

Building a Function Generator

CIEP

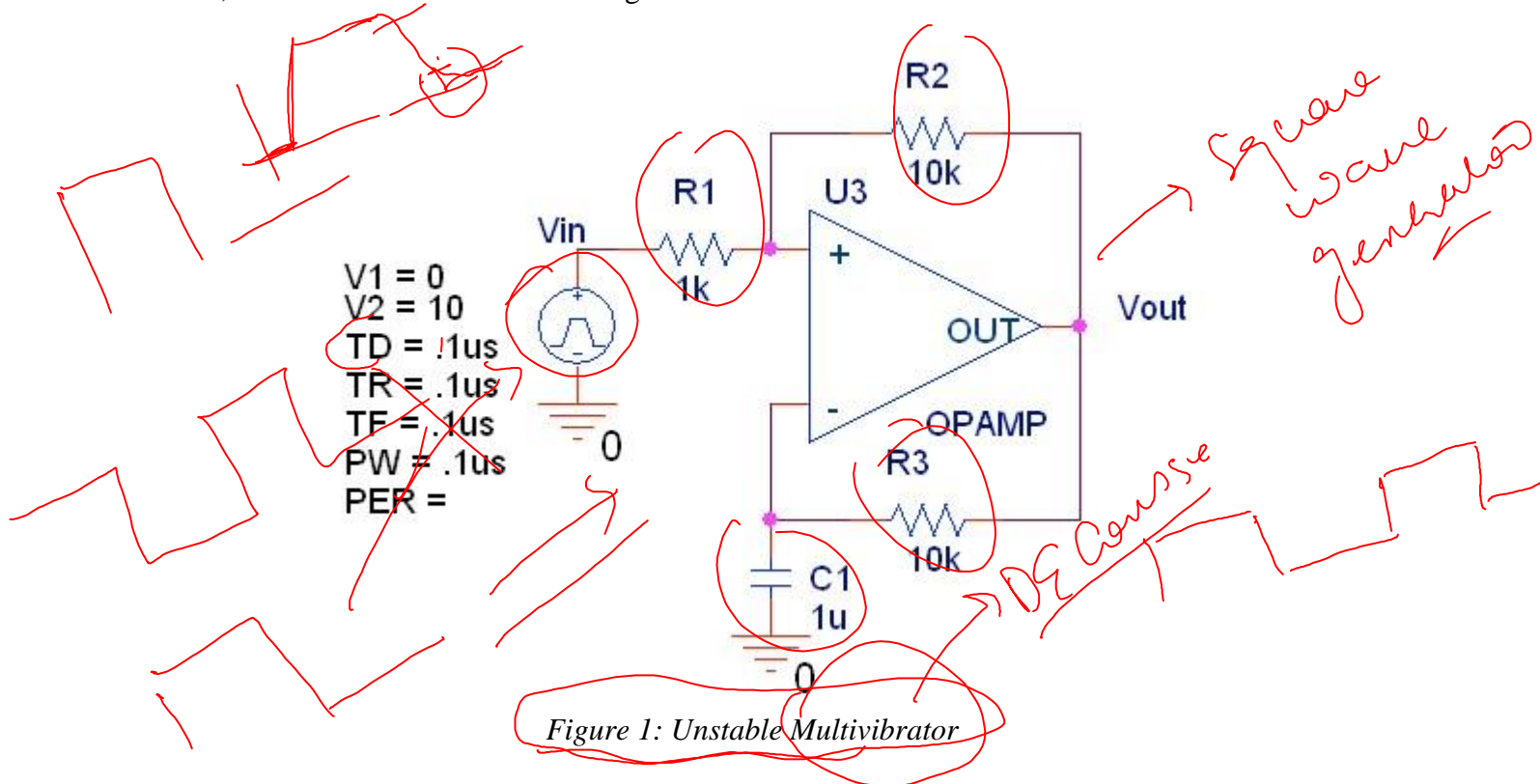
Objectives

The objective is to build a function generator capable of generating square, triangle, and sine waves. The frequency and amplitude of these waves will be controlled through potentiometers. There will be some frequency analysis to aid in the understanding of Laplace transforms, Bode plots, and Fourier series.

Simulation Part

Part One: Square Wave Generator

1. Simulate the unstable multivibrator shown in Figure 1. The voltage source in the diagram is a voltage pulse needed to start the circuit oscillating. Note that when you actually build the circuit in lab, noise in the circuit will be enough to start oscillations.



2. The period of the output square wave can be calculated as:

$$T = 2R_3C_1 \ln\left(\frac{1+\beta}{1-\beta}\right)$$

where $\beta = R_1/(R_1 + R_2)$.

Examine the frequency of the output voltage for the following component values:

R_3	C_1	Frequency (Calculated)	Frequency (Simulated)
10K	1 μ F	✓	✓
9K	1 μ F	✓	✓
7K	1 μ F	✓	✓
10K	2 μ F	✓	✓
10K	3 μ F	✓	✓

Table 1: Frequency Comparisons for Varying Resistances and Capacitances

To measure the frequency of the output waveform, measure the time it takes for the square wave to complete one full cycle, then take the reciprocal of this time.

