Abstract

In modern supply chain management, optimizing the distribution network is essential to minimize costs while maintaining efficiency and reliability. A critical decision in supply chain design is determining the optimal configuration of distribution centers, including whether to rent new Direct Distribution Centers (Direct DCs) or utilize existing ones, as well as how to allocate resources such as vehicles and containers effectively. The complexity of this decision-making process increases with multiple customer locations, variable transportation costs, and the diverse requirements for vehicle or container types.

This study addresses the challenge of determining the most cost-effective distribution strategy through a Mixed-Integer Linear Programming (MILP) model. The primary goal is to optimize the supply source for each customer and allocate resources for demand distribution, minimizing overall distribution costs. The model incorporates various factors, such as operational distribution costs, fuel costs, vehicle rental fees, and distribution time, all of which are crucial in the decision-making process. Additionally, constraints are established to ensure customer demand is met, vehicles and containers are efficiently utilized, and logistical and budgetary requirements are satisfied.

While the MILP model developed in this study provides a solid foundation for optimizing the distribution network, future extensions will incorporate reinforcement learning to adaptively adjust the number of warehouses based on changing demands and costs. This hybrid approach will enhance decision-making, further improving the distribution process in dynamic environments. By integrating operational costs, resource allocation, and flexibility, this research contributes to enhancing the overall efficiency of distribution networks in supply chain management.

Model component

Notation	Definition
Objective Function	Determining the capacity, and network to supply and distribute products (e.g., cartons) while minimizing total supply chain costs.
Performance Measure	Total supply chain cost, which consists of: • Capital expenses (e.g., vehicle rental, storage, and container costs) • Operational expenses (e.g., staff costs, fuel costs, vehicle rent costs)
Decision Variables	• Quantities of products supplied from supply points to customers. • Selection of transportation modes and their fraction of the total demand fulfilled. • Allocation of supply routes
Parameters	• **Supply Costs**: Costs related to sourcing goods from supply points to distribution centers. • **Transportation Costs**: Costs associated with vehicle and container usage, including fuel and rent • **Distance Metrics**: Relevant distances for all transportation routes • **Warehouse and Vehicle Capacity**: Constraints and limits on storage and transportation • **Fuel and Utilization**: Metrics related to fuel usage and vehicle utilization • **Staff and Labor**: Cost and capacity related to warehouse and driver staff • **Demand and Supply**: Parameters related to customer demands and supply chain needs • **Other Costs**: Additional costs related to supply chain and facility operations
Constraints	• Constraints focus on ensuring that the demand of customers is met by the available supply from direct DCs and depots. • Constraints ensure that depots are properly assigned and resources (containers, vehicles) are allocated efficiently.

- Constraints ensure that the fraction of resource utilization (vehicles and containers) remains within valid limits.
- Constraints ensure that variables for demand and supply are non-negative, and the assignment decisions are binary.

Mathematical model notation

Notation	Definition	Value
Objective Function		
Z	Total Monthly Supply and Distribution Cost	IDR/Month
Sets		
I	Sets of Direct DC units	$I = \{1, 2, \dots, 9\}$
J	Sets of Depo units	$J = \{1, 2, \dots, 10\}$
K	Sets of Customer points	$K = \{1, 2, \dots, k_{\|K\|}\}$
A	Sets of Container types	$A = \{1, 2\}$
B	Sets of Vehicle modes	$B = \{1, 2, 3, 4\}$
Indices		
i	Direct DC unit; $\forall i \in I$	
j	Depo unit; $\forall j \in J$	
k	Customer points; $\forall k \in K$	
a	Container types; $\forall a \in A$	
b	Vehicle modes; $\forall b \in B$	
Parameters		
SC_i	Supply cost to supply carton from SDC to Direct DC i	IDR/month
ODS_i	Operational Distribution Cost to distribute carton for direct DC i	IDR/month
ODS_j	Operational Distribution Cost to distribute carton from depo j	IDR/month
FC_i	Fuel Cost for all vehicle to distribute from direct DC i to customers	IDR/month
FC_j	Fuel Cost for all vehicle to distribute from depo j to customers	IDR/month
FC_{ij}	Fuel Cost for all vehicle to distribute from direct DC i to depo j	IDR/month
VRC_i	Vehicle Rent Cost for direct DC i to distribute Customer's demand for all vehicle	IDR/month
VRC_j	Vehicle Rent Cost for depo j to distribute Customer's demand for all vehicle	IDR/month
VRC_{ij}	Vehicle Rent Cost for direct DC j to supply Depo's supply for all vehicle	IDR/month
WSC_i	Total Direct DC i warehouse staff's cost	IDR/month
WSC_j	Total Depo j warehouse staff's cost	IDR/month
DSC_i	Total Direct DC i driver's cost	IDR/month

