기말고사

1. **문제**
    1. Entailment의 formal definition 을 만드시오

        α |= β if and only if, in every model in which α is true, β is also true. Using the notation just
        introduced, we can write

        α |= β if and only if M(α) ⊆ M (β) .

        (Note the direction of the ⊆ here: if α |= β, then α is a stronger assertion than β: it rules out more possible worlds.)

    2. Deduction theorem 을 기술하시오

        ▼ For any sentences α and β, α |= β if and only if the sentence (α ⇒ β) is valid.

        A |= B하면...A와 B를 |연산. 즉, or연산하여 그 결과값을A에 넣는다 입니다.

    3. Modus Ponens inference rule 이 무엇인지 설명하시오

        The best-known rule is called **Modus Ponens** (Latin for *mode that affirms*) and is written

        $$\frac{\alpha \Rightarrow \beta, \qquad \alpha}{\beta}$$

        The notation means that, whenever any sentences of the form α ⇒ β and α are given, then the sentence β can be inferred. For example, if *(WumpusAhead ∧WumpusAlive) ⇒ Shoot* and *(WumpusAhead ∧ WumpusAlive)* are given, then Shoot can be inferred.

    4. Full resolution rule 을 설명하시오

        The unit resolution rule can be generalized to the full resolution rule,

        $$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n}$$

        where li and mj are complementary literals. This says that resolution takes two clauses and produces a new clause containing all the literals of the two original clauses except the two complementary literals.

2. **문제**
    1. B < = > (P V Q)를 conjunctive normal form CNF로 converting 하시오 **(253~254)**

        ▼ 비슷한 해법

        B11 <=> (P12 v P21) 이라는 문장을 CNF로 변환합니다.

        1. <=>을 소거합니다. 즉, a<=>b 를 (a=>b)∧(b=>a) 으로 대체합니다.

        (B11 => (P12 v P21)) ∧ ((P12 v P21) => B11)

        2. =>를 제거합니다. 즉, a=>b를 ~a v b로 대체합니다.

        (~B11 v (P12 v P21)) ∧ (~(P12 v P21) v B11)

        3. CNF에는 ~이 리터럴에만 있어야합니다. 이를 위해 아래의 세개 방식 중 적절한 것을 적용하여 ~을 괄호 안으로 옮깁니다.

        $$\begin{array}{ll} \neg(\neg\alpha) \equiv \alpha & \text{(double-negation elimination)} \\ \neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta) & \text{(De Morgan)} \\ \neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta) & \text{(De Morgan)} \end{array}$$

        지금의 예에서는 세번째 규칙을 한번만 적용하면 됩니다.

        (~B11 v P12 v P21) ∧ ((~P12 ∧ ~P21) v B11)

        4. 이제 ∧, v 연산자와 리터럴들로 이루어진 부분들이 중첩된 형태의 문장이 나왔습니다. 이제 분배법칙을 적용해서 v를 ∧에 대해 적절히 분배합니다.

        (~B11 v P12 v P21) ∧ (~P12 v B11) ∧ (~P21 v B11)

        이렇게 해서 원래의 문장이 세개의 절의 논리곱 형태로 된 CNF로 변환되었습니다.

        이전보다 언어로써 표현하기는 힘들지만 완결적인 분해 절차의 입력으로써 사용될 수 있습니다.

        출처:

        https://doorbw.tistory.com/63

        [Tigercow.Door]

        ▼ CNF 설명

NP-완전 문제로 유명한 Satisfiability Problem(SAT), 또는 충족 가능성 문제라고 불리는 문제입니다.

SAT는 어떠한 변수들로 이루어진 논리식이 주어졌을 때, 그 논리식이 참이 되는 변수값이 존재하는지를 찾는 문제인데요. 논리식은 기본적으로 논리 변수와, 몇몇 논리 연산자 결합에 의해 만들어지는 유한한 식입니다.

논리 부정 : x가 참이면 거짓, 거짓이면 참

논리합 : x1이나 x2중 적어도 하나가 참이면 참, 나머지 경우는 거짓

논리곱 : x1과 x2가 모두 참이면 참, 나머지 경우는 거짓

이때 각각의 x 식을 리터럴(literal)이라고 부르고, 여러 리터럴의 논리합 꼴로 이루어진 식을 클로저(clause)라고 정의합니다. 클로저들의 논리곱으로 표현되어 있는 논리식을 CNF(논리곱 표준형)이라고 부릅니다.