Part Two: Integrator

1. Simulate the circuit shown in Figure 2.

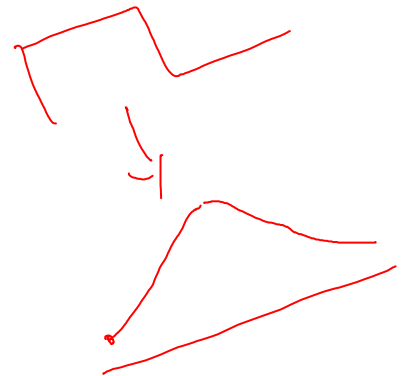
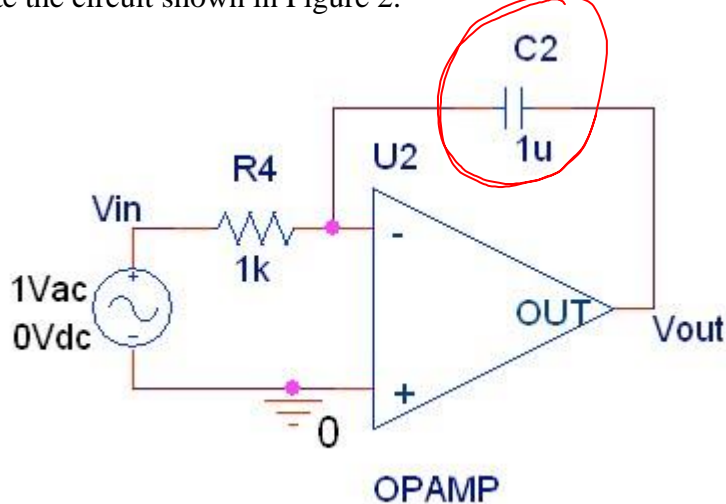


Figure 2: Integrator

2. Find the transfer function by hand, i.e., $V_{out}(s)/V_{in}(s)$. The input of the integrator is going to be the output of the square-wave generator, either +15V or -15V, DC. For the sake of analysis,

we can say that $v_{in}(t) = -15u(t)$ or $v_{in}(t) = 15u(t)$, depending on if the square wave is at its minimum or maximum value.

If we take the input as $-15u(t)$, what will the output of the integrator be in both the frequency and time domains (hint: the integral of a constant)? What will the output be in both the frequency and time domains if the input is $+15u(t)$?

$V_{out}(s)/V_{in}(s)$ _____

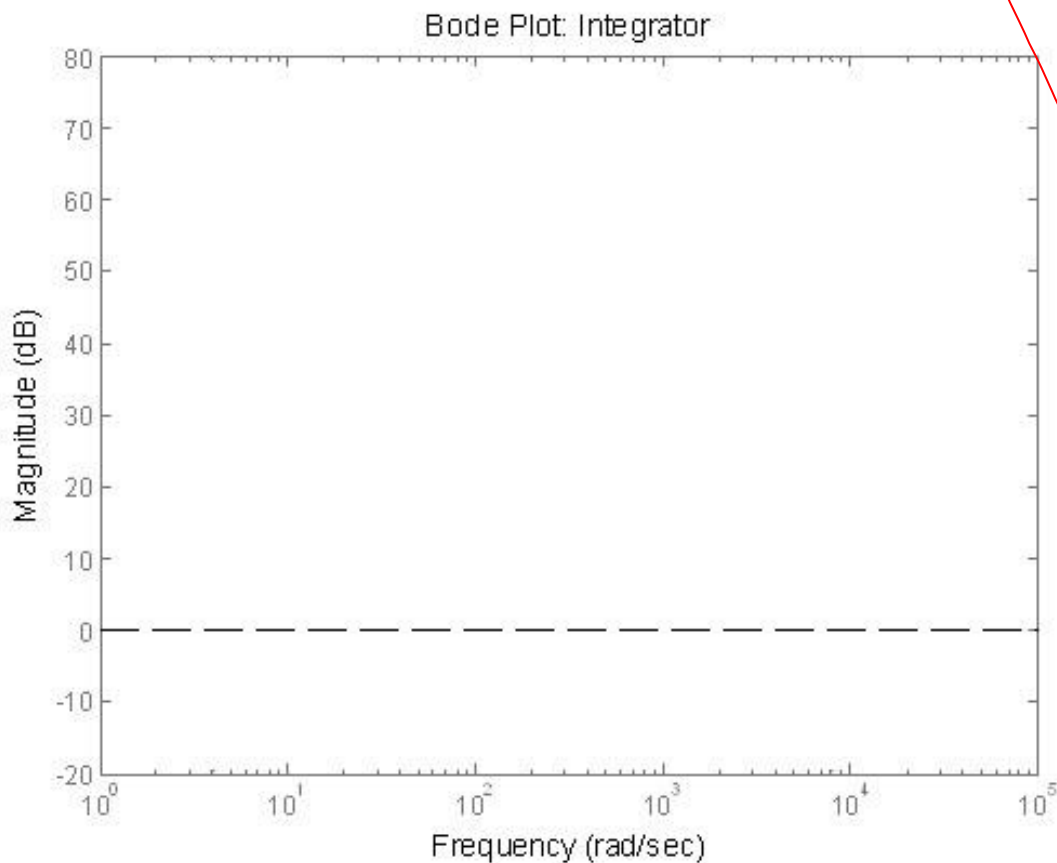
$V_{out}(s) (v_{in}(t) = -15u(t))$ _____

