \underline{s}^u, \end{aligned}$$

where v are the primal variables, s^l (s^u) are the slack variables of the soft lower (upper) constraints. The matrices J_{\dots} are made of rows from identity matrices. Furthermore, note that the constraint matrix with respect to v is the same for the upper and the lower constraints.

Chapter 3

OCP QP

The OCP QP is a QP in the form

$$\begin{aligned}
 \min_{x,u,s} \quad & \sum_{n=0}^N \frac{1}{2} \begin{bmatrix} u_n \\ x_n \\ 1 \end{bmatrix}^T \begin{bmatrix} R_n & S_n & r_n \\ S_n^T & Q_n & q_n \\ r_n^T & q_n^T & 0 \end{bmatrix} \begin{bmatrix} u_n \\ x_n \\ 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} s_n^l \\ s_n^u \\ 1 \end{bmatrix}^T \begin{bmatrix} Z_n^l & 0 & z_n^l \\ 0 & Z_n^u & z_n^u \\ (z_n^l)^T & (z_n^u)^T & 0 \end{bmatrix} \begin{bmatrix} s_n^l \\ s_n^u \\ 1 \end{bmatrix} \\
 \text{s.t} \quad & x_{n+1} = A_n x_n + B_n u_n + b_n, \quad n = 0, \dots, N-1, \\
 & \begin{bmatrix} \underline{u}_n \\ \underline{x}_n \\ \underline{d}_n \end{bmatrix} \leq \begin{bmatrix} J_{b,u,n} & 0 \\ 0 & J_{b,x,n} \\ D_n & C_n \end{bmatrix} \begin{bmatrix} u_n \\ x_n \end{bmatrix} + \begin{bmatrix} J_{s,u,n} \\ J_{s,x,n} \\ J_{s,g,n} \end{bmatrix} s_n^l, \quad n = 0, \dots, N, \\
 & \begin{bmatrix} J_{b,u,n} & 0 \\ 0 & J_{b,x,n} \\ D_n & C_n \end{bmatrix} \begin{bmatrix} u_n \\ x_n \end{bmatrix} - \begin{bmatrix} J_{s,u,n} \\ J_{s,x,n} \\ J_{s,g,n} \end{bmatrix} s_n^u \leq \begin{bmatrix} \bar{u}_n \\ \bar{x}_n \\ \bar{d}_n \end{bmatrix}, \quad n = 0, \dots, N, \\
 & s_n^l \geq \underline{s}_n^l, \quad n = 0, \dots, N, \\
 & s_n^u \geq \underline{s}_n^u, \quad n = 0, \dots, N,
 \end{aligned}$$

where u_n are the control inputs, x_n are the states, s_n^l (s_n^u) are the slack variables of the soft lower (upper) constraints and \underline{s}_n^l and \underline{s}_n^u are the lower bounds on lower and upper slacks, respectively. The matrices $J_{\dots,n}$ are made of rows from identity matrices. Note that all quantities can vary stage-wise. Furthermore, note that the constraint matrix with respect to u and x is the same for the upper and the lower constraints.

3.1 QP dimensions structure

3.1.1 Create structure

```
int d_ocp_qp_dim_memsize(int N);
```

```
void d_ocp_qp_dim_create(int N, struct d_ocp_qp *qp, void *memory);
```

3.1.2 Populate structure

Once created, an OCP QP dimensions structure can be populated using the global setter routine

```
void d_ocp_qp_dim_set_all(int *nx, int *nu,
    int *nbx, int *nbu, int *ng,
    int *nsbx, int *nsbu, int *nsg,
    struct d_ocp_qp_dim *dim);
```

which is useful when all structure fields have to be populated at once.
Alternatively, it is possible to set the individual structure fields with the setter routine

```
void d_ocrp_qp_dim_set(char *field, int *stage, int value,
    struct d_ocrp_qp_dim *dim);
```

where field can be one of nx, nu, nbx, nbu, ng, nsbx, nsbu, nsg.

3.2 QP structure

3.2.1 Create structure

```
int d_ocrp_qp_memsize(struct d_ocrp_qp_dim *dim);

void d_ocrp_qp_create(struct d_ocrp_qp_dim *dim, struct d_ocrp_qp *qp, void *memory);
```

3.2.2 Populate structure

Once created, an OCP QP structure can be populated using the global conversion routine

```
void d_ocrp_qp_set_all(double **A, double **B, double **b,
    double **Q, double **S, double **R, double **q, double **r,
    int **idxbx, double **lxb, double **ubx,
    int **idxbu, double **lbu, double **ubu,
    double **C, double **D, double **lg, double **ug,
    double **Zl, double **Zu, double **zl, double **zu,
    int **idxs, double **lls, double **lus,
    struct d_ocrp_qp *qp);
```

which is useful when all the structure fields have to be populated at once.
Alternatively, it is possible to set the individual structure fields with the setter routine

```
void d_ocrp_qp_set(char *field, int *stage, void *value,
    struct d_ocrp_qp *qp);
```

where field can be one of A, B, b, Q, S, R, q, r, idxb, lb, ub, Jbx, idxbx, lxb, ubx, Jbu, idxbu, lbu, ubu, C, D, lg, ug, Zl, Zu, zl, zu, idxs, lls, lus.

Bibliography

- [1] G. Frison, D. Kouzoupis, T. Sartor, A. Zanelli, and M. Diehl. BLASFEO: Basic linear algebra subroutines for embedded optimization. *ACM Transactions on Mathematical Software (TOMS)*, 2018. (accepted).