DSC_j	Total Depo j driver's cost	IDR/month
DV_i^b	Customer's Demand supplied by Vehicle b from Direct DC i to all Customer	Carton/month
DV_j^b	Customer's Demand supplied by Vehicle b from Depo j to all Customer	Carton/month
DV_{ij}^b	Customer's Demand supplied by Vehicle b from Direct DC i to Depo j	Carton/month
MDS_i	Monthly demand supply from direct DC i	Carton/month
d_{ij}	Distance from direct DC i to depo j	Km
d_{ik}	Distance from direct DC i to customer k	Km
d_{jk}	Distance from depo j to customer k	Km
WD_i	Weighted Distance from direct DC i to all Direct DC's Customer	Km
WD_j	Weighted Distance from Depo to Depo's Customer	Km
FU_i	Fuel Usage for all vehicle to distribute from direct DC i to customers	Liter
FU_j	Fuel Usage for all vehicle to distribute from depo j to customers	Liter
FU_{ij}	Fuel Usage for all vehicle to distribute from direct DC i to depo j	Liter
cV^b	Capacity of vehicle b	Carton/unit
cC^a	Capacity of Container a	Carton/unit
cWS	Capacity of warehouse staff to handle carton	Carton/person
cD	Capacity of driver to handle trips	Trip-unit/person
tDD_i^b	Total Depo's Distribution Time for direct DC i to distribute all customers for vehicle b	Hour

tDD_j^b	Total Depo's Distribution Time for depo j to distribute all customers for vehicle b	Hour
tDD_{ij}^b	Total Depo's Distribution Time for direct DC i to Depo j for vehicle b	Hour
tL	Time loading cartons	Hour
tUL	Time unloading cartons	Hour
tVU	Time Vehicle Utilization	Hour/Month
RTH_i	Round trip hour from direct DC i to all customer	Hour
RTH_j	Round trip hour from Depo j to all customer	Hour
RTH_{ij}	Round trip hour from direct DC i to Depo j	Hour
nV_i^b	Number of Depo's Vehicle in direct DC i to distribute all customer for vehicle mode b	Unit/month
nV_j^b	Number of Depo's Vehicle in depo j to distribute all customer for vehicle mode b	Unit/month
nV_{ij}^b	Number of Depo's Vehicle in direct DC i to supply depo j for vehicle mode b	Unit/month
nD_i	Total number of Driver for Direct DC i	Person/month
nD_j	Total number of Driver for Depo j	Person/month
nWS_i	Total number of warehouse staff for Direct DC i	Person/month
nWS_j	Total number of warehouse staff for Depo j	Person/month
nC_{0i}^a	Total number of containers a to supply from SDC to Direct DC i	Unit/month
VV	Vehicle Velocity	Km/hour
rV^b	Rate of vehicle b rent cost per month	Rp/unit-month

rWS_i	Rate of warehouse staff's cost at direct DC i	IDR/person
rWS_j	Rate of warehouse staff's cost at depo j	IDR/person
rCT_{0i}^{a}	Container's Rate per Trips for container a to supply from SDC to direct DC i	IDR/unit
rD_i	Rate of driver's cost at direct DC i	IDR/person
rD_j	Rate of driver's cost at depo j	IDR/person
rVC^b	Vehicle Consumption's Rate (km/liter)	Km/liter
T_i^b	Total trips from Direct DC i to Customer for vehicle b	Trip
T_j^b	Total trips from Depo j to Customer for vehicle b	Trip
T_{ij}^b	Total trips from Direct DC i to Depo for vehicle b	Trip
FP	Fuel price	IDR/liter
Decision Variable		
x_{ik}	Customer's Demand Carton served by Direct DC i to customer k	Carton/month
x_{jk}	Customer's Demand Carton served by Depo j to customer k	Carton/month
y_{ij}	Binary Decision for Depo j to be supplied from Direct DC i	-
z_{ik}	Binary Decision for Customer k to be supplied from Direct DC i	-
z_{jk}	Binary Decision for Customer k to be supplied from Depo j	-
fv_i^b	Fraction of Customer's Demand supplied by Vehicle b from Direct DC i	-
fv_j^b	Fraction of Customer's Demand supplied by Vehicle b from Depo j	-
fc_i^b	Fraction of T&W supply distributed by container a to direct DCj	-

