Deep Learning Foundations



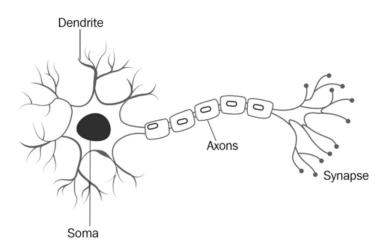
Story So Far...

- We learned the basic reinforcement learning algorithms
 - dynamic programming
 - Monte Carlo methods
 - temporal difference learning
- Now we are about to move on to Deep Reinforcement Learning (DRL), a
 method used for large and complex environments.
- DRL combines reinforcement learning with deep learning.
- Before going into DRL, we first look at the foundations of deep learning.
 - To those of you who already know and use deep learning, regard this as a refresher.



Biological Neuron

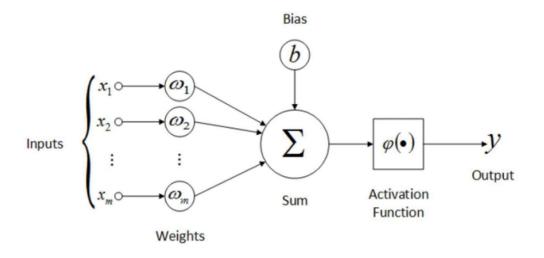
- Neurons are the fundamental units of our brain and nervous system.
 - Our brain has approximately 100 billion neurons.
- Every neuron is connected to one another through a structure called a synapse.
 - A synapse receives input from the external environment via sensory organs.
 - A synapse also sends motor instructions to our muscles.
- A neuron can also receive inputs from other neurons through a branchlike structure called dendrite.
- The inputs are strengthened or weakened according to their importance, and they
 are summed together in the cell body called soma.
- From the cell body, the summed inputs are processed and move through axons and are sent to other neurons.





Artificial Neuron

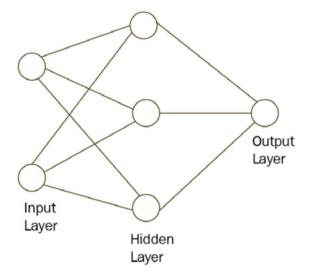
- A computational component modeled after a biological neuron
- An artificial neuron accepts one or multiple inputs x_i , and produces a single output y.
- The inputs are multipled by their respective weights w_i .
 - All inputs are not equally important in calculating the output.
 - Weights are used to strengthen or weaken inputs.
- The weighted inputs are summed together, and a bias b is added.
- After that, we apply a non-linear function to the result. The function is called an activation function or a transfer function.





Artificial Neural Network (ANN)

- To perform complex tasks, we need multiple neurons.
- The massive number of neurons in our brain are stacked in layers forming a network.
- Similarly, the artificial neurons are arranged in layers so that the information is passed from one layer to the other. This is called an artificial neural network (ANN).
- A typical ANN consists of the following layers
 - Input layer
 - Hidden layer(s)
 - Output layer





Artificial Neural Network (ANN)

Input layer

- The input layer is where we feed input to the network.
- The number of neurons in the input layer is the number of inputs.
- No computation is performed in the input layer; it is just used for passing information from the outside world to the network.

Hidden layer

- Any layer between the input layer and the output layer is called a hidden layer.
- A hidden layer is responsible for deriving complex relationships between input and output.
 - identifies patterns in the dataset
 - learns data representation
 - extracts features
- We choose how many hidden layers will be in the model.
- The network is called a deep neural network when we have many hidden layers.



Artificial Neural Network (ANN)

Output layer

- The output layer produces output of the model.
- The number of neurons in the output layer is based on the task we are trying to solve.
- For a regression problem, we could have a single neuron in the output layer.
- If we have a classification problem with five classes, the number of neurons in the output layer can be five.

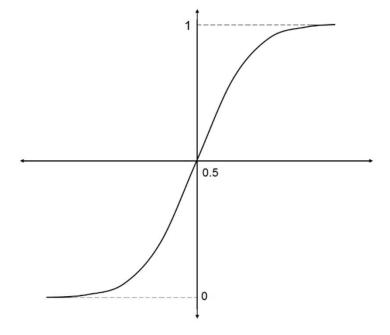


- A neuron adds all the weighted inputs and the bias, and then applies an
 activation function to calculate the output.
- The activation function adds non-linearity to the model.
- If we don't have activation function, the neural network becomes a linear model. A linear model is not effective in representing complex features.
- An activation function can be any function, but there are well-known and frequently used activation functions
 - sigmoid
 - tanh
 - ReLU
 - softmax



- The sigmoid function
 - scales the value between 0 and 1
 - differentiable
 - monotonic
 - also known as logistic function
 - used for predicting the probability of output

$$f(x) = \frac{1}{1 + e^{-x}}$$

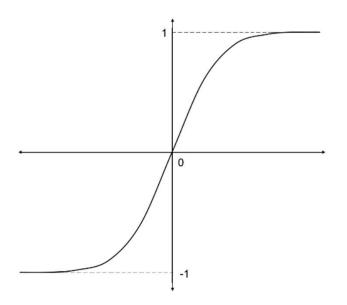




• The tanh function

- hyperbolic tangent function
- outputs a value between -1 and +1
- centered at 0

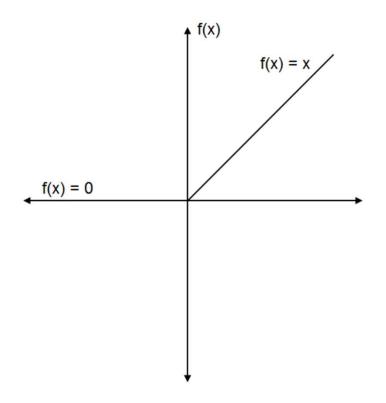
$$f(x) = \frac{1 - e^{-2x}}{1 + e^{-2x}}$$