$v_{out}(t) (v_{in}(t) = -15u(t))$ _____

$V_{out}(s) (v_{in}(t) = +15u(t))$ _____

$v_{out}(t) (v_{in}(t) = +15u(t))$ _____

3. Sketch the asymptotic Bode plot of the transfer function of the integrator:



3. In the simulation, make a Bode plot of the transfer function by conducting an AC sweep on the input voltage. In the simulation profile for AC Sweep, set the start frequency to 1Hz, the end frequency to 100,000Hz, and the points per decade to 100.

How much does the gain (value plotted on the y-axis) change per decade (a factor of 10, e.g., between 100rad/s and 1000rad/s)?

dB/decade _____

Part Three: Low-Pass Filter

1. Simulate the circuit shown in Figure 3.

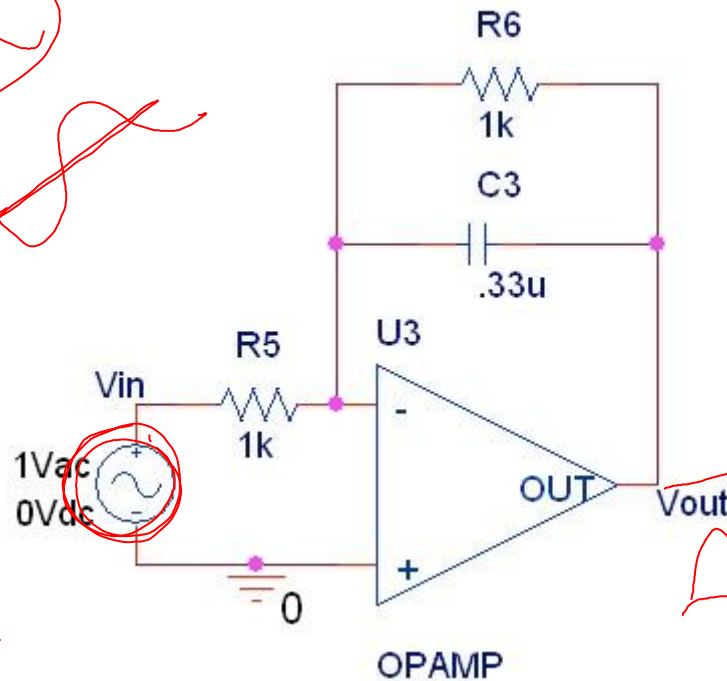
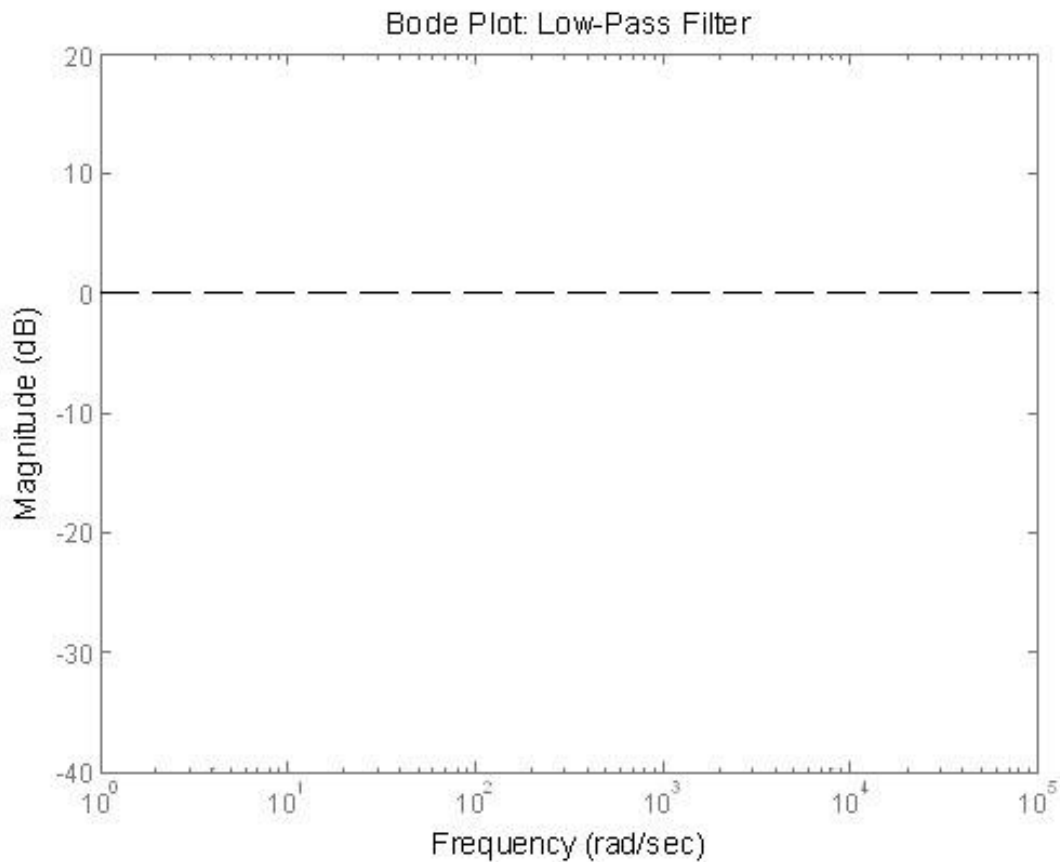