Mathematical Equation

$$
\begin{aligned}
\min \Bigg(& \sum_{i\in I}\sum_{a\in A}\left\lceil \frac{\sum_{k\in K}\left(z_{ik}D_k + \sum_{j\in J} z_{jk}D_k y_{ij}\right) fc_i^a}{cC^a} \right\rceil rCT_{0i}^a \\
& + \sum_{j\in J}\left(\sum_{b\in B}\left(\frac{\left(\sum_{k\in K} z_{jk}d_{jk}\right) fv_j^b}{rVC^b} \right) FP \right. \\
& + \sum_{b\in B}\left(\frac{\left\lceil \frac{\left(\sum_{k\in K} z_{jk}D_k\right) fv_j^b}{cV^b} \right\rceil}{tVU} \left(tL + tUL + \frac{\left(\sum_{k\in K} z_{jk}d_{jk}\right)}{VV} \right) rV^b \right) \\
& + \left(\left(\frac{\sum_{b\in B}\left\lceil \frac{\left(\sum_{k\in K} z_{jk}D_k\right) fv_j^b}{cV^b} \right\rceil}{cD} \right) rD_j + \left(\frac{\sum_{k\in K} z_{jk}D_k}{cWS} \right) rWS_j \right) \\
& + \sum_{i\in I}\left(\sum_{b\in B}\left(\frac{\left(\sum_{k\in K} z_{ik}d_{ik}\right) fv_i^b}{rVC^b} \right) FP \right. \\
& + \sum_{b\in B}\left(\left(\frac{\left\lceil \frac{\left(\sum_{k\in K} z_{ik}D_k\right) fv_i^b}{cV^b} \right\rceil}{tVU} \right) \left(tL + tUL + \frac{\left(\sum_{k\in K} z_{ik}d_{ik}\right)}{VV} \right) rV^b \right) \\
& + \left(\frac{\sum_{b\in B}\left\lceil \frac{\left(\sum_{k\in K} z_{ik}D_k\right) fv_i^b}{cV^b} \right\rceil}{cD} \right) rD_i + \frac{\sum_{k\in K} z_{ik}D_k}{cWS} rWS_i \\
& + \sum_{j\in J}\left(\left(\sum_{b\in B} \frac{y_{ij}d_{ij}fv_{ij}^b}{rVC^b} \right) FP \right. \\
& \left.\left.\left. + \sum_{b\in B}\left(\left(\frac{\left\lceil \frac{\sum_{k\in K} z_{jk}D_k y_{ij} fv_{ij}^b}{cV^b} \right\rceil \left(tL + tUL + \frac{y_{ij}d_{ij}}{VV} \right)}{tVU} \right) rV^b \right) \right) \right) \right) \Bigg)
\end{aligned}
\tag{1}
$$

subject to:

$$\sum_{i \in I} x_{ik} + \sum_{j \in J} x_{jk} = D_k \qquad \forall k \in K \qquad (2)$$

$$\sum_{i \in I} y_{ij} = 1 \qquad \forall j \in J \qquad (3)$$

$$\sum_{i \in I} z_{ik} + \sum_{j \in J} z_{jk} = 1 \qquad \forall k \in K \qquad (4)$$

$$\sum_{a \in A} fc_i^a = 1 \qquad \forall i \in I \qquad (5)$$

$$\sum_{b \in B} fv_i^b = 1 \qquad \forall i \in I \qquad (6)$$

$$\sum_{b \in B} fv_j^b = 1 \qquad \forall j \in J \qquad (7)$$

$$\sum_{b \in B} y_{ij}.fv_{ij}^b = 1 \qquad \forall i \in I,\ \forall j \in J \qquad (8)$$

