

Disclaimer

This summary is part of the lecture “ETH Semiconductor Devices” by Prof. Dr. Colombo Bolognesi (FS19). It is based on the lecture.

Please report errors to huettern@student.ethz.ch such that others can benefit as well.

The upstream repository can be found at <https://github.com/noah95/formulasheets>

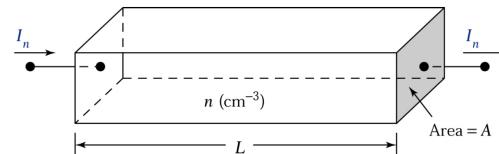
ETH Semiconductor Devices 2019

Noah Huetter

1. April 2019

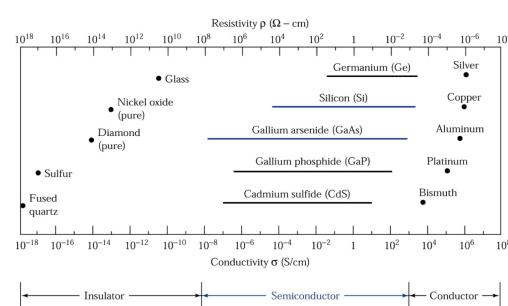
1 Introduction

1.1 Electric resistivity/conductivity

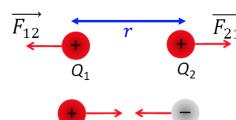


Conductivity σ is a material property describing how easily certain material can conduct electrical current. Resistivity $\rho = 1/\sigma$ describes how much a material opposes the current flow. The resistance of a square/round piece of metal is:

$$R = \rho \frac{l}{A} = \rho \frac{l}{r^2 \pi}$$



1.2 Electron motion



• Electric Force Definition

$$\vec{F}_e = Q \vec{E} \quad \vec{E} : \frac{\text{Newtons}}{C} = \frac{V}{m}$$

• Magneti Force Definition:

$$\vec{F}_m = Q \vec{v} \times \vec{B} \quad \text{not used in course}$$

• Definition of electric field:

$$F_{12} = F_{21} = k \frac{Q_1 Q_2}{r^2} \quad E_{21} = \frac{F_{21}}{Q_2} = k \frac{Q_1}{r^2}$$

• Definition: Current

$$I = \frac{dQ}{dt}$$

1.3 Current flow



1.4 Moore's Law

Gordon Moore predicted that the number of transistors on an integrated circuit doubles about every two years. This is described using exponential growth:

$$p(t) = p_0 \cdot b^{t/\tau}$$

Where

$p(t)$ = population at given time

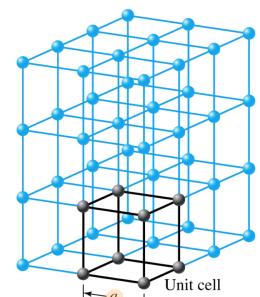
p_0 = initial population

b = growth rate per time constant

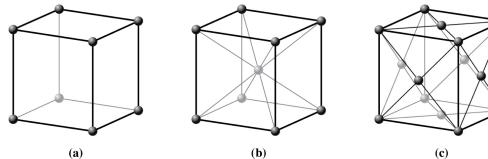
τ = time constant

2 Solid state physics

2.1 Crystal structures



Def (Coordination Number): Is the number of nearest neighbours any atom has in a given crystal lattice. By definition, a crystal lattice is periodic in 3D. a is the lattice constant.



- (a) SC: Simple cubic
- (b) BCC: Body-centered cubic
- (c) FCC: Face-centered cubic

Po
Li, Na, K,
Cr, Fe, NB
Al, Ar, Ni,
Cu, Kr, Pd

rections. Different crystal planes and directions generally have different properties.

