

4장 Panel Data 분석

Definition

- Panel data: Panel data contain observations on multiple phenomena observed over multiple time periods for the same entities (countries, firms, or individuals)
- Example)

country	year	Y	X1	X2	X3
1	2000	6.0	7.8	5.8	1.3
1	2001	4.6	0.6	7.9	7.8
1	2002	9.4	2.1	5.4	1.1
2	2000	9.1	1.3	6.7	4.1
2	2001	8.3	0.9	6.6	5.0
2	2002	0.6	9.8	0.4	7.2
3	2000	9.1	0.2	2.6	6.4
3	2001	4.8	5.9	3.2	6.4
3	2002	9.1	5.2	6.9	2.1

Model

- General linear panel model

$$y_{it} = \beta_{it}x_{it} + \alpha_{it} + u_{it}$$

⇒ Some restriction: $\beta_{it} = \beta$ and $\alpha_{it} = \alpha_i$

$$y_{it} = \beta x_{it} + \alpha_i + u_{it} \quad \text{with} \quad \text{Cov}(x_{it}, u_{it}) = 0 \quad \text{and} \quad \text{Cov}(\alpha_i, u_{it}) = 0$$

u_{it} : idiosyncratic error, usually $u_{it} \sim \text{ind } N(0, \sigma_u^2)$

α_i : time-invariant individual-specific component
individual heterogeneity

- Example) $\text{crime}_{it} = \beta \cdot \text{unemp}_{it} + \alpha_i + u_{it}$

$$\text{HP}_{it} = \beta_1 \cdot \text{crime}_{it} + \beta_2 \cdot \text{unemp}_{it} + \alpha_i + u_{it}$$

Estimation

- POLS (pooled OLS)

Disregard heterogeneity ($\alpha_i = \alpha$), and run OLS

If $Cov(\alpha_i, x_{it}) = 0$, Pools is consistent.

\Rightarrow Even if Pools is consistent, it is still inefficient in case of autocorrelation on u_{it} .

- LSDV (least square w/ dummy variable)

Put α_i as dummy variables, and run OLS

- FD (first difference)

Make the first difference in time (remove α_i from the model), and run OLS

$$\Delta y_{it} = \beta \Delta x_{it} + \Delta u_{it},$$

$$\text{with } \Delta y_{it} = y_{it} - y_{i,t-1}, \quad \Delta x_{it} = x_{it} - x_{i,t-1}$$

$$\text{Example) } \Delta \text{crime}_{it} = \beta \cdot \Delta \text{unemp}_{it} + \Delta u_{it}$$

Estimation

- FE (fixed effect)

$Cov(\alpha_i, X_{it}) \neq 0 \Rightarrow$ transform the model to remove α_i from the model

Example)

$$crime_{it} - \overline{crime_i} = \beta \cdot (unemp_{it} - \overline{unemp_i}) + (u_{it} - \overline{u_i})$$

$$HP_{it} - \overline{HP_i} = \beta_1 \cdot (crime_{it} - \overline{crime_i}) + \beta_2 \cdot (unemp_{it} - \overline{unemp_i}) + (u_{it} - \overline{u_i})$$

$$\overrightarrow{HP_{it}} = \beta_1 \cdot \overrightarrow{crime_{it}} + \beta_2 \cdot \overrightarrow{unemp_{it}} + \overrightarrow{u_{it}}$$

$$\hat{\alpha}_i = \overline{HP_i} - \hat{\beta}_1 \cdot \overline{crime_i} + \hat{\beta}_2 \cdot \overline{unemp_i}$$

Estimation

- FE (fixed effect)

Note 1) FE vs. FD: parameter estimates are identical if time period is two.

FE vs. LSDV

Note 2) All time-constant (invariant) effects are removed

⇒ Fixed-effect models are designed to study the causes of changes within a person (entity). Time invariant characteristic can not cause such a change because it is constant for each person.

⇒ Since it throws away information, FE estimator is inefficient.

Estimation

- RE (random effect)

$$\text{Cov}(\alpha_i, X_{it}) = 0$$

Error components model:

$$y_{it} = \beta x_{it} + \alpha_i + u_{it}, \text{ with } \alpha_i \sim N(0, \sigma_\alpha^2). \text{ Thus, } \alpha_i + u_{it} = \eta_{it}$$

Note 1) you can include time invariant variable as a predictor in the RE model.