$$x_{ik} \geq 0 \qquad \forall i \in I,\ \forall k \in K \qquad (9)$$

$$x_{jk} \geq 0 \qquad \forall j \in J,\ \forall k \in K \qquad (10)$$

$$y_{ij} \in \{0,1\} \qquad \forall i \in I,\ \forall j \in J \qquad (11)$$

$$0 \leq fv_i^b \leq 1 \qquad \forall i \in I,\ \forall b \in B \qquad (12)$$

$$0 \leq fv_j^b \leq 1 \qquad \forall j \in J,\ \forall h \in B \qquad (13)$$

$$0 \leq fv_{ij}^b \leq 1 \qquad \forall i \in I, \forall j \in J,\ \forall b \in B \qquad (14)$$

$$0 \leq fc_i^a \leq 1 \qquad \forall i \in I,\ \forall b \in B \qquad (15)$$

Model Verification

Conceptual model verification

Notation	Definition	Mathematical Model
Objective Function	Determining the capacity, and network to supply and distribute products (e.g., cartons) while minimizing total supply chain costs.	(1)
Decision Variables	• Quantities of products supplied from supply points to customers.	x_{ij}, x_{jk}
	• Selection of transportation modes and their fraction of the total demand fulfilled.	$fv_{ij}^b, fv_i^b, fv_j^b,$ fc_i^a
	• Allocation of supply routes	y_{ij}
Parameters	• **Supply Costs**: Costs related to sourcing goods from supply points to distribution centers.	SC_i
	• **Transportation Costs**: Costs associated with vehicle and container usage, including fuel and rent:	$ODS_i, ODS_j,$ $FC_i, FC_j, FC_{ij},$ $VRC_i, VRC_j,$ $VRC_{ij},$ $WSC_i, WSC_j,$ DSC_i, DSC_j
	• **Distance Metrics**: Relevant distances for all transportation routes	$d_{ij}, d_{jk}, d_{ik},$ WD_i, WD_j
	• **Warehouse and Vehicle Capacity**: Constraints and limits on storage and transportation	$cV^b,$ $cC^a,$
	• **Fuel and Utilization**: Metrics related to fuel usage and vehicle utilization	$FU_i, FU_{j,},$ $FU_{ij},$ $tVU,$ $tDD_i^b, tDD_j^b,$ $tDD_{ij}^b, tL, tUL,$ RTH_i, RTH_j, RTH_{ij} $nV_i^b, nV_j^b, nV_{ij}^b$ nC_{0i}^a

	• **Staff and Labor**: Cost and capacity related to warehouse and driver staff	cWS, cD, nD_i, nD_j, nWS_i, nWS_j
	• **Demand and Supply**: Parameters related to customer demands and supply chain needs	
	• **Other Costs**: Additional costs related to supply chain and facility operations	
Constraints	• Constraints focus on ensuring that the demand of customers is met by the available supply from direct DCs and depots.	
	• Constraints ensure that depots are properly assigned and resources (containers, vehicles) are allocated efficiently.	
	• Constraints ensure that the fraction of resource utilization (vehicles and containers) remains within valid limits.	
	• Constraints ensure that variables for demand and supply are non-negative, and the assignment decisions are binary.	

APPENDIX
(Unorganized)

Contain my train of thoughts and logic behind the design

1. Total Distribution Cost for distribution from SDC to all warehouse

$$TC = \sum_{i \in I} SC_i + \sum_{i \in I} ODC_i + \sum_{j = J} ODC_j$$

$$I = \{1, 2, \dots, 9\}$$

I: Sets of direct DC unit

i: Direct DC unit indices; $\forall i \in I$

$$J = \{1, 2, \dots, 10\}$$

J: Sets of depo unit

j: Depo unit indices; $\forall j \in J$

2. Supply cost to supply carton from SDC to Direct DC i

$$SC_i = \sum_{a \in A} nC_{0i}^a \times rCT_{0i}^a$$

$$A = \{1, 2\}$$

A: Sets of Container type

a: Container type indices; $\forall a \in A$

rCT_{0i}^a: Container's Rate per Trips for container a to supply from SDC to direct DC i (Rp/Trip)