- The Rectified Linear Unit Function (ReLU)
 - outputs a value from 0 to infinity
 - a piecewise function

$$f(x) = \begin{cases} 0 & for \ x < 0 \\ x & for \ x \ge 0 \end{cases}$$





The Softmax function

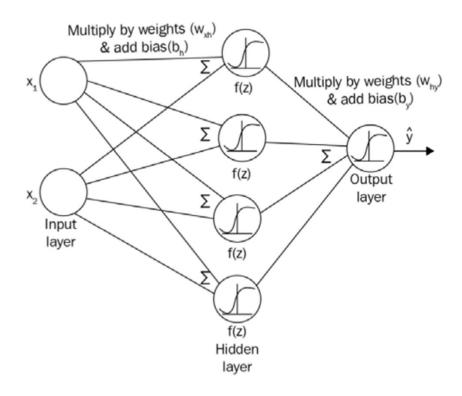
- a generalization of the sigmoid function
- usually applied to the final layer of the network while performing multiclass classification tasks
- the sum of softmax values equals 1

$$f(x_i) = \frac{e^{x_i}}{\sum_j e^{x_j}}$$

$$\begin{bmatrix}
0.5 \\
1.3 \\
1.1
\end{bmatrix}$$
Softmax
$$\begin{bmatrix}
0.198 \\
0.440 \\
0.360
\end{bmatrix}$$



- Suppose we have a network with one input layer, one hidden layer, and one output layer.
- We have two neurons in the input layer, four in the hidden layer, and one in the output layer.



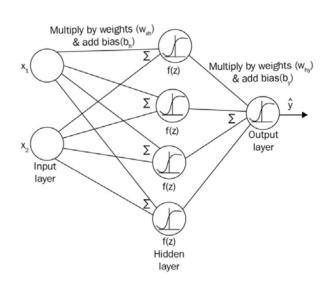


• To move from input layer to hidden layer, we multiply the inputs by the weights and add the bias.

$$z_1 = XW_{xh} + b_h$$

- W_{xh} is a weight matrix between the input and the hidden layer.
 - Its size is 2x4.
- b_h is a bias vector of 4 elements.
- Then, we apply the activation function.

$$a_1 = \sigma(z_1)$$





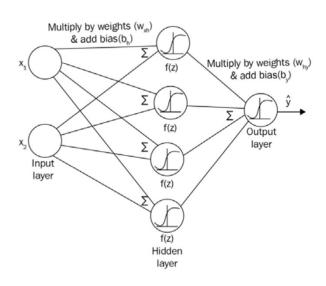
Now we propagate from the hidden layer to the output layer.

$$z_2 = a_1 W_{hy} + b_y$$

- W_{hy} is a weight matrix between the hidden and the output layer.
 - Its size is 4x1.
- b_{v} is a bias vector of 1 element.
- Then, we apply the activation function.

$$\hat{y} = \sigma(z_2)$$

We have the output value.





• The whole process

$$z_1 = XW_{xh} + b_h$$

$$a_1 = \sigma(z_1)$$

$$z_2 = a_1W_{hy} + b_y$$

$$\hat{y} = \sigma(z_2)$$

• In python code

```
def forward_prop(X):
    z1 = np.dot(X,Wxh) + bh
    a1 = sigmoid(z1)
    z2 = np.dot(a1,Why) + by
    y_hat = sigmoid(z2)
    return y_hat
```



The Cost Function

- In the neural network model we used, there were two weight matrices and two bias vectors: $W_{\chi h}$, b_h , $W_{h\nu}$, b_{ν} .
- They are called model parameters.
- When we first create the model, we do not know what values we should use for the parameters. So we just use random values.
- Because of that, when we propagate an input through the model, the output will be far from what we want.
- If we know the "true" output (label), we can measure how far the predicted output (model output) is different from the true output, using a cost function. A cost function is also called a loss function.



The Cost Function

A popular cost function is the mean squared error (MSE) function.

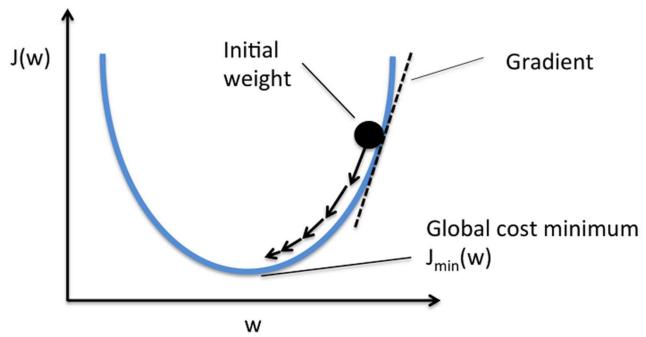
$$J = \frac{1}{2n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

- n is the number of training samples.
- y_i is the true output.
- \hat{y}_i is the predicted output (model output).
- If *J* is large, it means the model is not producing outputs close to their true outputs; the model is not performing well.
- If *J* is small, it means the model is performing well.

- We want to iteratively update the model parameters (weights and biases) so that the cost function is minimized.
- We do this by applying a technique called gradient descent.



- Let us consider a single scalar value w.
- We want to update w so that the cost function J is reduced.
- What we can do is calculate the gradient of J at w.
- If the gradient is positive at w, we increase w.
- If the gradient is negative at w, we decrease w.
- Since gradient changes according to w, we change the value of w in steps.

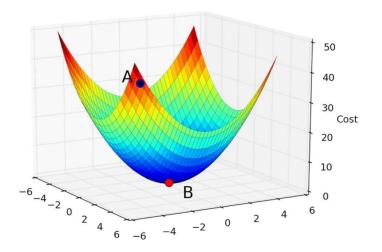




- In a neural network models, we have vectors as parameters.
- We do similar things and calculate $\frac{\partial J}{\partial W}$. It gives us the direction in which we need to move.
- Then, we update the weights according to the following equation.

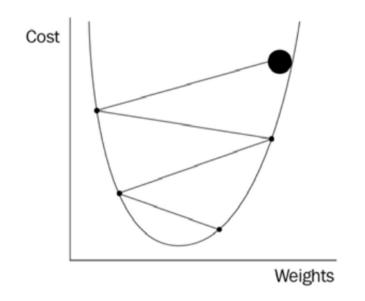
$$W = W - \alpha \frac{\partial J}{\partial W}$$

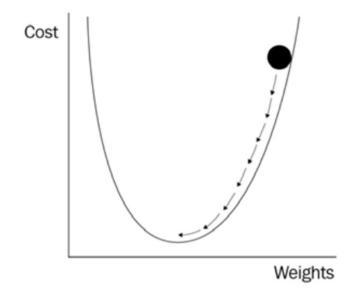
• α is the learning rate. It determines the step we move towards the direction.