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.
$(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$.

$(B \Rightarrow (P \vee Q)) \wedge ((P \vee Q) \Rightarrow B)$.

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$:
$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$.

$(\neg B \vee P \vee Q) \wedge (\neg(P \vee Q) \vee B)$.

3. CNF requires ¬ to appear only in literals, so we “move ¬ inwards” by repeated application of the following equivalences from Figure 7.11:

$\neg(\neg\alpha) \equiv \alpha$ (double-negation elimination)
$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$ (De Morgan)
$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$ (De Morgan)

In the example, we require just one application of the last rule:

$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$.

$(\neg B \vee P \vee Q) \wedge ((\neg P \wedge \neg Q) \vee B)$.

4. Now we have a sentence containing nested $\wedge$ and $\vee$ operators applied to literals. We apply the distributivity law from Figure 7.11, distributing $\vee$ over $\wedge$ wherever possible.
$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$.

$(\neg B \vee P \vee Q) \wedge (\neg P \vee B) \wedge (\neg Q \vee B)$.

The original sentence is now in CNF, as a conjunction of three clauses. It is much harder to read, but it can be used as input to a resolution procedure.

2. 13.a에서 얻어지는 3개의 clauses와 함께, ㄱB와 P의 두 clauses를 추가하여 만들어지는 a set of clauses의 resolution closure가 empty clause를 포함한다는 것을 보이시오 (254, 255 page)

3. Ground resolution theorem을 기술하시오 (255)

   If a set of clauses is unsatisfiable, then the resolution closure of those clauses contains the empty clause.

   Ground resolution은 두 ground term에 대해서 진행하는 해결 규칙이다. ground term이란 자유 변수(free variable)이 포함되지 않은 term을 말하는데 즉, 함수(function), 관계(relation), 그리고 상수(constant)[4]로만 이루어 진 것이다. ground resolution은 0차 논리의 propositional resolution과 거의 동일하다.

3. **다음의 Horn clauses의 set 에 대응하는 AND-OR graph를 draw 하시오 (258p)**

   P ⇒ Q,   L∧M⇒P,   B∧L⇒M,   A∧P⇒L,   A∧B⇒L, A, B

   ▼ 캡쳐

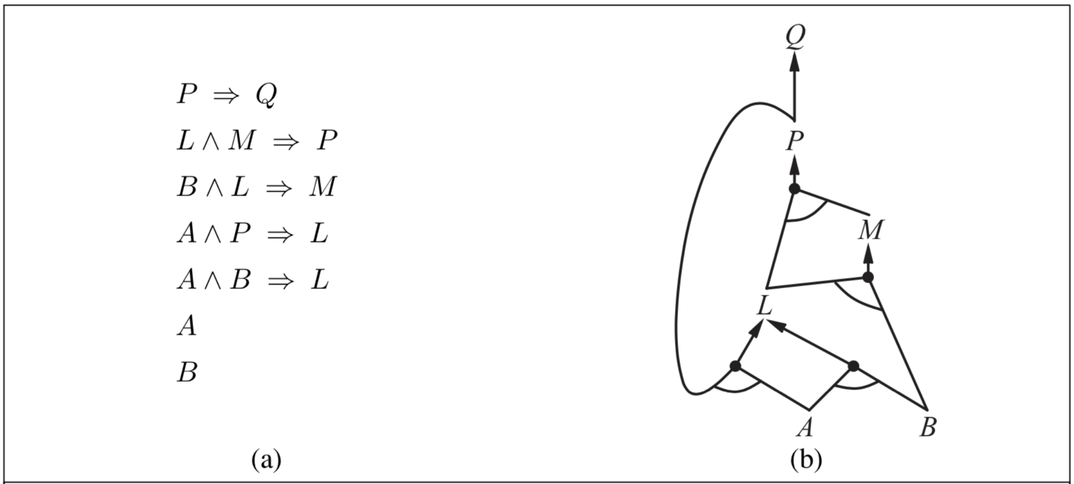

Figure 7.16 (a) A set of Horn clauses. (b) The corresponding AND–OR graph.