Figure 3: Low-Pass Filter

2. Find the transfer function by hand, i.e., $V_{out}(s)/V_{in}(s)$.

$V_{out}(s)/V_{in}(s)$ _____

3. Sketch the asymptotic Bode Plot of the transfer function of the low-pass filter:



- Fourier Theorem*
4. The theory behind Fourier series is that any periodic function, $f(t)$, with period T , can be written as the sum of weighted sinusoids. The frequency of each sinusoid is a positive integer multiple of ω_0 , the frequency in rad/s of $f(t)$ ($\omega_0 = 2\pi/T$). Each sinusoid also has a coefficient multiplying it, determining how strongly that sine (or cosine) wave contributes to the function $f(t)$. Derive the coefficients (a_0 , a_n , and b_n) of the Fourier series representation of the following signal:

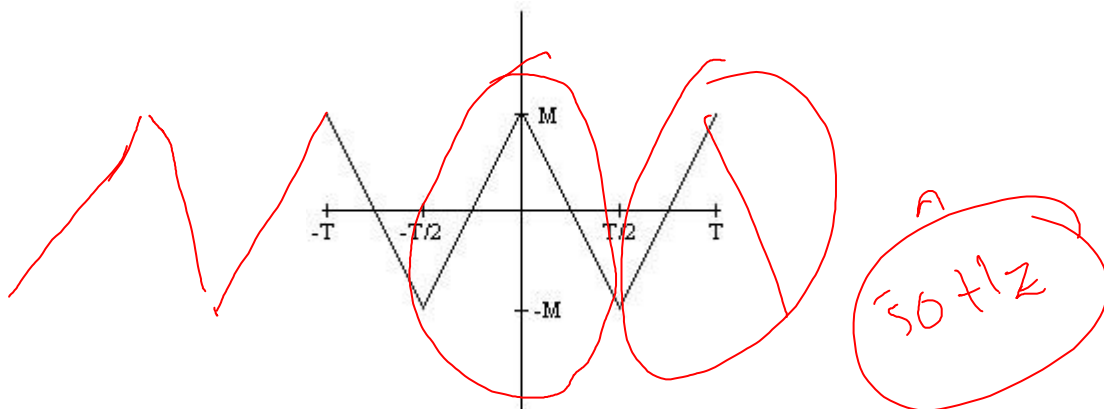


Figure 4: Triangle-Wave Input (to the Low-Pass Filter)

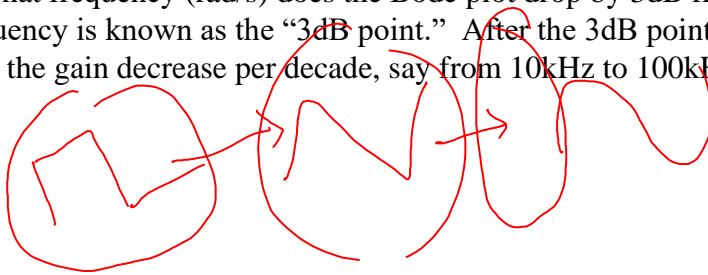
Here is a hint to aid you in finding the coefficients: Is this function even or odd? For even functions, the function is comprised entirely of cosine waves. For odd functions, the function is comprised entirely of sine waves. Find a_0 , a_n and b_n .

a_0 _____
 a_n _____
 b_n _____

4. For $M = 15V$ and $T = 3ms$, write down the first three terms in the Fourier series expansion of the triangle wave shown in Figure 4.

5. Make a Bode plot of the transfer function by conducting an AC sweep on the input voltage. In the simulation profile for AC Sweep, set the start frequency to 1Hz, the end frequency to 100,000Hz and the points per decade to 100.

At what frequency (rad/s) does the Bode plot drop by 3dB from its constant value? This frequency is known as the "3dB point." After the 3dB point, what is the rolloff (i.e., how much does the gain decrease per decade, say from 10kHz to 100kHz)?