- Learning rate is a hyper-parameter which the operator chooses.
- If learning rate is too high, may fail to find the optimal point.
- If learning rate is too small, learning takes too much time.







- In order to update all the parameters, we need the gradients of J with respect to $W_{\chi h}$, b_h , W_{hy} , b_y .
- Recall the equations for forward propagation

$$z_1 = XW_{xh} + b_h$$

$$a_1 = \sigma(z_1)$$

$$z_2 = a_1W_{hy} + b_y$$

$$\hat{y} = \sigma(z_2)$$

• To calculate $\frac{\partial J}{\partial W_{hy}}$, we use the chain rule.

$$\frac{\partial J}{\partial W_{hy}} = \frac{\partial J}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial z_2} \cdot \frac{\partial z_2}{\partial W_{hy}}$$

We calculate the three terms here and multiply them.



• Calculating $\frac{\partial J}{\partial W_{hy}}$, $\frac{\partial J}{\partial b_y}$

$$\frac{\partial J}{\partial \hat{y}} = -(y - \hat{y}) \qquad \frac{\partial \hat{y}}{\partial z_2} = \sigma'(z_2) \qquad \frac{\partial z_2}{\partial W_{hy}} = a_1 \qquad \frac{\partial z_2}{\partial b_y} = 1$$

• σ' is a derivative of the sigmoid function.

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$
 $\sigma'(z) = \frac{e^z}{(1 + e^{-z})^2}$

• Thus, substituting all terms in the chain rule equation, we get:

$$\frac{\partial J}{\partial W_{hy}} = -(y - \hat{y}) \cdot \sigma'(z_2) \cdot a_1$$

$$\frac{\partial J}{\partial b_y} = -(y - \hat{y}) \cdot \sigma'(z_2)$$

• Calculating $\frac{\partial J}{\partial W_{xh}}$, $\frac{\partial J}{\partial b_h}$

$$\frac{dz_1}{dW_{xh}} = X$$

Chain rule

$$\frac{\partial J}{\partial W_{xh}} = \frac{\partial J}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial z_2} \cdot \frac{\partial z_2}{\partial a_1} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial W_{xh}}$$

• We already know the first two terms. The last three terms are:

$$\frac{\partial z_2}{\partial a_1} = W_{hy} \qquad \frac{\partial a_1}{\partial z_1} = \sigma'(z_1) \qquad \frac{\partial z_1}{\partial W_{xh}} = x \qquad \frac{\partial z_1}{\partial b_h} = 1$$

Substituting all terms, we get:

$$\frac{\partial J}{\partial W_{xh}} = -(y - \hat{y}) \cdot \sigma'(z_2) \cdot W_{hy} \cdot \sigma'(z_1) \cdot x \qquad \frac{\partial J}{\partial b_y} = -(y - \hat{y}) \cdot \sigma'(z_2) \cdot W_{hy} \cdot \sigma'(z_1)$$

 Once we calculate the gradients, we can update the parameters using gradient descent.

$$\bullet \quad W_{hy} = W_{hy} - \alpha \frac{\partial J}{\partial W_{hy}}$$

•
$$b_y = b_y - \alpha \frac{\partial J}{\partial b_y}$$

•
$$W_{xh} = W_{xh} - \alpha \frac{\partial J}{\partial W_{xh}}$$

•
$$b_y = b_h - \alpha \frac{\partial J}{\partial b_h}$$

```
def backword_prop(y_hat, z1, a1, z2):
    delta2 = np.multiply(-(y-y_hat), sigmoid_derivative(z2))
    dJ_dWhy = np.dot(a1.T, delta2)

delta1 = np.dot(delta2, Why.T)*sigmoid_derivative(z1)
    dJ_dWxh = np.dot(X.T, delta1)

Wxh = Wxh - alpha * dJ_dWhy
    Why = Why - alpha * dJ_dWxh

return Wxh, Why
```



Variants of Gradient Descent Methods

- Stochastic gradient descent (SGD)
 - Instead of getting the gradient from the whole dataset, we choose a small random subset of the dataset and calculate gradient.
 - Reduces computational burden, achieving faster iterations in trade for a lower convergence rate.

Momentum

- Uses a notion of momentum
- remembers the previous update Δw , and determines the next update as a linear combination of the gradient and the previous update

$$egin{aligned} \Delta w &:= lpha \Delta w - \eta
abla Q_i(w) \ w &:= w + \Delta w \end{aligned}$$

tends to keep traveling in the same direction, preventing oscillations



Variants of Gradient Descent Methods

AdaGrad

- Uses per-parameter learning rate
- Increases learning rate for sparser parameters and decreases learning rate for ones that are less sparse
- works well when data is sparse and sparse parameters are more informative.

RMSProp

- The problem of AdaGrad is that in the later stages, the learning rate becomes too small.
- RMSProp revises the equation so that the learning does not stop
- Adam (Adaptive Moment Estimation)
 - A combination of momentum and RMSProp
 - Uses the concept of momentum, as well as the concept of adaptive learning rate



Learning Process

- Prepare a training set
 - Do any necessary preprocessing
- Define a model
 - Choose the model architecture. E.g.) # of layers, # of neurons, activation function
 - Initialize parameters (weights and biases)
- Define the gradient descent method
 - Choose hyperparameters such as learning rate and momentum factor
- Choose a batch size for stochastic gradient descent
 - Divide the training set into batches of the given size
- Choose how many epochs we are going to run training.
 - One epoch is equal to one forward pass and one backward pass for all training samples.
- For each batch, we calculate the forward pass
 - Feed the batch samples into the network and get the output
- Then, calculate the backward pass and update the parameters
 - Perform backpropagation



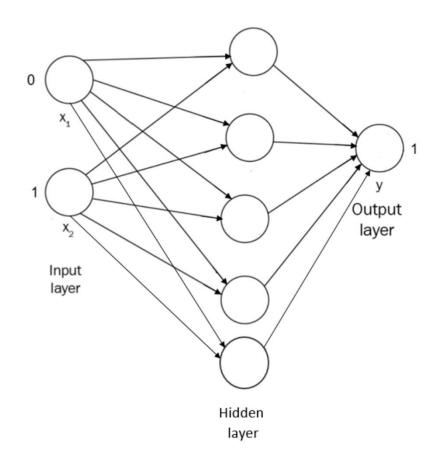
Our goal is to train a neural network, so that it acts like an XOR gate.