4. **문제3의 horn clauses의 set에 ㄱQ를 추가해 만든 a set of clauses S가 만들어 진다고 가정하자. 그 때 S 가 unsatisfiable임을 보이시오**

5. **Represent the following sentences in First-order logic, using a consistent vocabulary(which you must define)**

▼ example

Vocabulary:

Student(x), Person(x), Man(x), Barber(x), Expensive(x), Agent(x), Insured(x), Smart(x),Politician(x): predicates satisfied by members of the
corresponding categories
F, G: French and German courses
x > y: x is greater than y;
Take(x, c, s): student x, course c, semester s
Pass(x, c): student x passes course c
Score(x, c): the score obtained by student x in course c in semester s;
Subject(c, f): the subject of course c is field f;

Buys(x, y, z): x buys y from z
Sells(x, y, z): x sells y to z
Shaves(x, y): person x shaves person y
Parent(x, y): x is a parent of y
Citizen(x, c, r): x is a citizen of country c for reason r
Resident(x, c): x is a resident of country c
Birthplace(x, u): person x born in country u
Citizen(x, u): person x is a citizen of country u
Parent(x, z): z is a parent of x
Fools(x, y, t): person x fools person y at time t

a) Some students took French in spring 2001.
$\exists x\ Student(x) \land Takes(x, F, Spring2001).$

b) Every student who takes French passes it.
$\forall\ x, s\ Student(x) \land Takes(x, F, s) \Rightarrow Passes(x, F, s).$

c) Only one student took Greek in spring 2001.
$\exists x\ Student(x) \land Takes(x,G, Spring2001) \land \forall y\ y \neq x \Rightarrow \neg$
$Takes(y,G,Spring2001).$

d) The best score in Greek is always higher than the best score in French.
$\forall s\ \exists x\ \forall y\ Score(x,G,s) > Score(y,F, s).$

e) Every person who buys a policy is smart.
$\forall x\ Person(x) \land (\exists y, z\ Policy(y) \land Buys(x, y, z)) \Rightarrow Smart(x).$

f) No person buys an expensive policy.

$\forall$ x, y, z Person(x) $\wedge$ Policy(y) $\wedge$ Expensive(y) $\Rightarrow$ $\neg$Buys(x, y, z).

g) There is an agent who sells policies only to people who are not insured.

$\exists$x Agent(x) $\wedge\forall$y, z Policy(y) $\wedge$ Sells(x, y, z) $\Rightarrow$ (Person(z)$\wedge\neg$ Insured(z)).

h) There is a barber who shaves all men in town who do not shave themselves.

$\exists$x $\forall$y Barber(x) $\wedge$ Man(y) $\wedge$ $\neg$Shaves(y, y) $\Rightarrow$ Shaves(x, y).

i) A person born in the UK, each of whose parents is a UK citizen or a UK resident, is a UK citizen by birth.

$\forall$x Person(x)$\wedge$Born(x,UK)$\wedge$($\forall$y Parent(y, x) $\Rightarrow$ (($\exists$ r Citizen(y,UK, r))$\vee$ Resident(y,UK))) $\Rightarrow$ Citizen(x,UK,Birth).

j) A person born outside the UK, one of whose parents is a UK citizen by birth, is a UK citizen by descent.

$\forall$x Person(x)$\wedge$ $\neg$Born(x,UK) $\wedge$ ($\exists$y Parent(y, x) $\wedge$ Citizen(y,UK,Birth)) $\Rightarrow$ Citizen(x,UK,Descent).

k) Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time.

$\forall$ x Politician(x) $\Rightarrow$ ($\exists$ y $\forall$t Person(y) $\wedge$ Fools(x, y, t)) $\wedge$ ($\exists$ t $\forall$y Person(y) $\Rightarrow$ Fools(x, y, t)) $\wedge\neg$($\forall$ t $\forall$y Person(y) $\Rightarrow$ Fools(x, y, t))

1. Richard's father is married to John's mother

   Married(Father(Richard), Mother(John))

2. Brothers are siblings

   $\forall$x $\forall$y Brother(x,y) $\Rightarrow$ Sibling(x,y)

3. Everybody loves somebody

   $\forall$x $\exists$y Loves(x,y)

4. Everyone likes ice cream

   $\forall$x Likes(x,IceCream)

5. There is no one who does not like ice cream

   $\neg\exists$x $\neg$Likes(x,IceCream)

