Signal processing

Covers signals, systems, impulse responses, and filtering. The autocorrelation is introduced and the sampling process discussed.

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Convolution

Remember from last lecture

Convolution:

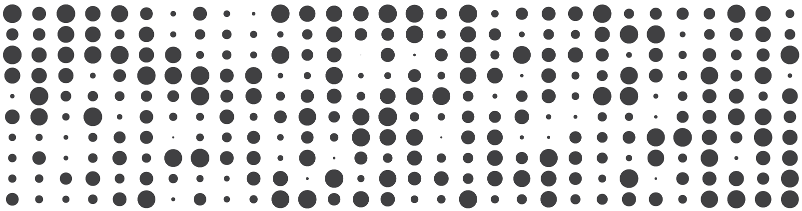
$$f(t) * g(t) := \int_{-\infty}^{\infty} f(\tau) g(t-\tau) \, \mathrm{d}\tau$$

Convolution theorem:

$$\mathcal{F}\{f * g\} = \mathcal{F}\{f\} \cdot \mathcal{F}\{g\}$$
$$\mathcal{F}\{f \cdot g\} = \mathcal{F}\{f\} * \mathcal{F}\{g\}$$

Signals and systems

A signal x is a function or sequence of values that represents information



Classification of signals:

- dimensionality
- continuous - discrete
- analogue - digital
- real-valued - complex-valued
- finite - infinite
- deterministic - stochastic

$$x = x(\mathbf{n}), \mathbf{n} \in \mathbb{R}^N$$

Signals and systems

A system $\mathcal{S}$ maps an input signal x onto an output signal y

$$x \longrightarrow \boxed{\mathcal{S}} \longrightarrow y$$

with $y = \mathcal{S}\{x\}$.

Classification of systems:

- dimensionality $\quad x = x(\mathbf{n}), y = y(\mathbf{n})$
- linear $\quad \mathcal{S}\{\lambda x_1 + x_2\} = \lambda\mathcal{S}\{x_1\} + \mathcal{S}\{x_2\}$
- linear time-shift invariant (LTI) $\quad \mathcal{S}\{x(t-\tau)\} = y(t-\tau)$

LTI System

Impulse response contains all relevant system properties

The output signal $y(t)$ of an LTI system is given as

$$y(t) = x(t) * h(t)$$

with the impulse response $h(t) := \mathcal{S}\{\delta(t)\}$.

Fourier transformation together with convolution theorem leads to

$$Y(\omega) = X(\omega)H(\omega)$$

with the transfer function $H(\omega) = \mathcal{F}\{h(t)\}$.

Autocorrelation

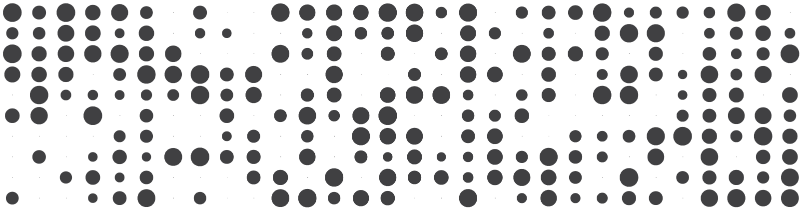
Measure correlation of a signal with itself

$$\varphi_{xx}(\tau) := \int_{-\infty}^{\infty} x(t)\, x(t+\tau)\, \mathrm{d}t$$

- Symmetric $\varphi_{xx}(\tau) = \varphi_{xx}(-\tau)$
- Maximum of $\varphi_{xx}(\tau)$ at $\tau = 0$
- Measure of periodicity of signal

Filter

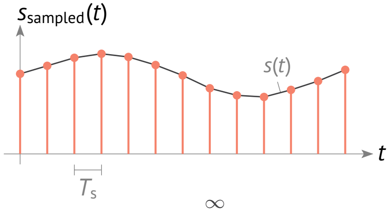
A filter is a system for manipulating signals in a desired way

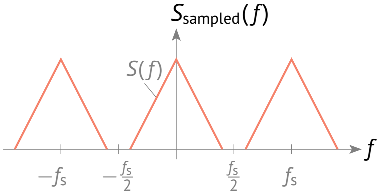


- Filter allow frequency dependent attenuation
- Manipulation in frequency domain have consequences for time domain
- Filter design accounts for this

Sampling

Sample signal at discrete time values





$$s_{\text{sampled}}(t) = s(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

- Sampling with constant frequency $f_s = \frac{1}{T_s}$
- Multiplication with Dirac comb

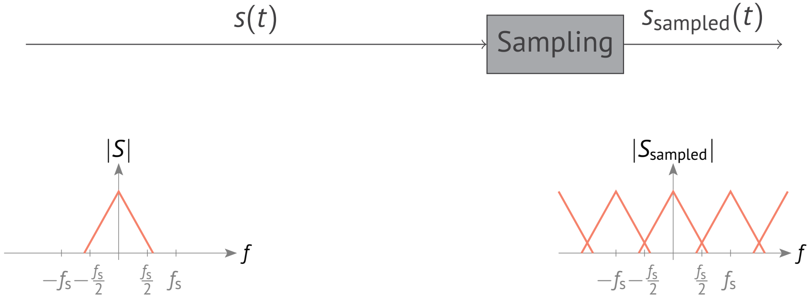
$$S_{\text{sampled}}(f) = S(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$

- Repetitions of signal spectrum
- Perfect reconstruction possible if no spectral overlap

Nyquist-Shannon sampling theorem: $f_s \geq 2f_{\max}$

Sampling

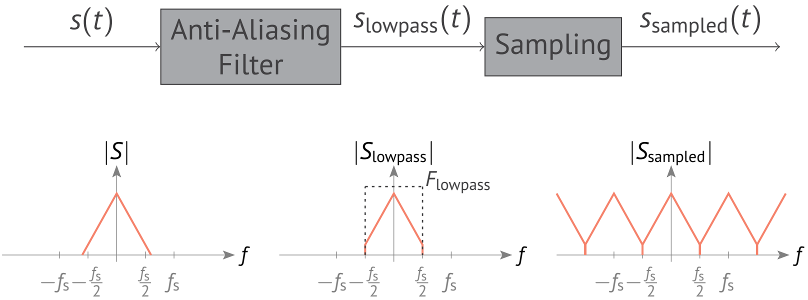
Sampling process



- If the sampling theorem is violated, aliasing occurs

Sampling

Sampling process



- If the sampling theorem is violated, aliasing occurs
- Can be avoided by anti-aliasing filter

Sampling

Reconstruction process