Input(x)		Output(y)
\mathbf{x}_1	X ₂	y
0	0	0
0	1	1
1	0	1
1	1	0

- We can see that the number of inputs is 2, and the number of outputs is 1.
- The four different samples will be our training set.



- We will use a model with a single hidden layer.
- We will use sigmoid as our activation function.





• Import libraries

```
import numpy as np
import matplotlib.pyplot as plt
```

Dataset

```
X = np.array([[0,1], [1,0], [1,1], [0,0]])
y = np.array([[1], [1], [0], [0]])
```

Model architecture

```
num_input = 2
num_hidden = 5
num_output = 1
```



Initialize model parameters

```
Wxh = np.random.randn(num_input, num_hidden)
bh = np.zeros((1, num_hidden))
Why = np.random.randn(num_hidden, num_output)
by = np.zeros((1, num_output))
```

• Define sigmoid function

```
def sigmoid(z):
    return 1 / (1+np.exp(-z))
```

Define derivative of sigmoid

```
def sigmoid_derivative(z):
    return np.exp(-z)/((1+np.exp(-z))**2)
```



Define function for forward propagation

```
def forward_prop(x, Wxh, Why):
    z1 = np.dot(x, Wxh) + bh
    a1 = sigmoid(z1)
    z2 = np.dot(a1, Why) + by
    y_hat = sigmoid(z2)
    return z1, a1, z2, y_hat
```

Define function for backward propagation

```
def backward_prop(y_hat, z1, a1, z2):

    delta2 = np.multiply(-(y-y_hat), sigmoid_derivative(z2))
    dJ_dWhy = np.dot(a1.T, delta2)
    dJ_dby = delta2
    delta1 = np.dot(delta2, Why.T) * sigmoid_derivative(z1)
    dJ_dWxh = np.dot(X.T, delta1)
    dJ_dbh = delta1

return dJ_dWxh, dJ_dWhy, dJ_dbh, dJ_dby
```



• Define cost function

```
def cost_function(y, y_hat):
    J = 0.5*sum((y-y_hat)**2)
    return J
```

Code for training (updating model parameters)

```
alpha = 0.01
num_iterations = 10000

cost = []
for i in range(num_iterations):
    z1, a1, z2, y_hat = forward_prop(X, Wxh, Why)
    dJ_dWxh, dJ_dWhy, dJ_dbh, dJ_dby = backward_prop(y_hat, z1, a1, z2)

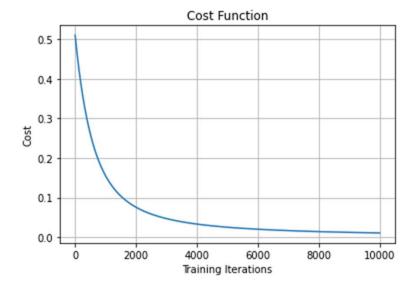
Wxh = Wxh - alpha * dJ_dWxh
    Why = Why - alpha * dJ_dWhy
    bh = bh - alpha * dJ_dbh
    by = by - alpha * dJ_dby

c = cost_function(y, y_hat)
    cost.append(c)
```



Plotting the cost function after each epoch

```
plt.grid()
plt.plot(range(num_iterations), cost)
plt.title('Cost Function')
plt.xlabel('Training Iterations')
plt.ylabel('Cost')
plt.show()
```

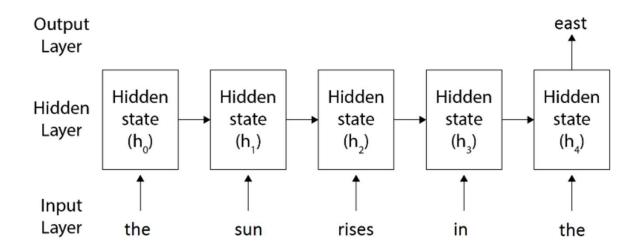




Recurrent Neural Networks

- A recurrent neural network (RNN) is a type of ANN commonly used for sequential data or time series data, such as the ones used in speech recognition or natural language processing.
- RNNs recognize data's sequential characteristics and use patterns to predict the next likely scenario.

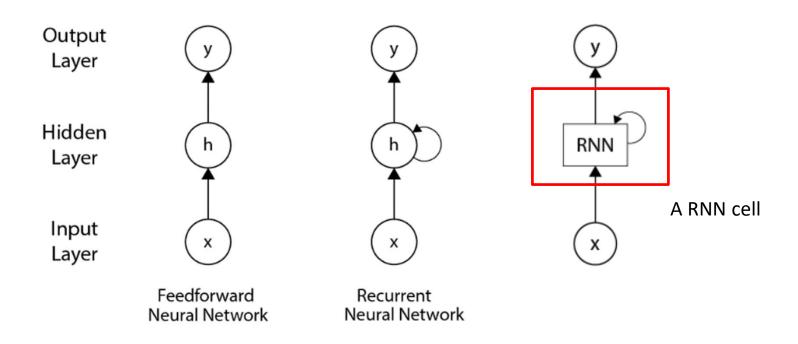
The sun rises in the ____.





RNN vs. Feed-Forward Networks

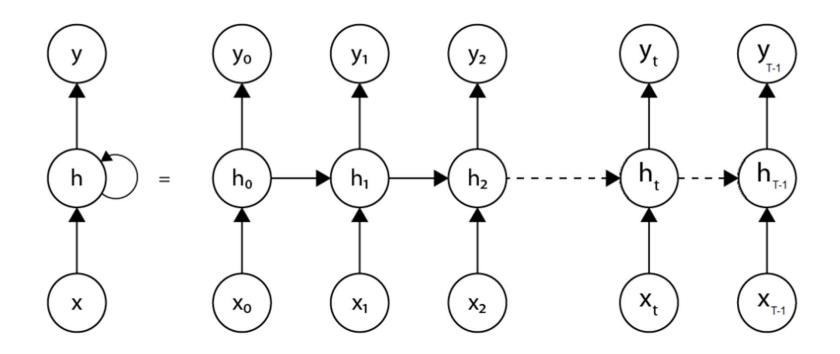
- In a feed-forward network, connections do not form a cycle. In other words, output of the previous input is not used in the processing of the current input.
- In a recurrent network, the output of the previous input is used in the processing of the current input. It is like having a memory of previous inputs.