Note 2) RE estimation is more efficient than FE or Pols.

Note 3) FGLS (feasible GLS) estimation is applied (based on quasi-demeaning framework).

Estimation

- RE (random effect)

Example)

$$HP_{it} - \theta \overline{HP}_i = \beta_0(1 - \theta) + \beta_1 \cdot (crime_{it} - \theta \overline{crime}_i) + \beta_2 \cdot (unemp_{it} - \theta \overline{unemp}_i) + (\eta_{it} - \theta \overline{\eta}_i)$$

$$, \text{ where } \theta = 1 - \left(\frac{\sigma_u^2}{\sigma_u^2 + T\sigma_\alpha^2} \right)^{1/2}$$

If $\sigma_\alpha^2 = 0 \Rightarrow \theta = 0$, RE \approx OLS

If $T\sigma_\alpha^2 = \infty \Rightarrow \theta = 1$, RE \approx FE

If $0 < \theta < 1$, RE may not be OLS or FE

\Rightarrow Estimate θ first, then run OLS.

Test

- Hausman test

$Y_{it} = \beta_0 + \beta_1 \cdot X_{it} + \alpha_i + u_{it}$, where α_i : between-entity error, u_{it} : within-entity error

$H_0 : Cov(\alpha_i, X_{it}) = 0$: use random effect model

$H_1 : Cov(\alpha_i, X_{it}) \neq 0$: use fixed effect model

If $Cov(\alpha_i, X_{it}) = 0$, both $\hat{\beta}_{RE}$ and $\hat{\beta}_{FE}$ are consistent. $se(\hat{\beta}_{RE}) < se(\hat{\beta}_{FE})$ (efficiency)

If $Cov(\alpha_i, X_{it}) \neq 0$, only $\hat{\beta}_{FE}$ is consistent

$$W = \frac{(\hat{\beta}_{FE} - \hat{\beta}_{RE})^2}{\sqrt{V(\hat{\beta}_{FE}) - V(\hat{\beta}_{RE})}} \sim \chi_1^2$$

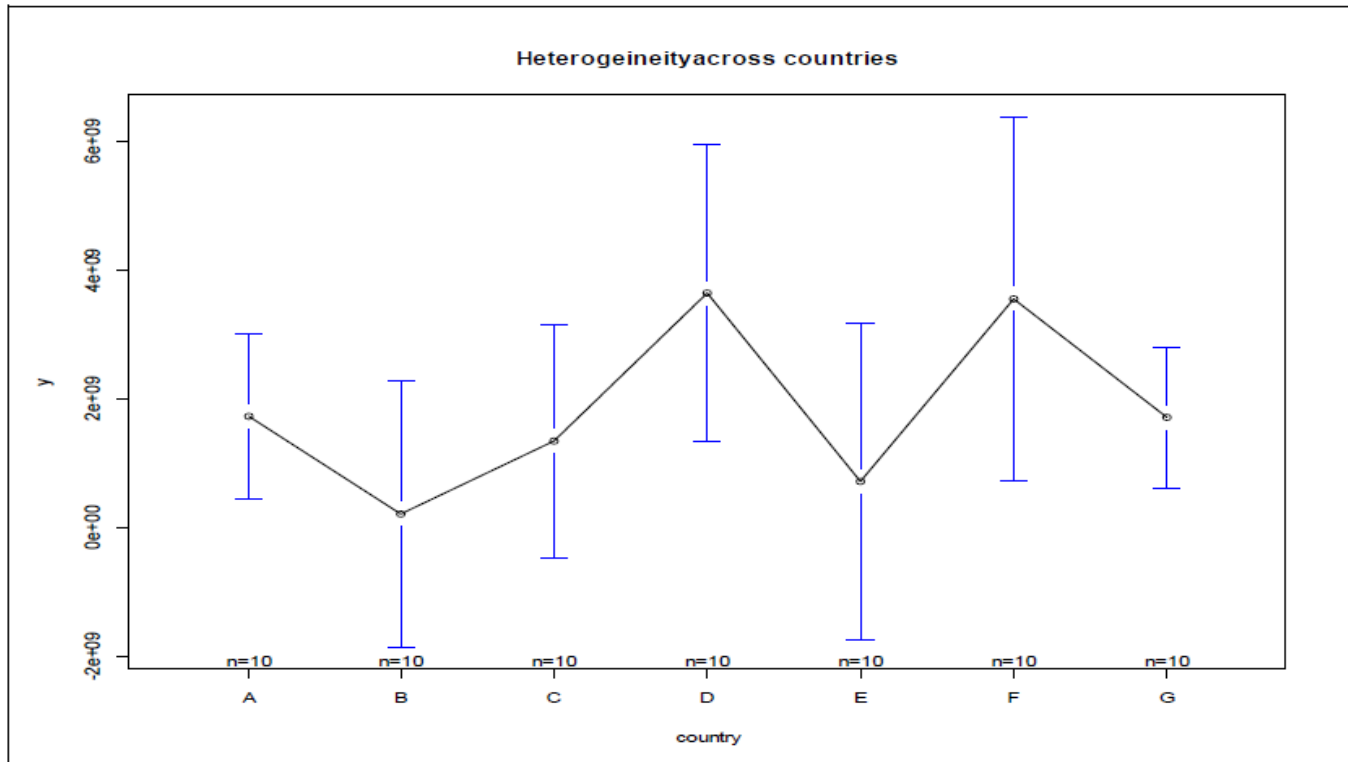
R code example

```
library(foreign)
Panel <- read.dta("http://dss.princeton.edu/training/Panel101.dta")
coplot(y ~ year|country, type="l", data=Panel) # Lines
coplot(y ~ year|country, type="b", data=Panel) # Points and lines
Panel[1:10,]
```