3dB point (rad/s) _____

Rolloff (dB/decade) _____

Part Four: Function Generator

1. Cascade the three previous op-amp circuits to simulate the circuit shown in Figure 5.

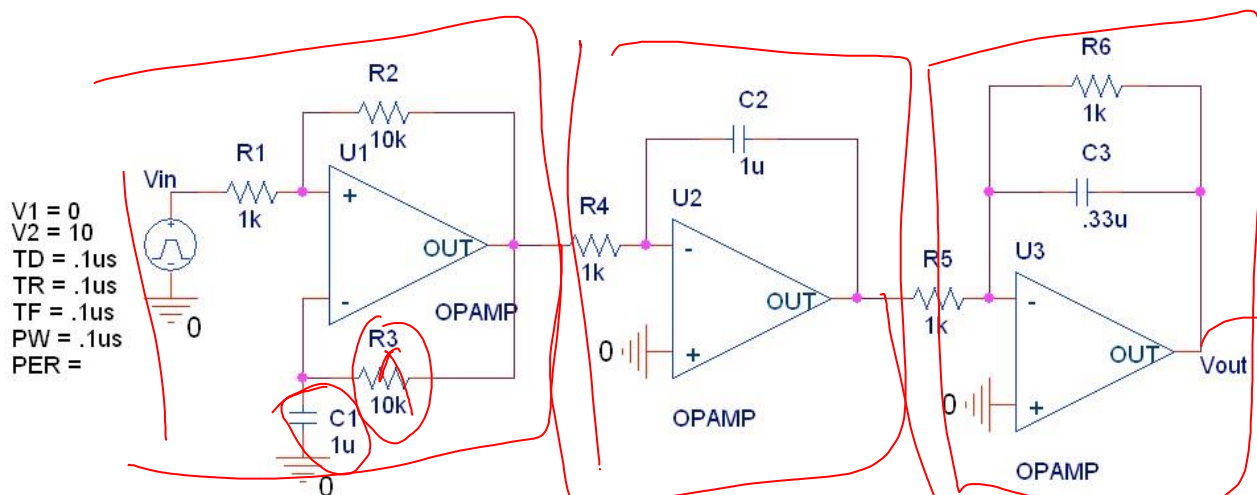


Figure 5: Function Generator

2. Plot the output of each op-amp (on the same plot). Change R_3 to 7K and print the three waveforms again (on another plot).

2. Using the results from the previous sections, explain in a paragraph or two how this function generator works. Discuss how the triangle and sine waves are generated (sketching graphs of the outputs may be useful). Be sure to mention the role of the integrator, as well as the low-pass filter in relation to the Fourier series representation of the triangle wave. ✓

Also comment on the following: Bode plots are useful in showing how the gain of a transfer function changes with the frequency of the input. Using this knowledge, respond to the question of why the amplitude of the triangle wave and the sine wave decrease in amplitude as R_3 is decreased from 10K to 7K (hint: look at the equation on the top of page 2, and also the bode plots of each op-amp stage).

Hardware Part

Part One: Build the Function Generator

1. Using either three 741 op-amp chips (shown in Figure 6), or one 324 op-amp chip (shown in Figure 7), wire the three stage op-amp circuit shown in Figure 8. The first op-amp stage generates a square wave. The second op-amp stage integrates the square wave to produce a triangle wave. The final op-amp stage is a low-pass filter, removing the higher frequency components of the triangle wave so that it closely resembles a sine wave with the same fundamental frequency. It is recommended to wire one stage at a time (starting with the square wave) and ensure the proper functionality of that stage before wiring subsequent stages. The output of each op-amp is sent to one of the switches on the 4-switch dual in-line package (DIP), which selects the output function when one turns on the appropriate switch.