Miller indices

$$[abc] = \left[\frac{1}{p}, \frac{1}{q}, \frac{1}{r} \right]$$

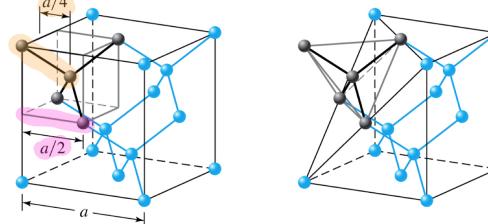
Where p, q, r are the intersections with the x, y, z axis. Miller indices describe the crystal plane.

2.4 Elements

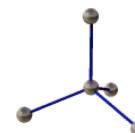
Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	H	S ²⁻																
2	Li	B ³⁺	N ³⁻	O ²⁻	F ⁻													
3	Na	Mg ²⁺																
4	Al	Si	Ge	Sn	Pb													
5	Si	Ge	Zn	Cd	Sn	As	P	S	Se	Te	Bi	As	Se	Te	Ge	As	Se	Te
6	Ge	Zn	As	Ge	As	As	As	As	As	As	As	As	As	As	As	As	As	As
7	Zn	As	Ge	As	Ge	As												
Lanthanides																		
Actinides																		

Non-metals
Halogens
Elementary or Compound Semiconductors (III-V, II-VI)

2.2 Silicon

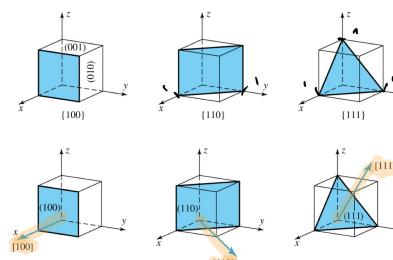


Diamond unit cell: (a) the cubic unit cell, and (b) the inherent tetrahedral structure. The diamond crystal structure is especially important in semiconductors. For Silicon, the nuclear diameter is $7.2\text{ fm} = 7.2 \cdot 10^{-16}\text{ nm}$: Matter is impressively "empty".



Tetrahedral:

2.3 Crystal Planes and Directions



Things don't look the same in all directions. The crystal has different periodicities in different directions, i.e. it does not look the same in all directions.

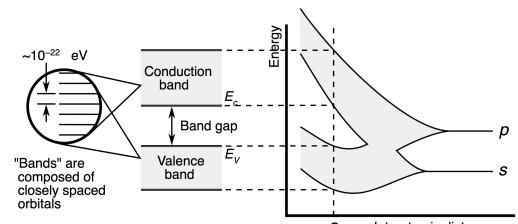
In solids, we are concerned about the atoms' outermost (i.e. valence) electrons because they determine the bonding and the electronic properties.

On the far right in the figure, atoms are well separated and non-interacting. Their energy states are sharp (atomic-like). As they get closer, the outermost (valence) electrons begin to interact and their energy levels start to shift with respect to the isolated value. **The number of states is conserved**. This is a consequence of the **Pauli Exclusion Principle**. Bands can be separated by energy gaps where no electron is permitted to exist.

Partially Filled / Empty Band

Are associated with electrical conduction. As atoms approach each other to form solid, valence electron distributions overlap. Equilibrium distance maximum density of electrons for isolated atoms. Lowering of potential barriers between atoms allows electrons to move freely.

Band Gap



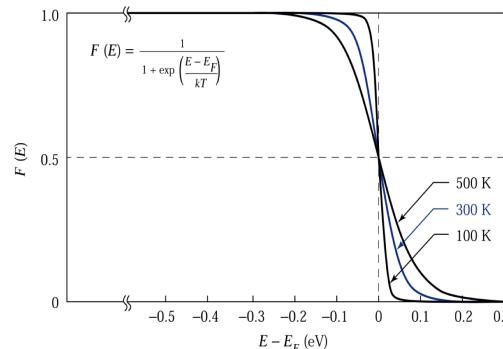
Full Band

Isolation, electrons are there but no net current. At equilibrium atomic separation, bands are separated by a forbidden energy gap. At low T, valence band full, conduction band empty: *i.e.* no current.