3. Container's Quantity used for selected Container a to supply from SDC to Direct DC i

$$nC_{0i}^a = \left\lceil \frac{MDS_i \times fc_i^a}{cC^a} \right\rceil$$

fc_i^a: Fraction of T&W supply distributed by container a

$$\sum_{a \in A} fc_i^a = 1, \forall i$$

$$fc_i^a \in [0,1], \forall i \in I, \forall a \in A$$

cC^a: Container capacity of container a (Carton)

4. Monthly demand supply that need to be supplied to direct DC i

$$MDS_i = \sum_{k \in K} \left(x_{ik} + \sum_{j \in J} x_{jk} \times y_{ij} \right)$$

$$K = \{1, 2, \dots, k_{|K|}\}$$

K: Sets of Customer unit

k: Customer indices; $\forall k \in K$

y_{ij}: Decision whether the Depo j supplied by Direct DC j or not

$$y_{ij} \in \{0,1\}\ i \in I, j \in J$$

5. Operational Distribution Cost to distribute carton from depo j

$$ODS_j = FC_j + VRC_j + DSC_j + WSC_j$$

6. Fuel Cost for depo j to distribute customer demand

$$FC_j = FU_j \times FP$$

FP: Fuel Price (Rp/km)

7. Fuel Usage for depo j to distribute customer demand

$$FU_j = \sum_{b \in B} \frac{WD_j \times fv_j^b}{rVC^b}$$

fv_j^b: Fraction of Customer's Demand supplied by selected Vehicle from Depo to Customer

$$\sum_{b \in B} fv_j^b = 1, \qquad \forall j \in J$$

$$fv_j^b \in [0,1], \forall j \in J, \forall b \in B$$

rVC^b: Vehicle Consumption's Rate (km/liter)

8. Weighted Distance from Depo to Depo's Customer

$$WD_j = \frac{\sum_{k \in K} x_{jk} \times d_{jk}}{\sum_{k \in K} x_{jk}}$$

d_{jk}: distance from Depo j to Customer k (km)

9. Vehicle Rent Cost for depo j

$$VRC_j = \sum_{b \in B} nV_j^b \times rV^b$$

rV^b: rate of vehicle's rate (Rp/unit-month)

10. Amount of Depo's Vehicle in depo j for vehicle mode b

$$nV_j^b = \left\lceil \frac{tDD_j^b}{tVU} \right\rceil$$

tVU: time Vehicle Utilization (Hour/month)

11. Total Depo's Distribution Time for depo i for vehicle b

$$tDD_j^b = \left\lceil \frac{DV_j^b}{cV^b} \right\rceil \times RTH_j$$

cV^b: capacity of vehicle b

12. Customer's Demand supplied by Vehicle b from Depo j to Customer k

$$DV_j^b = \left(\sum_{k \in K} x_{jk} \right) \times fv_j^b$$

13. Round trip Hour

$$RTH_j = tL + tUL + \frac{WD_j}{VV}$$

14. Operational Distribution Cost to distribute carton from direct DC i

$$ODS_i = FC_i + VRC_i + DSC_i + WSC_i + \sum_{j \in J} FC_{ij} + VRC_{ij}$$

15. Fuel Cost for direct DC i

$$FC_i = FU_i \times FP$$

FP: Fuel Price (Rp/km)

16. Fuel Usage for direct DC i to depo j

$$FU_i = \sum_{b \in B} \frac{WD_i \times fv_i^b}{rVC^b}$$

fv_i^b: Fraction of Customer's Demand supplied by selected Vehicle from direct DC i to Depo

$$\sum_{b \in B} fv_i^b = 1, \qquad \forall j \in I$$

$$fv_i^b \in [0,1], \forall i \in I, \forall b \in B$$

rVC^b: Vehicle Consumption's Rate (km/liter)

17. Weighted Distance from direct DC i to Direct's Customer

$$WD_i = \frac{\sum_{k \in K} x_{ik} \times d_{ik}}{\sum_{k \in K} x_{ik}}$$

d_{jk}: distance from Depo j to Customer k (km)

18. Vehicle Rent Cost for direct DC i

$$VRC_i = \sum_{b \in B} nD_i^b \times rV^b$$

rV^b: rate of vehicle's rate (Rp/unit-month)

19. Amount of Depo's Vehicle in direct DC i for vehicle mode b

$$nV_i^b = \left\lceil \frac{tDD_i^b}{tVU} \right\rceil$$

tVU: time Vehicle Utilization (Hour/month)