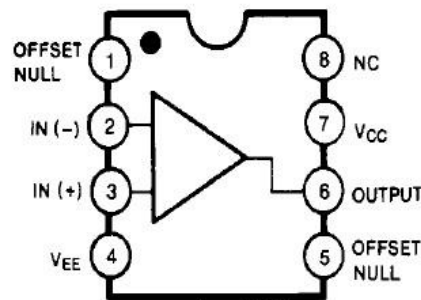


Figure 6: Pin Layout of the LM741 Op-Amp

$$V_{EE} = -15V$$

$$V_{CC} = 15V$$

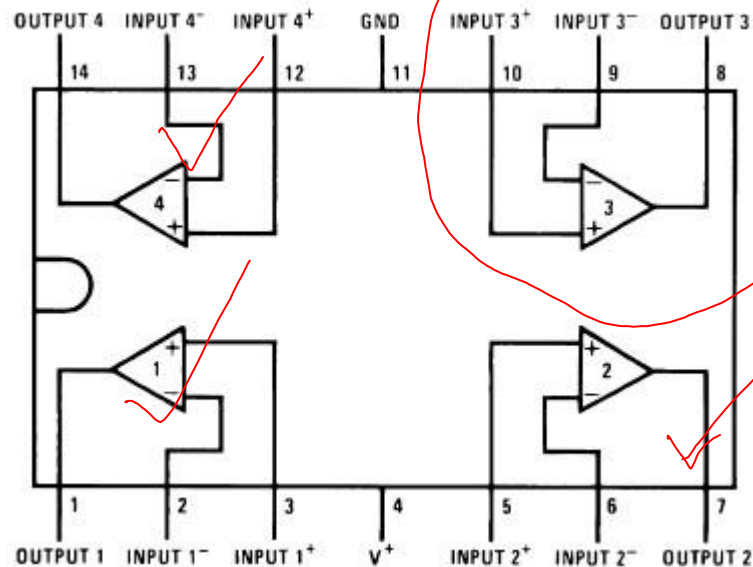


Figure 7: Pin Layout of the LM324 Op-Amp

$$V_{+} = 15V$$

$$GND = -15V$$

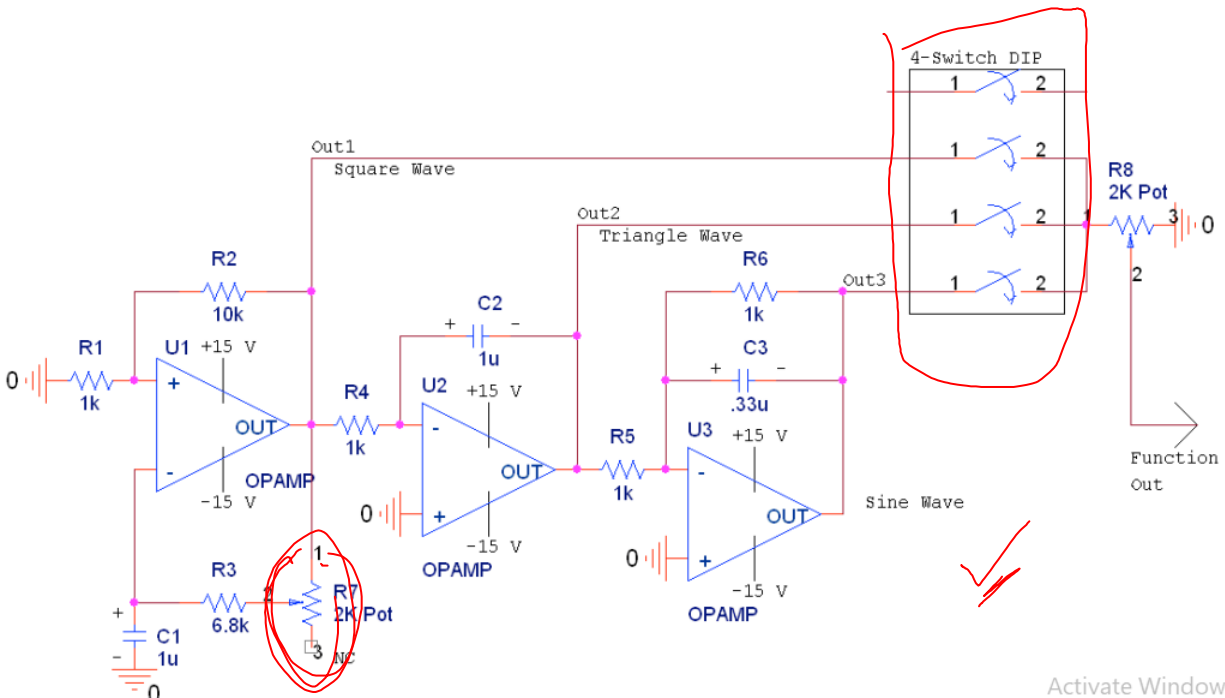


Figure 8: Circuit Diagram of the Function Generator

Part Two: Analyzing the Function Generator

1. Connect an oscilloscope probe to the point labeled “Function Out” in Figure 8. Select the output from the first op-amp (the square wave). You will now analyze the duty cycle of this square wave. Make sure that R8, the amplitude knob, is adjusted such that there is no resistance between the DIP-switch output and the oscilloscope probe (i.e. such that the amplitude of the output is at a maximum). The potentiometer R7 does not need to be turned to a specific resistance. The duty cycle of a square wave is defined as:

$$\text{Duty Cycle} = 100\% \times (\text{amount of time waveform is positive during one period} / \text{duration of one period})$$

For example, if a square wave has a period of 200ms and is positive for 50ms during each period, then the duty cycle is 25%. Record what happens to the duty cycle as you adjust V_{CC} , the positive voltage supply to the op-amp.

V_{EE}	V_{CC}	Time Positive per Period	Period	Duty Cycle
-15V	15V			
-15V	10V			
-15V	5V			

Table 2: Effect of V_{CC} on Duty Cycle

What happens to the positive voltage level of the square wave as V_{CC} is decreased? What happens to the negative voltage level?