2.6 Population of Electron States

Need to know the probability of finding electrons at a given energy to understand how they are distributed among the various states (*i.e.* conduction and valence band). This is done by the **Fermi Dirac Statistics (FD)**. FD statistics enforce the Pauli Exclusion Principle and the minimization of energy.

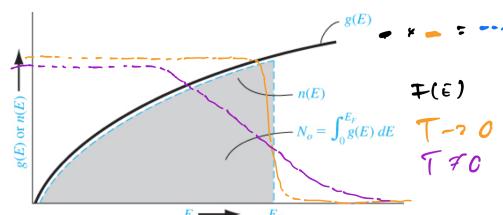
- Probability of finding e^- at energy E is $F(E)$
- Probability of finding a hole at energy E is $1 - F(E)$



$$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

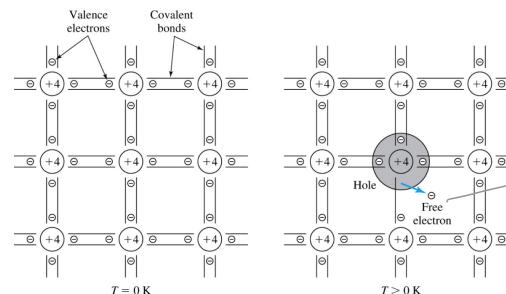
Fermi Level E_F : Energy at which the prob. of finding an e^- /hole is 50%

Density of states



Density of States $g(E)$: How many available states per volume at energy E .

2.7 Intrinsic carriers



Two types of carriers: electrons and holes.

Each Si atom is surrounded by 8 e^- at 0K, 4 come from the atom itself, 1 from each of its 4 nearest neighbours. As T increases, thermal energy eventually excites some e^- out of their bond. Equivalently, the FD distribution broadens around E_F , increasing the prob. that conduction band states become occupied. An empty state (a "hole") is left in the bonding (valence band) state. The valence band is then not full anymore, and it therefore becomes conductive too. **Consequence:** We now have two partially filled bands. Both the conduction and valence bands can now carry current because neither is completely full, not empty.

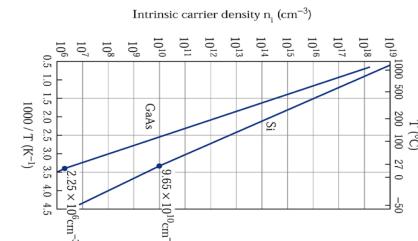
Detailed balance

In equilibrium, n_0 and p_0 are constant but the reaction still operates in both directions. This is called "detailed balance".

$$n_0 \cdot p_0 = K = K_0 \exp\left(\frac{-E_a}{kT}\right) = n_i^2$$

Free electron n_0 and hole p_0 density and activation energy E_a for bond breaking.

Intrinsic carrier density



The intrinsic carrier concentration depends exponentially on temperature T and on the energy gap of the material (Si: 1.12 eV, GaAs: 1.42 eV, @ 300K). Different energy gaps thus have a huge impact on n_i .

n_i becomes large at high T (limits high- T operation of devices)

Electric field

Electrons move opposite to electric field, holes move parallel to E-field.

3 Doping

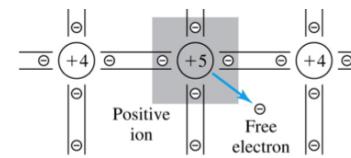
Doping is the introduction of impurities with different atomic valence in the pure crystal. These impurities are called *Extrinsic Carriers*.