	country	year	y	y_bin	x1	x2	x3	opinion
1	A	1990	1342787840	1	0.27790365	-1.1079559	0.28255358	Str agree
2	A	1991	-1899660544	0	0.32068470	-0.9487200	0.49253848	Disag
3	A	1992	-11234363	0	0.36346573	-0.7894840	0.70252335	Disag
4	A	1993	2645775360	1	0.24614404	-0.8855330	-0.09439092	Disag
5	A	1994	3008334848	1	0.42462304	-0.7297683	0.94613063	Disag
6	A	1995	3229574144	1	0.47721413	-0.7232460	1.02968037	Str agree
7	A	1996	2756754176	1	0.49980500	-0.7815716	1.09228814	Disag
8	A	1997	2771810560	1	0.05162839	-0.7048455	1.41590083	Str agree
9	A	1998	3397338880	1	0.36641079	-0.6983712	1.54872274	Disag
10	A	1999	39770336	1	0.39584252	-0.6431540	1.79419804	Str disag
...								

R code example

```
library(gplots)
plotmeans(y ~ country, main="Heterogeineityacross countries", data=Panel)
# plotmeansdraw a 95% confidence interval around the means
```



R code example

POLS

```
ols<-lm(y ~ x1, data=Panel)  
summary(ols)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	1.524e+09	6.211e+08	2.454	0.0167 *
x1	4.950e+08	7.789e+08	0.636	0.5272

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

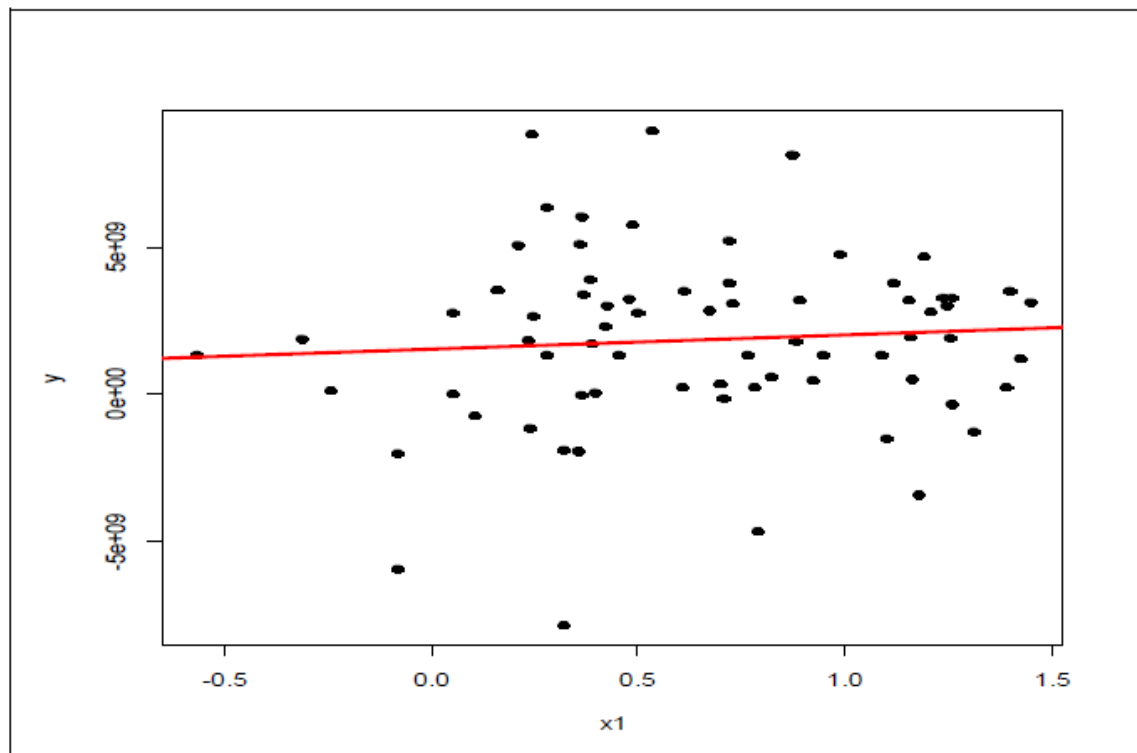
Residual standard error: 3.028e+09 on 68 degrees of freedom

Multiple R-squared: 0.005905, Adjusted R-squared: -0.008714

F-statistic: 0.4039 on 1 and 68 DF, p-value: 0.5272

R code example