20. Total Depo's Distribution Time for direct DC i for vehicle b

$$tDD_i^b = \left\lceil \frac{DV_i^b}{cV^b} \right\rceil \times RTH_i$$

cV^b: capacity of vehicle b

21. Customer's Demand supplied by Vehicle b from direct DC i to Customer k

$$DV_i^b = \left(\sum_{k \in K} x_{ik} \right) \times fv_i^b$$

22. Round trip Hour for overall vehicle in direct DC i

$$RTH_i = tL + tUL + \frac{WD_i}{VV}$$

23. Fuel Cost for direct DC i to Depo j

$$FC_{ij} = FU_{ij} \times FP$$

FP: Fuel Price (Rp/km)

24. Fuel Usage for direct DC i to Depo j (rawan)

$$FU_{ij} = \sum_{b \in B} \frac{d_{ij} \times fv_{ij}^b}{rVC^b}$$

fv_i^b: Fraction of Customer's Demand supplied by selected Vehicle from direct DC i to Depo j

$$\sum_{b \in B} fv_{ij}^b = 1, \qquad \forall j \in I$$

$$fv_{ij}^b \in [0,1], \forall i \in I, \forall j \in J, \forall b \in B$$

rVC^b: Vehicle Consumption's Rate (km/liter)

d_{ij}: distance from direct DC i to Depo j (km)

25. Vehicle Rent Cost for direct DC i to Depo j

$$VRC_{ij} = \sum_{b \in B} nV_{ij}^b \times rV^b$$

rV^b: rate of vehicle's rate (Rp/unit-month)

26. Amount of Depo's Vehicle in direct DC i to Depo j for vehicle mode b

$$nV_{ij}^b = \left\lceil \frac{tDD_{ij}^b}{tVU} \right\rceil$$

tVU: time Vehicle Utilization (Hour/month)

27. Total Depo's Distribution Time for direct DC i to Depo j for vehicle b

$$tDD_{ij}^{b} = \left\lceil \frac{DV_{ij}^{b}}{cV^{b}} \right\rceil \times RTH_{ij}$$

cV^{b}: capacity of vehicle b

28. Customer's Demand supplied by Vehicle b from direct DC i to Depo j

$$DV_{ij}^{b} = \sum_{k \in K} x_{jk} \times y_{ij} \times fv_{ij}^{b}$$

29. Round trip Hour for each direct DC i to Depo j

$$RTH_{ij} = tL + tUL + \frac{d_{ij}}{VV}$$

30. Total Depo to Customer Trips for vehicle b

$$T_{j}^{b} = \left\lceil \frac{DV_{j}^{b}}{cV^{b}} \right\rceil$$

31. Total number of Driver for Direct DC j

$$nD_{j} = \left\lceil \frac{\sum_{b \in B} T_{j}^{b}}{cD} \right\rceil$$

32. Total Depo driver's cost

$$DSC_{j} = nD_{j} \times rD_{j}$$

33. Total Direct DC to Customer Trips for vehicle b

$$T_{i}^{b} = \left\lceil \frac{DV_{i}^{b}}{cV^{b}} \right\rceil$$

34. Total Direct DC to Depo Trips for vehicle b

$$T_{ij}^{b} = \left\lceil \frac{DV_{ij}^{b}}{cV^{b}} \right\rceil$$

35. Total number of Driver for Direct DC j

$$nD_{i} = \left\lceil \frac{\sum_{b \in B} (T_{j}^{b} + \sum_{i \in I} T_{ij}^{b})}{cD} \right\rceil$$

36. Total Direct DC driver's cost

$$DSC_{i} = nD_{i} \times rD_{i}$$

37. Total number of warehouse staff in Depo j

$$nWS_{j} = \left\lceil \frac{\sum_{k \in K} x_{jk}}{cWS} \right\rceil$$

38. Total Depo j warehouse staff's cost

$$WSC_j = nWS_j \times rWS_j$$

39. Total number of warehouse staff in Direct DC i

$$nWS_i = \left\lceil \frac{MDS_i}{cWS} \right\rceil$$

40. Total Direct DC i warehouse staff's cost

$$WSC_i = nWS_i \times rWS_i$$