What do you think would happen if V_{EE} were changed from $-15V$ to $-10V$? Record the duty cycle and the negative voltage level of the square wave to see if you are correct.

V_{EE}	V_{CC}	Time Positive per Period	Period	Duty Cycle
$-10V$	$15V$			

Table 3: Effect of V_{EE} on Duty Cycle

2. Return V_{CC} and V_{EE} to their respective maximum absolute values. Select the triangle wave output to be displayed on the oscilloscope. This waveform may not look quite like a triangle wave at its peaks, so adjust V_{CC} and V_{EE} until it looks as close to a triangle wave as you can get it. Change the switch to display the square wave and then the sine wave to convince yourself that the function generator is working properly. Record the values of V_{CC} and V_{EE} that are needed.

V_{CC} _____

V_{EE} _____

3. The potentiometer labeled R7 in Figure 8 controls the frequency of the square wave. Because the triangle and sine waves will have the same frequency as the square wave, this potentiometer is the frequency knob for the function generator. Fill in the following table for the maximum and minimum values of the resistance of the potentiometer, R7, between pins 1 and 2.

	Square Wave		Triangle Wave		Sine Wave	
R7 =	0K	2K	0K	2K	0K	2K
Voltage (p-p)						
Voltage (RMS)						
Frequency						

Table 4: Effect of R7 on Frequency and Output Voltage

It is studied before that the frequency of the generated square wave is governed by the following equation:

$$T = 2(R_3 + R_7)C_1 \ln\left(\frac{1 + \beta}{1 - \beta}\right)$$

where $\beta = R_1 / (R_1 + R_2)$.

As R_2 increases in the above equation, what should happen to the frequency? Do your results in the table above support your claim? Explain.

~~Part Three: Connecting the Function Generator to a Speaker~~

1. Using the maximum peak-to-peak voltage of the sine wave (from Table 2), design a voltage divider to reduce this voltage to 10mV peak-to-peak. Label the resistor values you choose in the figure below:

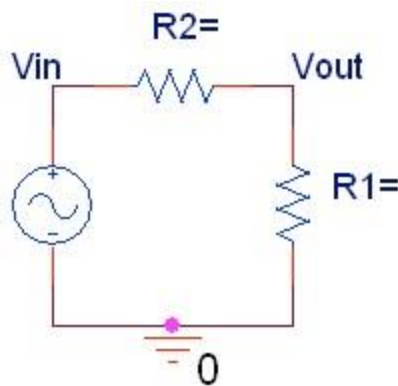
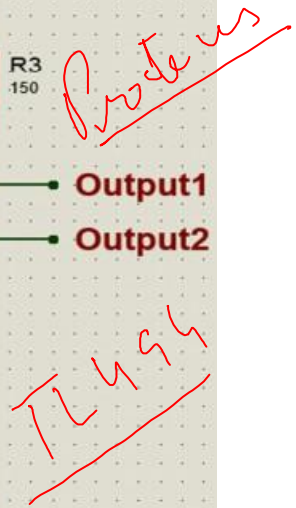


Figure 9: Voltage Divider to Reduce Function Generator Output to 2mV peak-to-peak

2. Connect the point labeled “Vout” in Figure 9 to one of the inputs of the stereo amplifier. Attach a speaker to the corresponding output channel, and play all three generated waveforms through the amplifier, varying the frequency (R_7) and amplitude (R_8) knobs to see how they affect the output signal you hear.

Figure below shows the circuit diagram for TLP494 for generation of different duty cycles at different frequencies. The circuit has been taken from its datasheet which should be read to understand the operation of the IC.



Tasks

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5. Observe the voltage across the oscillator capacitor and include the snapshot of waveform in your lab report.
6. Show waveforms for 20%, 40%, 60%, 80% and 90% duty cycles at all selected frequencies.
7. What is the function of dead time pin? How we can adjust the dead time? What is the dead time if that pin is grounded? Observe the dead time on the scope and include the snapshot in the lab report.
8. What is the function of output control pin? Configure it for push pull operation and observe the output waveforms then configure it for single ended operation and observe the output waveforms. Include the results in your lab report.
9. We have used variable voltage at the feedback pin using the potential divider. Configure error amplifiers to vary the duty cycle at the output terminals. Discuss all of your design in your lab report.
10. Use TL494 as a square wave generator in the above mentioned signal generator.