```
yhat <-ols$fitted  
plot(Panel$x1, Panel$y, pch=19, xlab="x1", ylab="y")  
abline(lm(Panel$y~Panel$x1),lwd=3, col="red")
```



R code example

#LSDV

```
fixed.dum <- lm(y ~ x1 + factor(country) - 1, data=Panel)
```

```
summary(fixed.dum)
```

```
lm(formula = y ~ x1 + factor(country) - 1, data = Panel)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
x1	2.476e+09	1.107e+09	2.237	0.02889 *
factor(country)A	8.805e+08	9.618e+08	0.916	0.36347
factor(country)B	-1.058e+09	1.051e+09	-1.006	0.31811
factor(country)C	-1.723e+09	1.632e+09	-1.056	0.29508
factor(country)D	3.163e+09	9.095e+08	3.478	0.00093 ***
factor(country)E	-6.026e+08	1.064e+09	-0.566	0.57329
factor(country)F	2.011e+09	1.123e+09	1.791	0.07821 .
factor(country)G	-9.847e+08	1.493e+09	-0.660	0.51190

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.796e+09 on 62 degrees of freedom

Multiple R-squared: 0.4402, Adjusted R-squared: 0.368

F-statistic: 6.095 on 8 and 62 DF, p-value: 8.892e-06

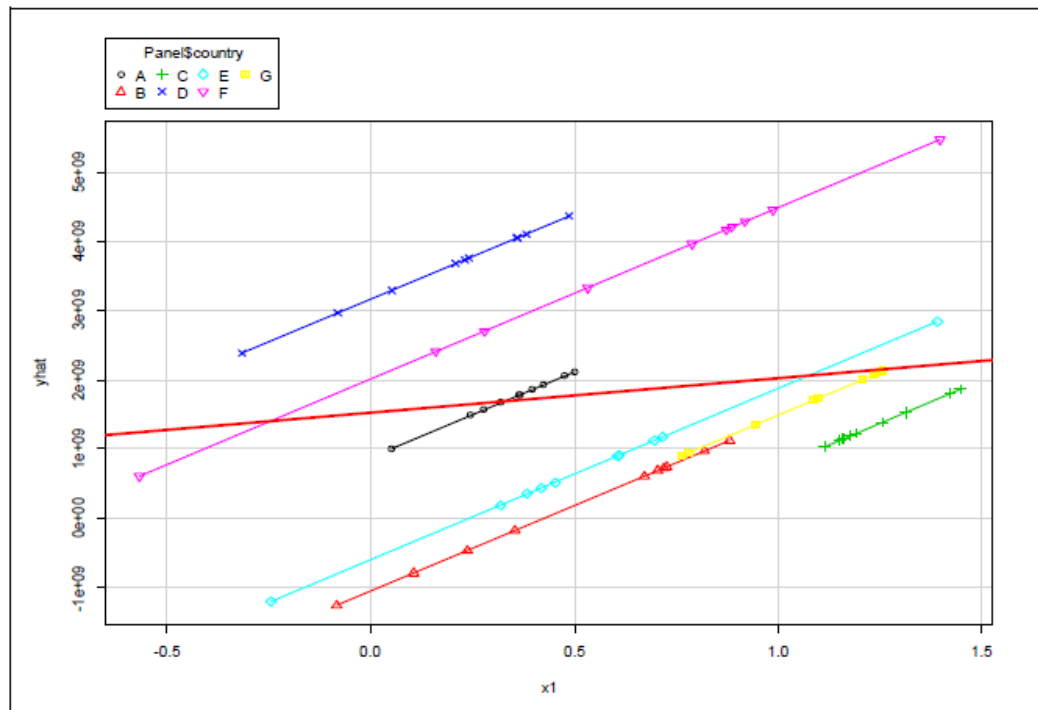
R code example

```
yhat<-fixed.dum$fitted
```

```
library(car)
```

```
scatterplot(yhat~Panel$x1|Panel$country, boxplots=FALSE, xlab="x1", ylab="yhat",smooth=FALSE)
```

```
abline(lm(Panel$y~Panel$x1),lwd=3, col="red")
```



