1. What is the Quaternion that would rotate this vector by 90 degrees around the zaxis?

$$q = cos(45) + 0sin(45)i + 0sin(45)j + 1sin(45)k$$
$$q = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}k$$

2. What quaternion would rotate this vector by 90 degrees around the y-axis?

$$q = cos(45) + 0sin(45)i + 1sin(45)j + 0sin(45)k$$
$$q = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j$$

3. What will we have by rotating vector v using the quaternion in question 1 and then using the quaternion in question 2?

$$\begin{aligned}
v' &= qvq^{-1} \\
&= (\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}k)(2i + 2j)(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}k) \\
&= (\sqrt{2}i + \sqrt{2}ki + \sqrt{2}j + \sqrt{2}kj)(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}k) \\
&= 2\sqrt{2}j(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}k) = 2j - 2jk = 2j - 2i \\
v' &= (\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j)(2j - 2i)(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j) \\
&= \sqrt{2}j + \sqrt{2}j^2 - \sqrt{2}i - \sqrt{2}ji(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j) \\
&= 2j + 2k
\end{aligned}$$

4. What is the quaternion that achieves the same rotation results by combining quaternion in question 1 and 2? In other words, what quaternion can we use to rotate the vector v to the result of question 3?

$$\begin{aligned}
q &= -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j \\
v' &= qvq' \\
&= (-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j)(2i + 2j)(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j) \\
&= -\sqrt{2}i + \sqrt{2}ji - \sqrt{2}j + \sqrt{2}j^2(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j) \\
&= i + k + j + 1 - ij + jk + j^2 + j \\
q' &= 2j + 2k
\end{aligned}$$