Recurrent Neural Networks

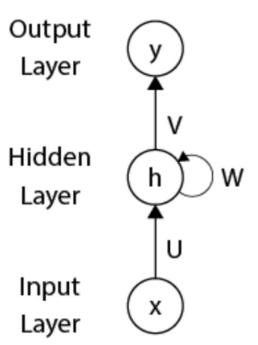
- Suppose we have an input sequence of T words.
- Then, we have T layers, one for each word.





Forward Propagation in an RNN

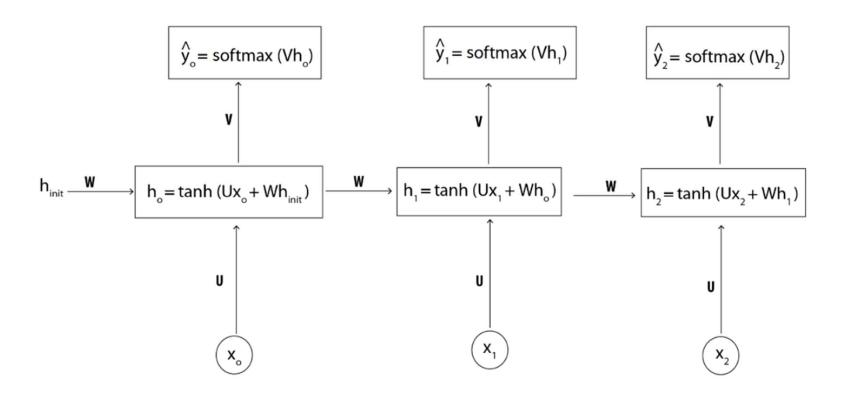
- Notations
 - *U*: input to hidden layer weight matrix
 - *W*: hidden to hidden layer weight matrix
 - V: hidden to output layer weight matrix
- The hidden state h at time step t
 - $h_t = \tanh(Ux_t + Wh_{t-1})$
 - The initial hidden state is set to random values
- The output at time step *t*
 - $\hat{y}_t = \operatorname{softmax}(Vh_t)$





Forward Propagation in RNN

Unrolled representation of RNN



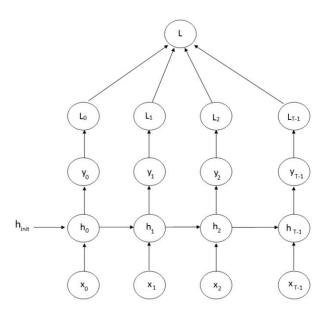


• In order to update the parameters, we first need to compute the loss function. Here we use the cross-entropy loss function.

$$L_t = -y_t \log(\hat{y}_t)$$

- cross-entropy is an alternative to mean squared error. CE is known to produce faster convergence compared to MSE.
- The final loss is a sum of the loss at all the time steps.

$$L = \sum_{j=0}^{T-1} L_j$$





- Our goal is to update parameters so that the loss is minimized.
- Equation for updating the weights

$$U = U - \alpha \frac{\partial L}{\partial U}$$
 $W = W - \alpha \frac{\partial L}{\partial W}$ $V = V - \alpha \frac{\partial L}{\partial V}$

- To update V, we use the following equation.
- For example, for the output at time t = 3,

$$\bullet \quad \frac{\partial E_3}{\partial V} = \frac{\partial E_3}{\partial \bar{y}_3} \cdot \frac{\partial \bar{y}_3}{\partial V}$$

• Now, we need to update W. Using the chain rule, we can say:

$$\frac{\partial E_3}{\partial W} = \frac{\partial E_3}{\partial \bar{y}_3} \cdot \frac{\partial \bar{y}_3}{\partial \bar{s}_3} \cdot \frac{\partial \bar{s}_3}{\partial W}$$

However, that is not all. We can also say,

$$\frac{\partial E_3}{\partial W} = \frac{\partial E_3}{\partial \bar{y}_3} \cdot \frac{\partial \bar{y}_3}{\partial \bar{s}_3} \cdot \frac{\partial \bar{s}_3}{\partial \bar{s}_2} \cdot \frac{\partial \bar{s}_2}{\partial W} \qquad \qquad \frac{\partial E_3}{\partial W} = \frac{\partial E_3}{\partial \bar{y}_3} \cdot \frac{\partial \bar{y}_3}{\partial \bar{s}_3} \cdot \frac{\partial \bar{s}_3}{\partial \bar{s}_2} \cdot \frac{\partial \bar{s}_2}{\partial \bar{s}_1} \cdot \frac{\partial \bar{s}_1}{\partial W}$$

• So we add up everything.

$$\frac{\partial E_3}{\partial W} = \frac{\partial E_3}{\partial \bar{y}_3} \cdot \frac{\partial \bar{y}_3}{\partial \bar{s}_3} \cdot \frac{\partial \bar{s}_3}{\partial W} + \frac{\partial E_3}{\partial \bar{y}_3} \cdot \frac{\partial \bar{y}_3}{\partial \bar{s}_3} \cdot \frac{\partial \bar{s}_3}{\partial \bar{s}_2} \cdot \frac{\partial \bar{s}_2}{\partial W} + \frac{\partial E_3}{\partial \bar{y}_3} \cdot \frac{\partial \bar{y}_3}{\partial \bar{s}_3} \cdot \frac{\partial \bar{s}_3}{\partial \bar{s}_2} \cdot \frac{\partial \bar{s}_2}{\partial \bar{s}_1} \cdot \frac{\partial \bar{s}_1}{\partial W}$$

 Since the backpropagation propagates back in time, it is called backpropagation through time (BPTT).



