No.

Date

Penurunan Rumus Double Pendulum

Diasumsikan batang tidak bermassa, Panjang l_1 dan l_2, massa m_1 & m_2 titik O yang bertepatan dengan titik Supensi pendulum atas

Koordinat pendulum ditentukan oleh :

$$x_1 = l_1 \sin \alpha_1 \ , \ x_2 = l_1 \sin \alpha_2$$

$$y_1 = -l_1 \cos \alpha_1 \ , \ y_2 = -l_1 \cos \alpha_2$$

Energi kinetik dan potensial pendulum (masing-masing T dan V) dinyatakan :

$$T = \frac{m_1 V_1^2}{2} + \frac{m_2 V_2^2}{2} = \frac{m_1(\dot{x}_1^2 + \dot{y}_1^2)}{2} + \frac{m_2(\dot{x}_2^2 + \dot{y}_2^2)}{2}$$

$$V = m_1 g y_1 + m_2 g y_2$$

Lagrangian :

$$L = T - V = T_1 + T_2 - (V_1 + V_2) = \frac{m_1}{2}(\dot{x}_1^2 + \dot{y}_1^2) + \frac{m_2}{2}(\dot{x}_2^2 + \dot{y}_2^2) - m_1 g y_1 - m_2 g y_2$$

$$\dot{x}_1 = l_1 \cos \alpha_1 \cdot \dot{\alpha}_1 \ , \quad \dot{x}_2 = l_1 \cos \alpha_1 \cdot \dot{\alpha}_1 + l_2 \cos \cdot \dot{\alpha}_2$$

$$\dot{y}_1 = l_1 \cos \alpha_1 \cdot \dot{\alpha}_1 \ , \quad y_2 = l_1 \sin \alpha_1 \cdot \dot{\alpha}_1 + l_2 \sin \alpha_2 \cdot \dot{\alpha}_2$$

Sehingga,

$$T_1 = \frac{m_1}{2}(x_1^2 + y_1^2) = \frac{m_1}{2}(l_1^2 \dot{\alpha}_1^2 \cos^2 \alpha_1 + l_1^2 \dot{\alpha}_1^2 \sin^2 \alpha_1) = \frac{m_1}{2} l_1^2 \dot{\alpha}_1^2$$

$$T_2 = \frac{m_2}{2}(\dot{x}_2^2 + y_2^2) = \frac{m_2}{2}\left[(l_1 \dot{\alpha}_1 \cos \alpha_1 + l_2 \dot{\alpha}_2 \cos \alpha_2)^2 + (l_1 \dot{\alpha}_1 \sin \alpha_1 + l_2 \dot{\alpha}_2 \sin \alpha_2)^2\right]$$

$$= \frac{m_2}{2}\Big[l_1^2 \dot{\alpha}_1^2 \cos^2 \alpha_1 + l_2^2 \dot{\alpha}_2^2 \cos^2 \alpha_2 + 2 l_1 l_2 \dot{\alpha}_1 \dot{\alpha}_2 \cos \alpha_1 \cos \alpha_2$$

$$+ l_1^2 \dot{\alpha}_1^2 \sin^2 \alpha_1 + l_2^2 \dot{\alpha}_2^2 \sin \alpha_2 + 2 l_1 l_2 \dot{\alpha}_1 \dot{\alpha}_2 \sin \alpha_1 \sin \alpha_2\Big]$$

$$= \frac{m_2}{2}\left[l_1^2 \dot{\alpha}_1^2 + l_2^2 \dot{\alpha}_2^2 + 2 l_1 l_2 \alpha_1 \alpha_2 \cos(\dot{\alpha}_1 - \dot{\alpha}_2)\right]$$

$$V_1 = m_1 g y_1 = -m_1 g l_1 \cos \alpha_1$$

$$V_2 = m_2 g y_2 = -m_2 g (l_1 \cos \alpha_1 + l_2 \cos \alpha_2)$$

Sehingga,

$$L = T - V = T_1 + T_2 - (V_1 + V_2) = \left(\frac{m_1}{2} + \frac{m_2}{2}\right) l_1^2 \dot{\alpha}_1^2 + \frac{m_2}{2} l_2^2 \dot{\alpha}_2^2 + m_2 l_1 l_2 \dot{\alpha}_1 \dot{\alpha}_2 \cos(\alpha_1 - \alpha_2)$$

$$+ (m_1 + m_2) g l_1 \cos \alpha_1 + m_2 g l_2 \cos \alpha_2.$$