E_D	Donor energy level/state
E_c	Conduction band edge
E_v	Valence band edge
E_F	Fermi energy
E_{Fi}	Intrinsic Fermi energy
E_g	Band gap width/energy
m^*	Effective mass of electron(n)/hole(p)
ϵ_0	Vacuum permittivity $8.854 \cdot 10^{-12} \text{ F/m}$
n_i	Intrinsic electron concentration
p_i	Intrinsic hole concentration
$n_i = p_i$	
n_0	Thermal-equilibrium e^- concentration
p_0	Thermal-equilibrium hole concentration
n_d	Concentration of e^- in donor state
p_a	Concentration of holes in acceptor state
N_d	Concentration of donor atoms
N_a	Concentration of acceptor atoms
N_c	Effective density of states
N_v	Effective density of states
N_d^+	Conc. of pos. charged (ionized) donors
N_a^-	Conc. of neg. charged (ionized) acceptors

3.1 Impurity doping

It is distinguished between negative and positive doping.

N-Doping

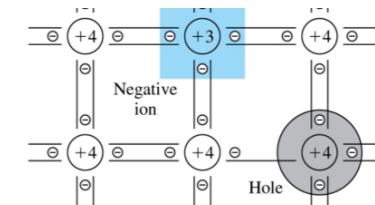


Taking donors from PT col V (P, As, Sb) that have 5 valence electrons. 4 make covalent bond, 5th is "not needed" and looks like a hydrogen atom. It is easily ionized/excited to conduction band. Its energy state is close to CB (E_c). Δ Overall, the solid is still charge neutral but with impurities Δ

Donors introduce an energy state E_D near the conduction band edge E_c . e^- easily promoted to conduction band because E_c , E_D are close compared to E_c , E_v (E_g). Only $E_c - E_D$ is needed to free the

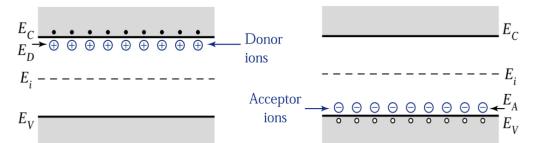
electron. **Extra electrons are added without adding holes.**

P-Doping



Same as n-doping but by adding extra holes. Acceptors from PT col III (B, Al, Ga, In)

Energy Band



n-type doping

donor
State E_D below conduction band

p-type doping

acceptor
State E_A above valence band

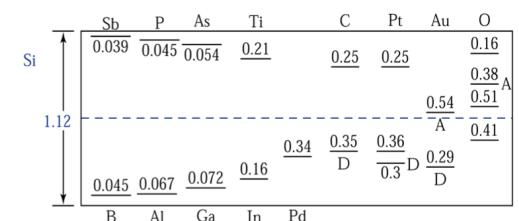


Image shows impurity levels in Silicon in eV. This amount of energy is required to move impurity to conductance/valence band.

Donors: treat as "Hydrogen-Like" atom inside solid. Its ionization energy E_D is modified by the dielectric constant and effective mass m^* .

$$E_d \approx \frac{1}{\epsilon_r^2 m_0} m^* E_H$$

$E_{D,Bohr} = 27 \text{ meV}$ a very good approximation.

Bohr radius

The most probable distance between nucleus and electron in a Hydrogen atom in ground state. For semiconductor we need to modify this value for ϵ_0 and m^* .

$$r'_{\text{Bohr}} = \frac{4\pi\epsilon_r\epsilon_0\hbar^2}{m^* m_0 q^2} = \frac{\epsilon_r}{m^*} 5.3 \cdot 10^{-9} = 31 \text{ \AA}$$

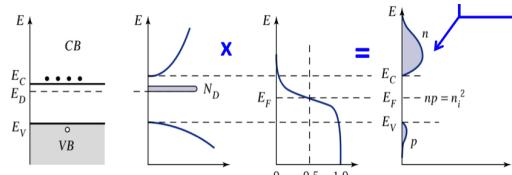
This sphere corresponds to about 6735 Si atoms or 841 unit cells. Is thus very loosely bound to its "parent" impurity atom.