Appendix A Fourier Analysis

Under steady-state conditions, the output voltage of power converters is, generally, a periodic function of time defined by

$$v_o(t) = v_o(t + T) \quad (\text{E.1})$$

where T is the periodic time. If f is the frequency of the output voltage in hertz, the angular frequency is

$$\omega = \frac{2\pi}{T} = 2\pi f \quad (\text{E.2})$$

and Eq. (E.1) can be rewritten as

$$v_o(\omega t) = v_o(\omega t + 2\pi) \quad (\text{E.3})$$

The Fourier theorem states that a periodic function $v_o(t)$ can be described by a constant term plus an infinite series of sine and cosine terms of frequency $n\omega$, where n is an integer. Therefore, $v_o(t)$ can be expressed as

$$v_o(t) = \frac{a_o}{2} + \sum_{n=1,2,\dots}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t) \quad (\text{E.4})$$

where $a_o/2$ is the average value of the output voltage $v_o(t)$. The constants a_o , a_n , and b_n can be determined from the following expressions:

$$a_o = \frac{2}{T} \int_0^T v_o(t) dt = \frac{1}{\pi} \int_0^{2\pi} v_o(\omega t) d(\omega t) \quad (\text{E.5})$$

$$a_n = \frac{2}{T} \int_0^T v_o(t) \cos n\omega t dt = \frac{1}{\pi} \int_0^{2\pi} v_o(\omega t) \cos n\omega t d(\omega t) \quad (\text{E.6})$$

$$b_n = \frac{2}{T} \int_0^T v_o(t) \sin n\omega t dt = \frac{1}{\pi} \int_0^{2\pi} v_o(\omega t) \sin n\omega t d(\omega t) \quad (\text{E.7})$$

If $v_o(t)$ can be expressed as an analytical function, these constants can be determined by a single integration. If $v_o(t)$ is discontinuous, which is usually the case for the output

of converters, several integrations (over the whole period of the output voltage) must be performed to determine the constants, a_o , a_n , and b_n .

$$\begin{aligned} & a_n \cos n\omega t + b_n \sin n\omega t \\ &= (a_n^2 + b_n^2)^{1/2} \left(\frac{a_n}{\sqrt{a_n^2 + b_n^2}} \cos n\omega t + \frac{b_n}{\sqrt{a_n^2 + b_n^2}} \sin n\omega t \right) \end{aligned} \quad (\text{E.8})$$

Let us define an angle ϕ_n , whose adjacent side is b_n , opposite side is a_n , and the hypotenuse is $(a_n^2 + b_n^2)^{1/2}$. As a result, Eq. (E.8) becomes

$$\begin{aligned} a_n \cos n\omega t + b_n \sin n\omega t &= (a_n^2 + b_n^2)^{1/2} (\sin \phi_n \cos n\omega t + \cos \phi_n \sin n\omega t) \\ &= (a_n^2 + b_n^2)^{1/2} \sin(n\omega t + \phi_n) \end{aligned} \quad (\text{E.9})$$

where

$$\phi_n = \tan^{-1} \frac{a_n}{b_n} \quad (\text{E.10})$$

Substituting Eq. (E.9) into Eq. (E.4), the series may also be written as

$$v_o(t) = \frac{a_o}{2} + \sum_{n=1,2,\dots}^{\infty} C_n \sin(n\omega t + \phi_n) \quad (\text{E.11})$$

where

$$C_n = (a_n^2 + b_n^2)^{1/2} \quad (\text{E.12})$$

C_n and ϕ_n are the peak magnitude and the delay angle of the n th harmonic component of the output voltage $v_o(t)$, respectively.

If the output voltage has a *half-wave symmetry*, the number of integrations within the entire period can be reduced significantly. A waveform has the property of a half-wave symmetry if the waveform satisfies the following conditions:

$$v_o(t) = -v_o\left(t + \frac{T}{2}\right) \quad (\text{E.13})$$

or

$$v_o(\omega t) = -v_o(\omega t + \pi) \quad (\text{E.14})$$

In a waveform with a half-wave symmetry, the negative half-wave is the mirror image of the positive half-wave, but phase shifted by $T/2$ s (or π rad) from the positive half-wave. A waveform with a half-wave symmetry does not have the even harmonics (i.e., $n = 2, 4, 6, \dots$) and possess only the odd harmonics (i.e., $n = 1, 3, 5, \dots$). Due

to the half-wave symmetry, the average value is zero (i.e., $a_o = 0$). Equations (E.6), (E.7), and (E.11) become

$$a_n = \frac{2}{T} \int_0^T v_o(t) \cos n\omega t dt = \frac{1}{\pi} \int_0^{2\pi} v_o(\omega t) \cos n\omega t d(\omega t), \quad n = 1, 3, 5, \dots$$

$$b_n = \frac{2}{T} \int_0^T v_o(t) \sin n\omega t dt = \frac{1}{\pi} \int_0^{2\pi} v_o(\omega t) \sin n\omega t d(\omega t), \quad n = 1, 3, 5, \dots$$

$$v_o(t) = \sum_{n=1,3,5,\dots}^{\infty} C_n \sin(n\omega t + \phi_n)$$

In general, with a half-wave symmetry, $a_o = a_n = 0$, and with a quarter-wave symmetry, $a_o = b_n = 0$.

A waveform has the property of quarter-wave symmetry if the waveform satisfies the following conditions:

$$v_o(t) = -v_o\left(t + \frac{T}{4}\right) \quad (\text{E.15})$$

or

$$v_o(\omega t) = -v_o\left(\omega t + \frac{\pi}{2}\right) \quad (\text{E.16})$$