R code example

#POLS vs. LSDV

```
library(apsrtable)
```

```
apsrtable(ols,fixed.dum, model.names= c("OLS", "OLS_DUM")) # Displays a table in Latex form
```

%	& OLS	& OLS_DUM	\\
(Intercept)	& 1524319070.05 ^*	&	\\
	& (621072623.86)	&	\\
x1	& 494988913.90	& 2475617827.10 ^*\\	
	& (778861260.95)	& (1106675593.60) \\	

R code example

```
# FE
```

```
library(plm)
```

```
fixed <- plm(y ~ x1, data=Panel, index=c("country", "year"), model="within")
```

```
summary(fixed)
```

```
plm(formula = y ~ x1, data = Panel, model = "within", index = c("country", "year"))
```

```
Balanced Panel: n=7, T=10, N=70
```

	Estimate	Std. Error	t-value	Pr(> t)
x1	2475617827	1106675594	2.237	0.02889 *

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Total Sum of Squares:    5.2364e+20
```

```
Residual Sum of Squares: 4.8454e+20
```

```
R-Squared:      0.074684
```

```
Adj. R-Squared: 0.066148
```

```
F-statistic: 5.00411 on 1 and 62 DF, p-value: 0.028892
```

R code example

```
fixef(fixed)      # Display the fixed effects (constants for each country)
```

A	B	C	D	E	F	G
880542404	-1057858363	-1722810755	3162826897	-602622000	2010731793	-984717493

```
pFtest(fixed, ols)  # Testing for fixed effects, null: OLS better than fixed
```

F test for individual effects

data: y ~ x1

F = 2.9655, df1 = 6, df2 = 62, p-value = 0.01307

alternative hypothesis: significant effects

R code example

#RE

```
random <- plm(y ~ x1, data=Panel, index=c("country", "year"), model="random")  
summary(random)
```

Oneway (individual) effect Random Effect Model

(Swamy-Arora's transformation)

```
plm(formula = y ~ x1, data = Panel, model = "random", index = c("country", "year"))
```

Balanced Panel: n=7, T=10, N=70

	var	std.dev	share	
idiosyncratic	7.815e+18	2.796e+09	0.873	
individual	1.133e+18	1.065e+09	0.127	
theta:	0.3611			
	Estimate	Std. Error	t-value	Pr(> t)
(Intercept)	1037014284	790626206	1.3116	0.1941
x1	1247001782	902145601	1.3823	0.1714

Total Sum of Squares: 5.6595e+20

Residual Sum of Squares: 5.5048e+20

R-Squared: 0.02733

Adj. R-Squared: 0.026549

F-statistic: 1.91065 on 1 and 68 DF, p-value: 0.17141

R code example

```
# Setting as panel data (an alternative way to run the above model)
Panel.set<-plm.data(Panel, index = c("country", "year"))
# Random effects using panel setting (same output as above)
random.set<-plm(y ~ x1, data = Panel.set, model="random")
summary(random.set)
```

#FE or RE?

Hausman test (H0: random effect model, H1: fixed effect model)

phptest(fixed, random)

Hausman Test

data: y ~ x1

chisq = 3.674, df = 1, p-value = 0.05527

alternative hypothesis: one model is inconsistent

R code example

Test for time fixed effect

