Homework 6

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1. For each pair of atomic sentences, give the most general unifier if it exists:

   (a) $P(A, B, B), P(x, y, z)$
   $\sigma = \{x \backslash A, y \backslash B, z \backslash B\}$

   (b) $Q(y, G(A, B)), Q(G(x, x), y)$
   Does not exist

   (c) $R(x, A, z), R(B, y, z)$
   $\sigma = \{x \backslash B, y \backslash A\}$

   (d) $\text{Older}(\text{Father}(y), y), \text{Older}(\text{Father}(x), \text{John})$
   $\sigma = \{x \backslash \text{John}, y \backslash \text{John}\}$

   (e) $\text{Knows}(\text{Father}(y), y), \text{Knows}(x, x)$
   Does not exist

2. Translate the following sentences into formulas in first-order logic:

   (a) There exists at most one x such that $P(x)$
   $\forall x \forall y ((P(x) \land P(y)) \Rightarrow x = y)$

   (b) There exists exactly one x such that $P(x)$
   $\exists x (P(x) \land \forall y (P(y) \Rightarrow x = y))$

   (c) There exists at least two x such that $P(x)$
   $\exists x \exists y (P(x) \land P(y) \land \neg(x = y))$

   (d) There exists at most two x such that $P(x)$
   $\forall x \forall y \forall z ((P(x) \land P(y) \land P(z)) \Rightarrow (x = y \lor y = z \lor x = z))$

   (e) There exists exactly two x such that $P(x)$
   $\exists x \exists y (P(x) \land P(y) \land x \neq y \land \forall z (P(z) \Rightarrow z = x \lor z = y))$

3. For each knowledge base, determine whether it is satisfiable. Justify your answer intuitively.

(a) $P(A), (\exists x)(\neg P(x))$

This knowledge base is satisfiable. It is only known that $P(x)$ returns true for A.

(b) $P(A), (\forall x)(\neg P(x))$

This knowledge base is not satisfiable. As it is given that $P(x)$ is true for A, it is impossible for $P(x)$ to always be false.

(c) $(\forall x)(\exists y)(P(x, y)), (\forall x)(\neg P(x, x)))$

This knowledge base is satisfiable. This is true if x and y are different in $P(x, y)$.

(d) $(\forall x)(P(x) \Rightarrow (\exists x)(P(x)))$

This knowledge base is satisfiable. This is trivially true as the validity of $P(x)$ for all x implies that there is some $P(x)$.

(e) $(\forall x)(P(x) \Rightarrow (\forall x)(P(x)))$

This knowledge base is satisfiable. This works in the case that x satisfies $P(x)$ and everyone else satisfies $P(x)$.

4. Consider the following sentences:

- John likes all kinds of food.
- Apples are food.
- Chicken is food.
- Anything anyone eats and isn't killed by is food.
- If you are killed by something, you are not alive.
- Bill eats peanuts and is still alive. *
- Sue eats everything Bill eats.

(a) Translate these sentences into formulas in first-order logic.

- $\forall x(\text{Food}(x) \Rightarrow \text{Likes}(\text{John}, x))$
- $\text{Food}(\text{Apples})$
- $\text{Food}(\text{Chicken})$
- $\forall x \forall y((\text{Eats}(x, y) \wedge \neg \text{Kills}(y, x)) \Rightarrow \text{Food}(y))$
- $\forall x \forall y(\text{Kills}(y, x) \Rightarrow \neg \text{Alive}(x))$
- $\text{Eats}(\text{Bill}, \text{Peanuts}) \wedge \text{Alive}(\text{Bill})$
- $\forall x(\text{Eats}(\text{Bill}, x) \Rightarrow \text{Eats}(\text{Sue}, x)$

(b) Convert the formulas of part (a) into CNF (also called clausal form).

