Homework 5

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CS 161

November 2, 2015

1. **(20 points)** Use truth tables (worlds) to show that the following pairs of sentences are equivalent:

   - P $\Rightarrow \neg$ Q, Q $\Rightarrow \neg$ P

P	Q	$\neg$ P $\vee \neg$ Q	**P $\Rightarrow \neg$ Q**	$\neg$ Q $\vee \neg$ P	**Q $\Rightarrow \neg$ P**
T	T	F	**F**	F	**F**
T	F	T	**T**	T	**T**
F	T	T	**T**	T	**T**
F	F	T	**T**	T	**T**

   - P $\Leftrightarrow \neg$ Q, ((P $\wedge \neg$ Q) $\vee$ ($\neg$ P $\wedge$ Q))

P	Q	**P $\Leftrightarrow \neg$ Q**	P $\wedge \neg$ Q	$\neg$ P $\wedge$ Q	**(P $\wedge \neg$ Q) $\vee$ ($\neg$ P $\wedge$ Q)**
T	T	**F**	F	F	**F**
T	F	**T**	T	F	**T**
F	T	**T**	F	T	**T**
F	F	**F**	F	F	**F**

2. **(30 points)** Consider the following sentences and decide for each whether it is valid, unsatisfiable, or neither:

   - (Smoke $\Rightarrow$ Fire) $\Rightarrow$ ($\neg$ Smoke $\Rightarrow \neg$ Fire)

Smoke	Fire	$\neg$ Smoke $\vee$ Fire	Smoke $\vee \neg$ Fire	$\neg$ ($\neg$ Smoke $\vee$ Fire) $\vee$ (Smoke $\vee \neg$ Fire)
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

   This sentence is neither valid nor unsatisfiable.

- (Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)

Smoke	Fire	Heat	¬ Smoke ∨ Fire	Smoke ∨ Heat	¬ (Smoke ∨ Heat) ∨ Fire
T	T	T	T	T	T
T	F	T	F	T	F
F	T	T	T	T	T
F	F	T	T	T	F
T	T	F	T	T	T
T	F	F	F	T	F
F	T	F	T	F	T
F	F	F	T	F	T

¬ (¬ Smoke ∨ Fire) ∨ (¬ (Smoke ∨ Heat) ∨ Fire)
T
T
T
F
T
T
T
T

This sentence is neither valid nor unsatisfiable.

- ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

Smoke	Fire	Heat	Smoke ∧ Heat	¬ (Smoke ∧ Heat) ∨ Fire	¬ Smoke ∨ Fire	¬ Heat ∨ Fire
T	T	T	T	T	T	T
T	F	T	T	F	F	F
F	T	T	F	T	T	T
F	F	T	F	T	T	F
T	T	F	F	T	T	T
T	F	F	F	T	F	T
F	T	F	F	T	T	T
F	F	F	F	T	T	T

(¬ Smoke ∨ Fire) ∨ (¬ Heat ∨ Fire)	((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))
T	T
F	T
T	T
T	T
T	T
T	T
T	T
T	T

This sentence is valid.

3. (50 points) Consider the following:

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned

(a) Represent the above information using a propositional logic knowledge base (set of sentences in propositional logic).

1. Mythical $\Rightarrow \neg$ Mortal
2. $\neg$ Mythical $\Rightarrow$ (Mortal $\wedge$ Mammal)
3. ($\neg$ Mortal $\vee$ Mammal) $\Rightarrow$ Horned
4. Horned $\Rightarrow$ Magical

(b) Convert the knowledge base into CNF.

($\neg$ Mythical $\vee \neg$ Mortal) $\wedge$ (Mythical $\vee$ Mortal) $\wedge$ (Mythical $\vee$ Mammal) $\wedge$ (Mortal $\vee$ Horned) $\wedge$ ($\neg$ Mammal $\vee$ Horned) $\wedge$ ($\neg$ Horned $\vee$ Magical)

(c) Can you use the knowledge base to prove that the unicorn is mythical? How about magical? Horned?

Mythical?

1.	$\neg$ Mythical $\vee \neg$ Mortal	
2a.	Mythical $\vee$ Mortal	
2b.	Mythical $\vee$ Mammal	
3a.	Mortal $\vee$ Horned	
3b.	$\neg$ Mammal $\vee$ Horned	
4.	$\neg$ Horned $\vee$ Magical	
5.	$\neg$ Mythical	
6.	Mortal	2a, 5
7.	Mammal	2b, 5
8.	Horned	3b, 7
9.	Magical	4, 8

We cannot prove that the unicorn is mythical.

Magical?

1.	¬ Mythical ∨ ¬ Mortal	
2a.	Mythical ∨ Mortal	
2b.	Mythical ∨ Mammal	
3a.	Mortal ∨ Horned	
3b.	¬ Mammal ∨ Horned	
4.	¬ Horned ∨ Magical	
5.	¬ Magical	
6.	¬ Horned	4, 5
7.	Mortal	3a, 6
8.	¬ Mythical	1, 7
9.	Mammal	2b, 8
10.	¬ Mammal	3b, 6

As there is a contradiction in the resolution, we can prove that the unicorn is magical.

Horned?

1.	¬ Mythical ∨ ¬ Mortal	
2a.	Mythical ∨ Mortal	
2b.	Mythical ∨ Mammal	
3a.	Mortal ∨ Horned	
3b.	¬ Mammal ∨ Horned	
4.	¬ Horned ∨ Magical	
5.	¬ Horned	
6.	Mortal	3a, 5
7.	¬ Mythical	1, 6
8.	Mammal	2b, 7
9.	¬ Mammal	3b, 5

As there is a contradiction in the resolution, we can prove that the unicorn is horned.