3.2 Electro neutrality

Sum of atoms making up the semiconductor are electrically neutral: The semiconductor thus has zero net charge. In general, both donor and acceptor impurities may be present.

$$n_0 + N_a^- = p_0 + N_d^+$$

3.3 Density of States (DOS)



Density of available quantum states times Fermi-Dirac distribution equals the density of electrons. $D(E_{\text{kin}})$ is the number of electronic states at energy E_{kin} in a range δE_{kin} per cm^3 .

$$D(E_{\text{kin}}) = \frac{8}{h^3} \sqrt{2\pi} (m^*)^{2/3} (E_{\text{kin}})^{1/2}$$

3.4 Population of Electron States

Concentration of electrons and holes ($g(E)$ int Neamen p. 109) is sum of all density of states times Fermi-Dirac probability:

$$\int_{E=0}^{\infty} f(E) \cdot D(E_{\text{kin}}) dE_{\text{kin}}$$

Learn by hard:

$$f(E) = \frac{1}{1 + \exp\left(\frac{E-E_F}{kT}\right)} \approx e^{-\frac{E-E_F}{kT}} \quad \text{if } E \gg E_F$$

$$n_0 = \frac{4\sqrt{2}(\pi m^* kT)^{2/3}}{h^3} e^{-(E_c - E_F)/(kT)} \\ = N_c \cdot e^{-(E_c - E_F)/(kT)}$$

For $E \gg E_F$ the Maxwell-Boltzmann approximation holds.

General case

$$n_0 = \int_{E=0}^{\infty} f(E) \times D(E_{\text{kin}}) dE_{\text{kin}}$$

$$x = \frac{E_{\text{kin}}/kT}{(E_F - E_C)/kT} \\ \eta = \frac{(E_F - E_C)/kT}{}$$

$$D(E_{\text{kin}}) = \frac{8\pi\sqrt{2}}{h^3} (m^*)^{3/2} (E_{\text{kin}})^{1/2} = g(E_{\text{kin}})$$

$$n_0 = \frac{8\sqrt{2}\pi}{h^3} (m^*)^{3/2} \int_{E_C=0}^{\infty} \frac{(E_{\text{kin}})^{1/2}}{1 + \exp[(E_{\text{kin}} - E_F)/kT]} dE_{\text{kin}}$$

$$n_0 = \frac{8\sqrt{2}\pi}{h^3} (m^* kT)^{3/2} \int_{x=0}^{\infty} \frac{(x)^{1/2}}{1 + \exp[x - \eta]} dx = \frac{2N_c F_{1/2}(\eta)}{\sqrt{\pi}} = N_c F_{1/2}(\eta)$$

When the Fermi level E_F is more than $3kT$ below E_c (or above E_v), the full Fermi-Dirac (FD) integral $F_{1/2}(\eta)$ is well approximated by the Maxwell-Boltzmann (MB) approximation e^η . When E_F is closer or even above E_c (below E_v), the material is said to be "degenerate".

Concentration of electrons and holes

Back to (MB).

$$n_0 = N_c \cdot e^{-\frac{E_c - E_F}{kT}} \quad N_c = \frac{4\sqrt{2}(\pi m_n^* kT)^{3/2}}{h^3}$$

$$p_0 = N_v \cdot e^{-\frac{E_F - E_v}{kT}} \quad N_v = \frac{4\sqrt{2}(\pi m_p^* kT)^{3/2}}{h^3}$$

We see that the intrinsic carrier concentration depends on the temperature and the energy gap of the semiconductor:

$$n_0 p_0 = N_v N_c \cdot e^{-\frac{E_c - E_v}{kT}} = n_i^2 = N_v N_c \cdot e^{-\frac{E_g}{kT}}$$