1. Eliminate $\Rightarrow$ and $\Leftrightarrow$
   - $\forall x(\neg \text{Food}(x) \vee \text{Likes}(\text{John}, x))$
   - $\text{Food}(\text{Apples})$
   - $\text{Food}(\text{Chicken})$
   - $\forall x \forall y(\neg(\text{Eats}(x, y) \wedge \neg \text{Kills}(y, x)) \vee \text{Food}(y))$

- $\forall x \forall y(\neg \text{Kills}(y, x) \vee \neg \text{Alive}(x))$
- $\text{Eats}(\text{Bill}, \text{Peanuts}) \wedge \text{Alive}(\text{Bill})$
- $\forall x(\neg \text{Eats}(\text{Bill}, x) \vee \text{Eats}(\text{Sue}, x)$

2. Move $\neg$ inwards
   - $\forall x(\neg \text{Food}(x) \vee \text{Likes}(\text{John}, x))$
   - $\text{Food}(\text{Apples})$
   - $\text{Food}(\text{Chicken})$
   - $\forall x \forall y(\neg \text{Eats}(x, y) \vee \text{Kills}(y, x) \vee \text{Food}(y))$
   - $\forall x \forall y(\neg \text{Kills}(y, x) \vee \neg \text{Alive}(x))$
   - $\text{Eats}(\text{Bill}, \text{Peanuts}) \wedge \text{Alive}(\text{Bill})$
   - $\forall x(\neg \text{Eats}(\text{Bill}, x) \vee \text{Eats}(\text{Sue}, x)$
3. Standardize variables
   - $\forall a(\neg \text{Food}(a) \vee \text{Likes}(\text{John}, a))$
   - $\text{Food}(\text{Apples})$
   - $\text{Food}(\text{Chicken})$
   - $\forall b \forall c(\neg \text{Eats}(b, c) \vee \text{Kills}(c, b) \vee \text{Food}(c))$
   - $\forall d \forall e(\neg \text{Kills}(e, d) \vee \neg \text{Alive}(d))$
   - $\text{Eats}(\text{Bill}, \text{Peanuts}) \wedge \text{Alive}(\text{Bill})$
   - $\forall f(\neg \text{Eats}(\text{Bill}, f) \vee \text{Eats}(\text{Sue}, f)$
4. Skolemization
   - $\forall a(\neg \text{Food}(a) \vee \text{Likes}(\text{John}, a))$
   - $\text{Food}(\text{Apples})$
   - $\text{Food}(\text{Chicken})$
   - $\forall b \forall c(\neg \text{Eats}(b, c) \vee \text{Kills}(c, b) \vee \text{Food}(c))$
   - $\forall d \forall e(\neg \text{Kills}(e, d) \vee \neg \text{Alive}(d))$
   - $\text{Eats}(\text{Bill}, \text{Peanuts}) \wedge \text{Alive}(\text{Bill})$
   - $\forall f(\neg \text{Eats}(\text{Bill}, f) \vee \text{Eats}(\text{Sue}, f)$
5. Get rid of universal quantifiers
   - $\neg \text{Food}(a) \vee \text{Likes}(\text{John}, a)$
   - $\text{Food}(\text{Apples})$
   - $\text{Food}(\text{Chicken})$
   - $\neg \text{Eats}(b, c) \vee \text{Kills}(c, b) \vee \text{Food}(c)$
   - $\neg \text{Kills}(e, d) \vee \neg \text{Alive}(d)$
   - $\text{Eats}(\text{Bill}, \text{Peanuts}) \wedge \text{Alive}(\text{Bill})$
   - $\neg \text{Eats}(\text{Bill}, f) \vee \text{Eats}(\text{Sue}, f)$
6. Distribute $\vee$ and $\wedge$
   - $\neg \text{Food}(a) \vee \text{Likes}(\text{John}, a)$
   - $\text{Food}(\text{Apples})$
   - $\text{Food}(\text{Chicken})$

- $\neg \text{Eats}(b, c) \vee \text{Kills}(c, b) \vee \text{Food}(c)$
- $\neg \text{Kills}(e, d) \vee \neg \text{Alive}(d)$
- $\text{Eats}(\text{Bill}, \text{Peanuts}) \wedge \text{Alive}(\text{Bill})$
- $\neg \text{Eats}(\text{Bill}, f) \vee \text{Eats}(\text{Sue}, f)$

(c) Prove that John likes peanuts using resolution.