- Say we are at t=10. How far should we go back in time when calculating the gradient for W?
- As gradients are multiplied, its value becomes very small. This is called the vanishing gradient problem.
- So we can only backpropagate through a limited amount of time.

$$\frac{\partial E_3}{\partial W} = \frac{\partial E_3}{\partial \bar{y}_3} \cdot \frac{\partial \bar{y}_3}{\partial \bar{s}_3} \cdot \frac{\partial \bar{s}_3}{\partial \bar{s}_2} \cdot \frac{\partial \bar{s}_2}{\partial \bar{s}_1} \cdot \frac{\partial \bar{s}_1}{\partial W}$$

Similar calculations apply to *U*.

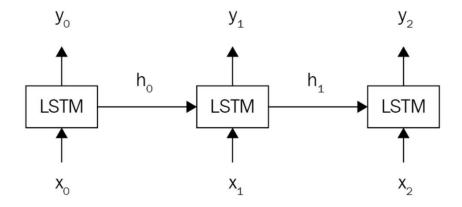
$$\frac{\partial E_3}{\partial U} = \frac{\partial E_3}{\partial \bar{y}_3} \cdot \frac{\partial \bar{y}_3}{\partial \bar{s}_3} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial E_3}{\partial \bar{y}_3} \cdot \frac{\partial \bar{y}_3}{\partial \bar{s}_3} \cdot \frac{\partial \bar{s}_3}{\partial \bar{s}_2} \cdot \frac{\partial \bar{s}_2}{\partial U} + \frac{\partial E_3}{\partial \bar{y}_3} \cdot \frac{\partial \bar{y}_3}{\partial \bar{s}_3} \cdot \frac{\partial \bar{s}_3}{\partial \bar{s}_2} \cdot \frac{\partial \bar{s}_2}{\partial \bar{s}_1} \cdot \frac{\partial \bar{s}_2}{\partial U} + \frac{\partial \bar{s}_3}{\partial \bar{s}_2} \cdot \frac{\partial \bar{s}_3}{\partial \bar{s}_3} \cdot \frac{\partial \bar{s}_3}{\partial \bar{s}_2} \cdot \frac{\partial \bar{s}_2}{\partial \bar{s}_1} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} \cdot \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U} + \frac{\partial \bar{s}_3}{\partial U$$



The problem of RNN is that it cannot retain long sequences in the memory.
 Thus, it is not good at predicting the blank term in:

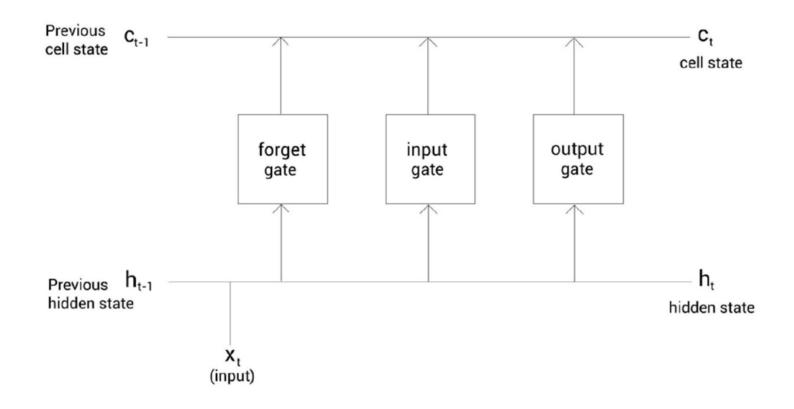
Archie lived in China for 13 years. He loves listening to good music. He is a fan of comics. He is fluent in _____.

- LSTM is a variant of RNN that can retain information in the memory as long as it is required.
- In LSTM network, the RNN cells are replaced with LSTM cells.



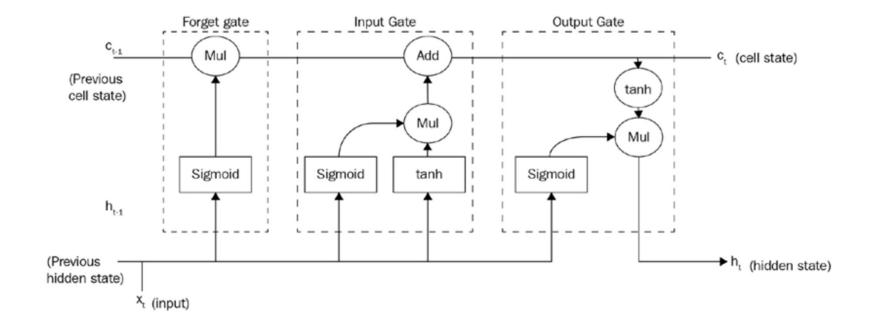


• The LSTM cell



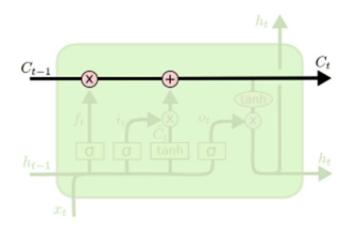


Internals of LSTM gates





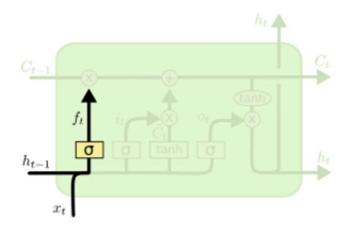
- Cell state
 - LSTM maintains a cell state which acts as a long-term memory





Forget gate

- A gate responsible for deciding what to discard from the previous cell state.
- Suppose we are reading a sentence, and the previous subject was a male.
- Now, the new input is another subject.
- At this point, we would like to forget the gender of the previous subject.
- If f_t is close to 0, it means "forget everything".
- If f_t is close to 1, it means "remember everything".

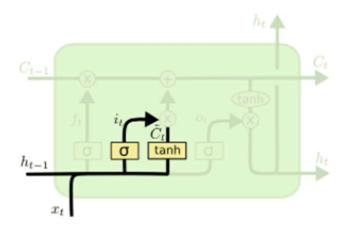


$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] + b_f\right)$$



Input gate

- A gate responsible for deciding what information should be stored in the memory.
- Since we have a new subject, we might want to remember the gender of the new subject.
- The sigmoid line decides which information to update.
- The tanh line calculates the new cell state \tilde{C}_t .