Remember:

$$n_0 p_0 = n_i^2$$

3.5 Fermi Level & Doping

3.6 Book Chapter 4 Summary

Materials at 300K

Material	Si	Ge	GaAs
$n_i^2 (\text{cm}^{-6})$	$9.3 \cdot 10^{19}$	$5.76 \cdot 10^{26}$	$3.24 \cdot 10^{12}$
$N_c (\text{cm}^{-3})$	$2.86 \cdot 10^{19}$	$1.04 \cdot 10^{19}$	$4.7 \cdot 10^{17}$
$N_v (\text{cm}^{-3})$	$1.04 \cdot 10^{19}$	$6.0 \cdot 10^{18}$	$7.0 \cdot 10^{18}$
$E_g (\text{eV})$	1.12	0.66	1.42
m_n^*/m_0	1.08	0.067	0.55
m_p^*/m_0	0.56	0.48	0.37

Intrinsic Band Gap

$$E_g = -kT \ln \frac{n_i^2}{N_c N_v} \quad n_i^2 = N_v N_c e^{-\frac{E_g}{kT}}$$

$$n_0 = N_c e^{-\frac{(E_c - E_F)}{kT}} \quad p_0 = N_v e^{-\frac{(E_F - E_v)}{kT}}$$

Effective Mass

$$N_c = 2 \left(\frac{2\pi m_n^* kT}{h^2} \right)^{3/2} \quad N_v = 2 \left(\frac{2\pi m_p^* kT}{h^2} \right)^{3/2}$$

Intrinsic Carrier Concentration

$$n_i^2 = N_c N_v e^{-\frac{(E_c - E_v)}{kT}} = N_c N_v e^{-\frac{E_g}{kT}}$$

Intrinsic Fermi Level

$$E_{F,i} - E_{\text{midgap}} = \frac{3}{4} kT \ln \left(\frac{m_p^*}{m_n^*} \right) \\ E_{F,i} = \frac{E_v + E_c}{2} + \frac{kT}{2} \ln \left(\frac{N_v}{N_c} \right)$$

Equilibrium distribution of el/holes

$$n_0 = n_i e^{-\frac{E_F - E_{F,i}}{kT}} \quad p_0 = n_i e^{-\frac{(E_F - E_{F,i})}{kT}}$$

The $n_0 p_0$ product

$$n_0 p_0 = n_i^2$$

Statistics of Donors and Acceptors

$$n_d = \frac{N_d}{1 + \frac{1}{2} e^{\frac{E_d - E_F}{kT}}} \quad p_d = \frac{N_a}{1 + \frac{1}{2} e^{\frac{E_F - E_a}{kT}}}$$

Thermal-Equilibrium El. Concentration

n_0 for $N_d > N_a$ (n-type), p_0 for $N_a > N_d$ (p-type).

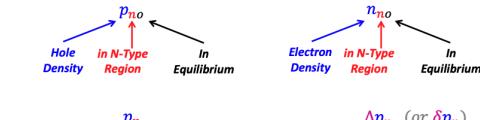
$$n_0 = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2} \right)^2 + n_i^2} \\ p_0 = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2} \right)^2 + n_i^2}$$

Position of Fermi Energy Level

Use the n_0 formula for n-type, the p_0 formula for p-type.

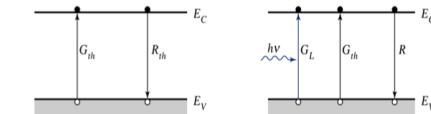
$$E_F - E_{F,i} = kT \ln \left(\frac{n_0}{n_i} \right) \quad E_{F,i} - E_F = kT \ln \left(\frac{p_0}{n_i} \right)$$

4 Excess Carriers



G_{th}	Thermal generation rate
R_{th}	Thermal recombination rate
G_L	Photonic generation rate
τ_p, τ_n	Minority carrier lifetime
Δ_p, Δ_n	Excess carrier concentration to equi.
N_t	Density of recombination centers
σ	Recomb. center cross section
σ conductivity	Carrier mobility in thermal equilibrium
v_{th}	El. indirect capture and emission rate
R_a, R_b	Hole indirect capture and emission rate
R_c, R_d	El./Hole indirect emission probability
e_n, e_p	Net recombination rate
U	Diffusion current dens.
J_{diff}	Diffusion constant
D	electron charge
q	Electron diffusion current dens.
J_n	Electron/hole drift velocity
$v_{dr,n/p}$	Electron/hole drift current dens.
$J_{dr,n/p}$	Electron/hole mobility
$\mu_{n/p}$	Electron/hole total current dens.
$J_{n/p}$	

4.1 Direct Generation/Recombination



Thermal (spontaneous) and external generation (e.g. light) across the energy gap.