1.	$\neg \text{Food}(a) \vee \text{Likes}(\text{John}, a)$	
2.	$\text{Food}(\text{Apples})$	
3.	$\text{Food}(\text{Chicken})$	
4.	$\neg \text{Eats}(b, c) \vee \text{Kills}(c, b) \vee \text{Food}(c)$	
5.	$\neg \text{Kills}(e, d) \vee \neg \text{Alive}(d)$	
6.	$\text{Eats}(\text{Bill}, \text{Peanuts})$	
7.	$\text{Alive}(\text{Bill})$	
8.	$\neg \text{Eats}(\text{Bill}, f) \vee \text{Eats}(\text{Sue}, f)$	
9.	$\neg \text{Likes}(\text{John}, \text{Peanuts})$	
10.	$\neg \text{Food}(\text{Peanuts})$	1, 9
11.	$\neg \text{Kills}(e, \text{Bill})$	5, 7
12.	$\neg \text{Eats}(\text{Bill}, c) \vee \text{Food}(c)$	4, 11
13.	$\text{Food}(\text{Peanuts})$	6, 12

Contradiction between statements 10 and 13. Therefore John likes peanuts.

(d) Use resolution to answer the question "What food does Sue eat?"

1.	$\neg \text{Food}(a) \vee \text{Likes}(\text{John}, a)$	
2.	$\text{Food}(\text{Apples})$	
3.	$\text{Food}(\text{Chicken})$	
4.	$\neg \text{Eats}(b, c) \vee \text{Kills}(c, b) \vee \text{Food}(c)$	
5.	$\neg \text{Kills}(e, d) \vee \neg \text{Alive}(d)$	
6.	$\text{Eats}(\text{Bill}, \text{Peanuts})$	
7.	$\text{Alive}(\text{Bill})$	
8.	$\neg \text{Eats}(\text{Bill}, f) \vee \text{Eats}(\text{Sue}, f)$	
9.	$\neg \text{Eats}(\text{Sue}, \text{Peanuts})$	
10.	$\text{Eats}(\text{Sue}, \text{Peanuts})$	6, 8

Contradiction between statements 10 and 13. Therefore Sue eats peanuts.

(e) Use resolution to answer (d) if, instead of the axiom marked with an asterisk above we had:

- If you don't eat you die.
- If you die, you are not alive.
- Bill is alive.

Converted to first-order logic:

- $\forall x \forall y (\neg \text{Eats}(x, y) \Rightarrow \text{Dies}(x))$
- $\forall x (\text{Dies}(x) \Rightarrow \neg \text{Alive}(x))$
- $\text{Alive}(\text{Bill})$

Converted to CNF:

- $\text{Eats}(g, h) \vee \text{Dies}(g)$
- $\neg\text{Dies}(i) \vee \neg\text{Alive}(i)$
- $\text{Alive}(\text{Bill})$

New knowledge base:

1.	$\neg\text{Food}(a) \vee \text{Likes}(\text{John}, a)$	
2.	$\text{Food}(\text{Apples})$	
3.	$\text{Food}(\text{Chicken})$	
4.	$\neg\text{Eats}(b, c) \vee \text{Kills}(c, b) \vee \text{Food}(c)$	
5.	$\neg\text{Kills}(e, d) \vee \neg\text{Alive}(d)$	
8.	$\neg\text{Eats}(\text{Bill}, f) \vee \text{Eats}(\text{Sue}, f)$	
9.	$\text{Eats}(g, h) \vee \text{Dies}(g)$	
10.	$\neg\text{Dies}(i) \vee \neg\text{Alive}(i)$	
11.	$\text{Alive}(\text{Bill})$	
12.	$\neg\text{Eats}(\text{Sue}, \text{Apples})$	
13.	$\text{Dies}(\text{Sue})$	9, 12
14.	$\neg\text{Alive}(\text{Sue})$	10, 13
15.	$\neg\text{Kills}(e, \text{Bill})$	5, 11
16.	$\neg\text{Eats}(\text{Bill}, \text{Apples})$	8, 12
17.	$\text{Dies}(\text{Bill})$	9, 16
18.	$\neg\text{Alive}(\text{Bill})$	10, 17

Contradiction between statements 11 and 18. Therefore Sue eats apples.