6. ~~문제~~

1. ~~Unification process를 설명하시오~~
2. ~~Most general unifier를 설명하고 예를 보이시오~~

7. **From "Horses are animals." it follows that "The head of a horse is the head of an animal.". Demonstrate that this inference is valid by carrying out the following steps:**
    1. translate the premise and the conclusion into the language of first-order logic. Use three predicates: HeadOf(h,x) (meaning "h is the head of x"), Horses(x), and Animal(x)
    2. Negate the conclusion, and convert the premise and the negated conclusion into conjunctive normal form.
    3. user resolution to show that the conclusion follows from the premise.
8. ~~**Employs(IBM, Richard), Employs(x, Richard), Employs(IBM, y), Employs(x,y) 의 queries가 있다고 할 때, 이들로부터 Subsumption lattice를 만드시오**~~
9. ~~**"Everyone who loves all animals is loved by someone"와 같은 sentence가 다음과 같이 First-order logic으로 표현될 수 있다:**~~

    ~~∀x [∀y Animal(y) ⇒ Loves(x,y)] ⇒ [∃y Loves(y,x)]~~

    ~~이를 CNF로 변환시키시오. 해당 step들을 함께 보이시오 (346page)~~
10. ~~**Men are mortal. The father of a man is a man. Therefore, John's father is mortal. Given the above sentences;**~~
    1. ~~make a set of clauses S to prove that "John's father is mortal" is "true" using resolution.~~
    2. ~~Find the Herbrand universe of S~~
    3. ~~Find the Herbrand base of S~~
    4. ~~State Herbrand's theorem~~
    5. ~~Show that the Herbrand's theorem holds "true" for the above example.~~

참고자료

▼ 원본 이미지

인공지능 학기말고사 2017.12.20.(수)

1. a) Entailment의 formal definition을 만드시오.

   b) Deduction theorem을 기술하시오. c) Modus Ponens inference rule이 무엇인지 설명하시오.

   d) Full resolution rule을 설명하시오.

2. a) B <=> (P V Q)를 conjunctive normal form(CNF) 로 converting 하시오.

   b) 문제 13.a에서 얻어지는 3개의 clauses와 함께, ㄱB 와 P의 두 clauses를 추가하여 만들어지는 a set of clauses 의 resolution closure가 empty clause를 포함한다는 것을 보이시오.

   c) Ground resolution theorem을 기술하시오.

3 다음의 Horn clauses 의 set 에 대응하는 AND-OR graph를 draw 하시오.

   $P \Rightarrow Q \quad L \wedge M \Rightarrow P \quad B \wedge L \Rightarrow M \quad A \wedge P \Rightarrow L \quad A \wedge B \Rightarrow L \quad A \quad B$

4. 문제3의 Horn clauses의 set 에 ㄱQ를 추가해 만든 a set of clauses S 가 만들어 진다고 가정하자. 그때 S 가 unsatisfiable 임을 보이시오.

5. Represent the following sentences in First-order logic, using a consistent vocabulary(which you must define)

   a. Richard's father is married to John's mother

   b. Brothers are siblings

   c. Everybody loves somebody

   d. Everyone likes ice cream

   e. There is no one who does not like ice cream.

6. a. Unification process 가 무엇인지 설명하시오.

   b. Most general unifier 가 무엇인지 설명하고 실제 예를 보이시오.

7. From "Horses are animals." it follows that "The head of a horse is the head of an animal." Demonstrate that this inference is valid by carrying out the following steps:

   a. Translate the premise and the conclusion into the language of first-order logic. Use three predicates: HeadOf(h,x) (meaning "h is the head of x"), Horses(x), and Animal(x)

   b. Negate the conclusion, and convert the premise and the negated conclusion into conjunctive normal form.

   c. Use resolution to show that the conclusion follows from the premise.

8. Employs(IBM, Richard), Employs(x, Richard), Employs(IBM, y), Employs(x, y) 의 queries가 있다고 할 때, 이들로 부터 Subsumption lattice를 만드시오.

9. "Everyone who loves all animals is loved by someone"와 같은 sentence가 다음과 같

11:02 LTE

인공지능-2017...

9. "Everyone who loves all animals is loved by someone"와 같은 sentence가 다음과 같

이 First-order logic으로 표현될 수 있다:

$\forall x\ [\forall y\ Animal(y) \Rightarrow Loves(x,y)] \Rightarrow [\exists y\ Loves(y,x)]$

이를 CNF로 변환시키시오. 해당 step 들을 함께 보이시오.

10. Men are mortal. The father of a man is a man. Therefore, John's father is mortal. Given the above sentences;

a. make a set of clauses S to prove that "John's father is mortal" is "true" using resolution.

b. Find the Herbrand universe of S

c. Find the Herbrand base of S

d. State Herbrand's theorem

e. Show that the Herbrand's theorem holds "true" for the above example.