In steady state, at a given T: Electrons are continually generated due to thermal energy. Some electrons recombine with holes, so that on average n_0 and p_0 are constant. Rate is proportional to $n_0 \cdot p_0$.

$$G_{th} = R_{th} = \beta(n_0 \cdot p_0) = \beta n_i^2$$

This is because we always need 1 electron and 1 hole to meet for each recombination event.

$$R = \beta np \quad G = G_L + G_{th} \quad \Delta n = \Delta p$$

$$R = \beta \cdot n_n p_n = \beta(n_{n0} + \Delta n)(p_{n0} + \Delta p)$$

$$\frac{d}{dt} p_n = G - R$$

Mass action law ($n_0 p_0 = n_i^2$) states for n-type doping that:

$$p_{n0} \ll n_{n0}$$

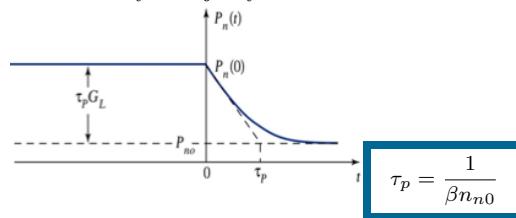
Low level injection is when

$$\Delta p \ll n_{n0}$$

Then the net recombination rate can be written as:

$$U = \beta(n_{n0} + p_{n0} + \Delta p)\Delta p \approx \beta n_{n0}\Delta p = \frac{\Delta p}{\tau_p}$$

The minority carrier lifetime describes how fast the excess carrier concentration decays towards equil. when excitation ends. Determined by majority carrier concentration.



Light on:

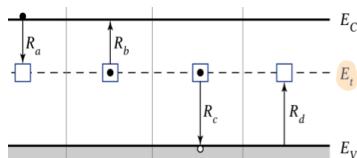
$$G_L = U = \frac{p_n - p_{n0}}{\tau_p} \quad p_n(t \leq 0) = p_{n0} + \tau_p G_L t$$

Light off:

$$\begin{aligned} \frac{dp_n}{dt} &= G_{th} - R = -U = -\frac{p_n - p_{n0}}{\tau_p} \\ p_n(t) &= p_{n0} + \tau_p G_L \exp\left(-\frac{t}{\tau_p}\right) \end{aligned}$$

Generally: $\Delta n = \tau_{n0} R_n$

4.2 Indirect Generation/Recombination



Energy trap near midgap.

$$U \approx \frac{v_{th}\sigma_0 N_t \cdot \Delta p}{1 + \frac{2n_i}{n_{n0}} \cosh \frac{E_t - E_i}{kT}} \approx \frac{\Delta p}{\tau_p} \approx \frac{p_n - p_{n0}}{\tau_p}$$

Where $N_t v_{th} \sigma$ are the recombination events taking place per unit time, $v_{th} \sigma$ a cylindrical column in material per unit time.

$$\frac{1}{2} m_n v_{th}^2 = \frac{3}{2} kT \quad v_{th} \approx 10^7 \text{ cm} \cdot \text{s}^{-1}$$

Electron capture (R_a) and emission (R_b) rate must be equal in therm. equi.

$$R_a = n N_t (1-f) \cdot v_{th} \sigma_n \quad R_b = e_n N_t f$$

The emission probability increases exponentially as E_t gets closer to conduction band edge:

$$e_n = \frac{v_{th} \sigma_n n (1-f)}{f} = v_{th} \sigma_n n e^{(E_t - E_i)/kT}$$