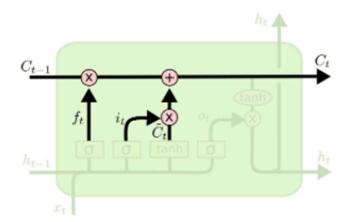


$$i_t = \sigma \left(W_i \cdot [h_{t-1}, x_t] + b_i \right)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$



- Updating the cell state
 - We multiple f_t to the previous cell state C_{t-1} .
 - Then, we add the calculated cell state \tilde{C}_t multipled by the weight i_t to get the new cell state C_t .

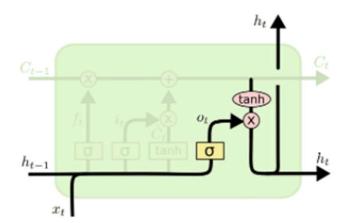


$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$



Output gate

- A gate responsible for deciding the output of the cell
- Since the input was a subject, the next word may be a verb.
- Depending on what it remembers, it will output a proper verb.
- tanh is applied to the new cell state \mathcal{C}_t , and is multipled by the value from the sigmoid line.



$$o_t = \sigma \left(W_o \left[h_{t-1}, x_t \right] + b_o \right)$$
$$h_t = o_t * \tanh \left(C_t \right)$$



• CNN is a widely used model architecture for various computer vision tasks such as image recognition.

Suppose we have an image containing a horse.

- When we feed the image to a model, we convert it to a matrix of pixel values.
- The dimension of the matrix is [width x height x channels]
- For color images, number of channels is 3 (RGB).
- We can use a simple ANN architecture for image recognition, but its performance is poor.
 - The model does not consider spatial structure of the image.



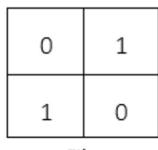
 In CNN, we use convolution layers and pooling layers to capture visual features from an image.

Convolution layers

- used to extract important features from the image
- a convolution layer uses a kernel or a filter, which is used to perform elementwise multiplication on the input.
- The size of the kernel is pre-determined.
- The values in the kernel are learnable parameters.

0	13	13
7	7	7
9	11	11

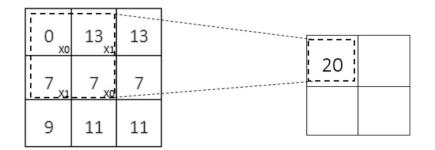




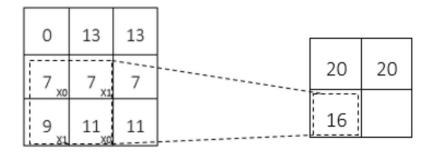
Filter

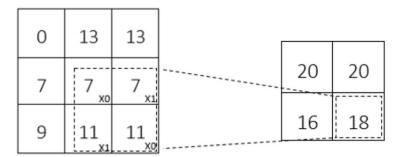


- Convolution layers
 - When propagating through a convolution layer, we perform convolution operation.



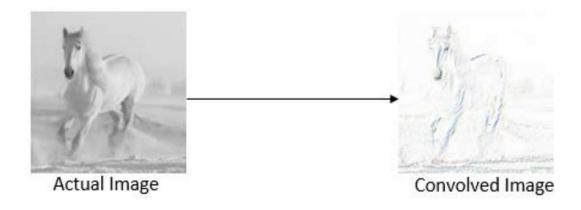
0	13 xo	13 x1	 	
7	7	7 x0	 20	20
9	11	11		







- Convolution layers
 - The output of a convolution layer is called a feature map or an activation map.
 - The feature map holds important features from the image.



 Instead of calculating a single feature map, we can use multiple filters to create multiple feature maps.

Feature map of depth 7



Strides

 When doing convolution, the number of pixels we slide over the input matrix is called a stride.

17 xo	80 x1	14	63		17	80	14 xo	63 x1
13 xı	11 xo	43	79	Stride of 2	13	11	43 xı	79 xo
27	33	7	4		27	33	7	4
255	69	77	63		255	69	77	63



- Padding
 - At the border of the image, the kernel may not fit into the image.

17	80	14	63 xo	X1
13	11	43	79 <u>x1</u>	<u>x</u> o,
27	33	7	4	
255	89	77	63	

We use padding to resolve this boundary case.



- Types of padding
 - zero padding: pad the borders with zero values
 - valid padding: if the kernel does not fit in a region, simply discard the region.

17	80	14	63 xo	0 X1
13	11	43	79 x1	0
27	33	7	4	
255	89	77	63	

 17
 80
 14
 63

 13
 11
 43
 79

 27
 33
 7
 4

 255
 89
 77
 63

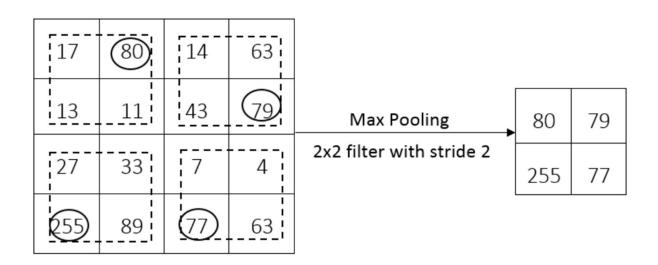
zero padding

valid padding



Pooling layers

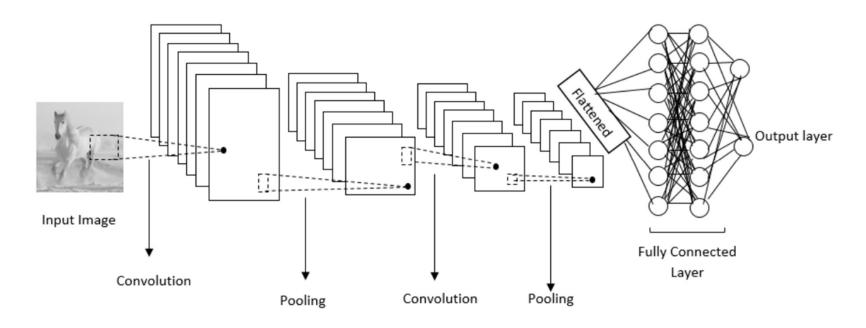
- After convolution, we often use pooling layers to downsample an image to a smaller image.
- In a max pooling, we choose the maximum value from a certain region and use it as a representative value for that region.
- In an average pooling, we average the values from a region and use that value.
- If we use a 2x2 pooling with stride 2, the resulting image is halved in both width and height.