```
library(plm)
fixed <- plm(y ~ x1, data=Panel, index=c("country", "year"), model="within")
fixed.time <- plm(y ~ x1 + factor(year), data=Panel, index=c("country", "year"), model="within")
summary(fixed.time)
```

	Estimate	Std. Error	t-value	Pr(> t)
x1	1389050354	1319849567	1.0524	0.29738
factor(year)1991	296381559	1503368528	0.1971	0.84447
factor(year)1992	145369666	1547226548	0.0940	0.92550
factor(year)1993	2874386795	1503862554	1.9113	0.06138 .
factor(year)1994	2848156288	1661498927	1.7142	0.09233 .
factor(year)1995	973941306	1567245748	0.6214	0.53698
factor(year)1996	1672812557	1631539254	1.0253	0.30988
factor(year)1997	2991770063	1627062032	1.8388	0.07156 .
factor(year)1998	367463593	1587924445	0.2314	0.81789
factor(year)1999	1258751933	1512397632	0.8323	0.40898

R code example

```
# Testing time-fixed effects. The null is that no time-fixed effects needed  
pFtest(fixed.time, fixed)
```

F test for individual effects

data: $y \sim x1 + \text{factor}(\text{year})$

$F = 1.209$, $df1 = 9$, $df2 = 53$, p-value = 0.3094

alternative hypothesis: significant effects

```
plmtest(fixed, c("time"), type=("bp"))
```

Lagrange Multiplier Test - time effects (Breusch-Pagan)

data: $y \sim x1$

$\text{chisq} = 0.16532$, $df = 1$, p-value = 0.6843

alternative hypothesis: significant effects

R code example

Testing for random effects: Breusch-Pagan Lagrange multiplier (LM)

Regular OLS (pooling model) using plm

```
pool <- plm(y ~ x1, data=Panel, index=c("country", "year"), model="pooling")  
summary(pool)
```

```
plm(formula = y ~ x1, data = Panel, model = "pooling", index = c("country", "year"))
```

Balanced Panel: n=7, T=10, N=70

	Estimate	Std. Error	t-value	Pr(> t)
(Intercept)	1524319070	621072624	2.4543	0.01668 *
x1	494988914	778861261	0.6355	0.52722

Total Sum of Squares: 6.2729e+20

Residual Sum of Squares: 6.2359e+20

R-Squared: 0.0059046

Adj. R-Squared: 0.0057359

F-statistic: 0.403897 on 1 and 68 DF, p-value: 0.52722

R code example

```
# Breusch-Pagan Lagrange Multiplier for random effects. Null is no panel effect (i.e. OLS better).  
plmtest(pool, type=c("bp"))
```

Lagrange Multiplier Test - (Breusch-Pagan)

data: $y \sim x1$

chisq = 2.6692, df = 1, p-value = 0.1023

alternative hypothesis: significant effects

```
#The LM test helps you decide between a random effects regression and a simple OLS regression.  
# The null hypothesis in the LM test is that variances across entities is zero. This is, no evidence of  
# significant difference across units (i.e. no panel effect)
```


R code example

Testing for cross-sectional dependence/contemporaneous correlation:

using Breusch-Pagan LM test of independence and Pasaran CD tes

```
fixed <- plm(y ~ x1, data=Panel, index=c("country", "year"), model="within")  
pcdtest(fixed, test = c("lm"))
```

Breusch-Pagan LM test for cross-sectional dependence in panels

data: formula

chisq = 28.914, df = 21, p-value = 0.1161

alternative hypothesis: cross-sectional dependence

```
pcdtest(fixed, test = c("cd"))
```

Pesaran CD test for cross-sectional dependence in panels

data: formula

z = 1.1554, p-value = 0.2479

alternative hypothesis: cross-sectional dependence

The null hypothesis in the B-P/LM and Pasaran CD tests of independence is that residuals across
entities are not correlated. B-P/LM and Pasaran CD (cross-sectional dependence) tests are used to
test whether the residuals are correlated across entities*. Cross-sectional dependence can lead to
bias in tests results (also called contemporaneous correlation).

R code example

Testing for serial correlation

```
pbgtest(fixed)
```

Breusch-Godfrey/Wooldridge test for serial correlation in panel models

data: $y \sim x1$

chisq = 14.137, df = 10, p-value = 0.1668

alternative hypothesis: serial correlation in idiosyncratic errors

The null is that there is not serial correlation