For holes:

$$R_c = p N_t f \cdot v_{th} \sigma_p \quad R_d = e_p N_t (1-f)$$

$$e_p = v_{th} \sigma_p n_i e^{(E_i - E_t)/kT}$$

These lead to the equation for $U = \dots$

4.3 Diffusion

Result of concentration gradients. Equal probability of moving in any direction. Fick's first law of diffusion:

$$J_{\text{diff}} = -D \Delta N = -D \left(\frac{\partial N}{\partial x} \vec{x}_u + \frac{\partial N}{\partial y} \vec{y}_u \dots \right)$$

In thermal equi. and uniform distribution, free charge carriers are in constant motion. Net current is thus zero. Statistical mechanics show that particles at temp. T have avg. thermal energy of $3kT/2$. For a particle of mass m this corresponds to an avg. thermal velocity

$$\frac{1}{2} m v_{th}^2 = \frac{3}{2} kT$$

In a crystal we must use the **effective mass**. The electron diffusion current:

$$J_{\text{diff},n} = -qF = q D_n \frac{dn}{dx}$$

4.4 Drift

Result of an electric field as driving force. Zero field: Electrons move thermally (randomly) in all directions (no net flow). Non-zero field: There is a net drift of electrons, opposite to the E-field. We can then define a drift velocity of electrons $v_{dr,n}$. Electrons do not accelerate indefinitely due to collisions

$$J_{dr,n} = -q n v_{dr,n} \quad v_{dr,n} = -\mu_n E$$

$$J_{dr,p} = q p v_{dr,h} \quad v_{dr,h} = \mu_p E$$

$$J_{dr,tot} = \sigma E \quad \sigma = q(n\mu_n + p\mu_p)$$

4.5 Total current

Total current = drift + diffusion current = electron + hole current.

$$\text{Electrons: } J_n = n q \mu_n \vec{E} + q D_n \frac{dn}{dx}$$

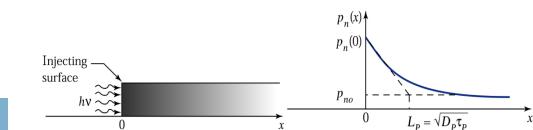
$$\text{Holes: } J_p = p q \mu_p \vec{E} - q D_p \frac{dp}{dx}$$

Einstein relation

$$D_p = \frac{kT}{q} \mu_p$$

$$D_n = \frac{kT}{q} \mu_n$$

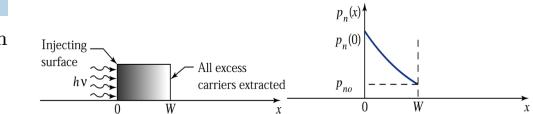
Semi-infinite sample



$$p(x) = p_{n0} + (p_n(0) - p_{n0}) \exp\left(-\frac{x}{L_p}\right)$$

$$L_p = \sqrt{D_p \tau_p}$$

Finite sample

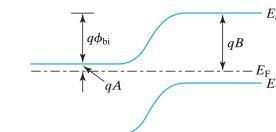


$$p_n(x) = p_{n0} + (p_n(0) - p_{n0}) \frac{\sinh\left(\frac{W-x}{L_p}\right)}{\sinh\left(\frac{W}{L_p}\right)}$$

If $W \ll L_p$ then recombination is weak, few holes recombine in the time required for them to cross the region W . The excess carrier profile becomes linear:

$$p_n(x) = (p_n(0) - P_{n0}) \frac{W-x}{W}$$

5.2 Equilibrium: constant fermi level



In equilibrium, the Fermi level must be constant to balance transfer rates so that no net current flows.

$$E_F1 = E_F2$$

5.3 Doping: Band bending

Near the transition region, the distance between the Fermi level and conduction/valence band edge changes. Far away from the junction, the material does not "know" there is a junction.