Fully-connected layers

- The convolution and pooling layers extract features from the image and produce feature maps.
- We still need to classify images based on the feature maps.
- The fully-connected layers is used to classify images.
- This layer is basically a layer in the ANN.





- Generative adversarial networks are networks used to generate new data points.
- It is used to generate realistic human faces, converting grayscale images to colored images, translating text descriptions to realistic images, and so on.





- Training a GAN is essentially playing a two-player game where one tries to defeat the other.
- There is a counterfeiter who tries to create fake money.
- There is a police who tries to find fake money.
- The goal of the counterfeiter is to create a very realistic fake money so that it cannot be differentiated from the real money.
- The goal of the police is to discriminate the fake money from the real money.



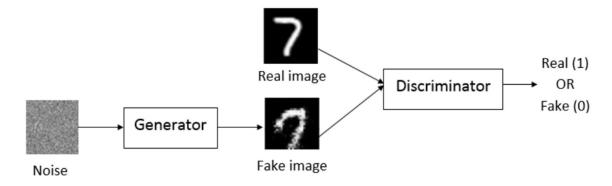
- A GAN consists of two components
 - Generator (counterfeiter)
 - Discriminator (police)

Generator

- The generator learns the distribution of images in the dataset.
- Once it learns the distribution, it can convert a random noise into a new image of the learned distribution

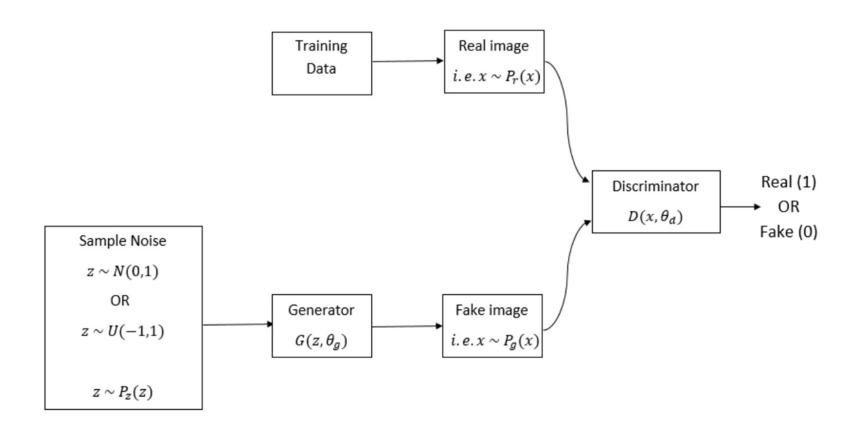
Discriminator

- The discriminator performs a classification task with two classes: real and fake.
- A "real" image is an original image in the dataset, and a "fake" image is one created by the generator.





Architecture of a GAN





Terms in the architecture

- $G(z; \theta_g)$: the generator
- z: input to the generator; a random noise
- P_z : distribution of the random noise
- N(0,1): a Gaussian distribution with mean 0 and std 1.
- U(-1,1): a Uniform distribution from -1 to 1.
- θ_q : parameters of the generator network
- P_g : distribution of samples created by the generator
- P_r : distribution of the real dataset
- $D(x; \theta_d)$: the discriminator
- x: input to the discriminator; an image
- θ_d : parameters of the discriminator network



- Loss functions
 - discriminator loss

$$\max_{d} L(D,G) = \mathbb{E}_{x \sim P_r(x)}[\log D(x;\theta_d)] + \mathbb{E}_{z \sim P_z(z)}\left[\log\left(1 - D\big(G\big(z;\theta_g\big);\theta_d\big)\right)\right]$$

- first term: log-likelihood that the discriminator will say "real" to the real images.
- second term: log-likelihood that the discriminator will say "fake" to the generated images.
- generator loss

$$\min_{g} L(D, G) = \mathbb{E}_{z \sim P_{z}(z)} \left[\log \left(1 - D(G(z; \theta_{g}); \theta_{d}) \right) \right]$$

• first term: log-likelihood that the discriminator will correctly determine the fake images.



Total loss

$$\min_{G} \max_{D} L(D, G) = \mathbb{E}_{x \sim P_r(x)}[\log D(x)] + \mathbb{E}_{z \sim P_z(z)}[\log \left(1 - D(G(z))\right)]$$

- Our objective function is a min-max objective function.
- a maximization for the discriminator and a minimization for the generator
- Gradient Descent
 - maximization on the discriminator

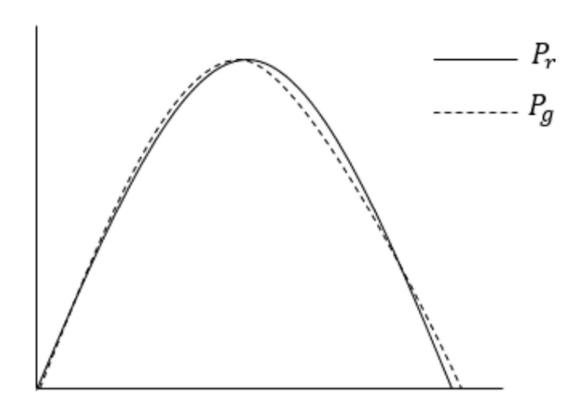
$$\nabla \theta_d \frac{1}{m} \sum_{i=1}^m \left[\log D(x^{(i)}) + \log \left(1 - D\left(G(z^{(i)})\right) \right) \right]$$

minimization on the generator

$$\nabla \theta_g \frac{1}{m} \sum_{i=1}^m \log \left(1 - D\left(G(z^{(i)}) \right) \right)$$



- Goal of training
 - $-\,\,$ Generator learns the real data distribution, so that P_g and P_r become very similar.





End of Chapter

- Do you understand the concept of deep learning as well as algorithms such as backpropagation?
- Do you understand various methods of deep learning such as RNN, CNN, and GAN?
- Can you write [ex015] yourself?



End of Class

Questions?

Email: jso1@sogang.ac.kr

