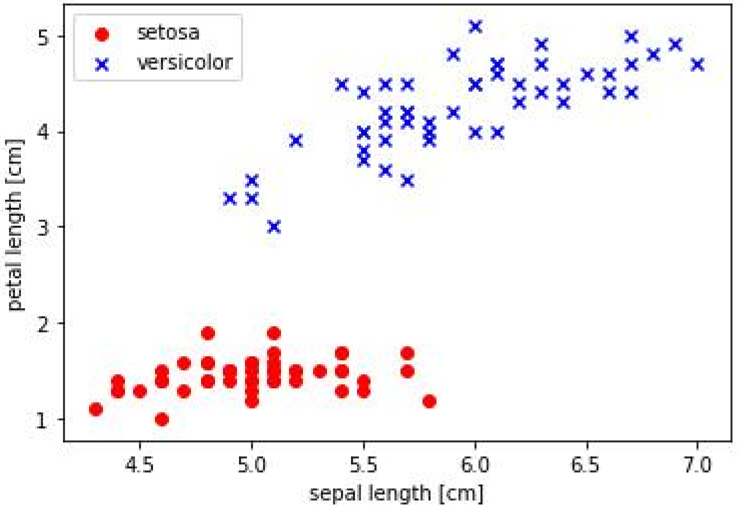
❑ Reading-in the Iris data

```
import os
import pandas as pd


s = 'https://archive.ics.uci.edu/ml/machine-learning-databases/iris/iris.data'
print('URL:', s)

df = pd.read_csv(s,
                 header=None,
                 encoding='utf-8')

df.tail()
```

	0	1	2	3	4
145	6.7	3.0	5.2	2.3	Iris-virginica
146	6.3	2.5	5.0	1.9	Iris-virginica
147	6.5	3.0	5.2	2.0	Iris-virginica
148	6.2	3.4	5.4	2.3	Iris-virginica
149	5.9	3.0	5.1	1.8	Iris-virginica

❑ Plotting the Iris data

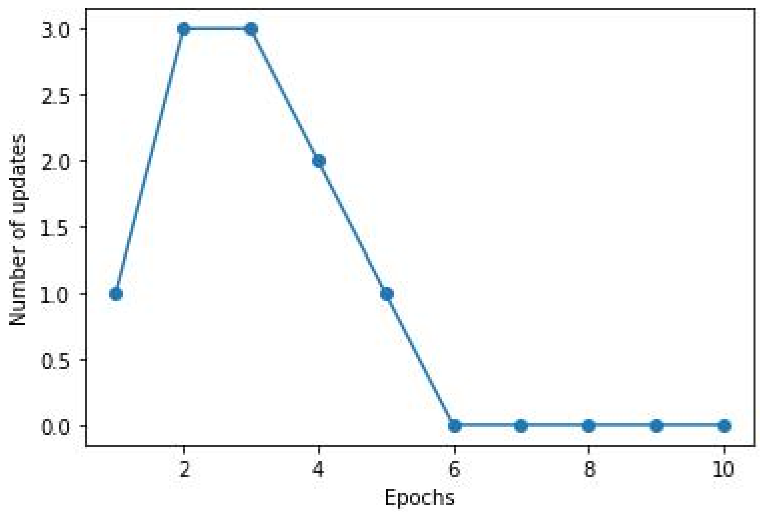
```
%matplotlib inline
import matplotlib.pyplot as plt
import numpy as np

# select setosa and versicolor
y = df.iloc[0:100, 4].values
y = np.where(y == 'Iris-setosa', -1, 1)

# extract sepal length and petal length
X = df.iloc[0:100, [0, 2]].values

# plot data
plt.scatter(X[:50, 0], X[:50, 1],
            color='red', marker='o', label='setosa')
plt.scatter(X[50:100, 0], X[50:100, 1],
            color='blue', marker='x', label='versicolor')

plt.xlabel('sepal length [cm]')
plt.ylabel('petal length [cm]')
plt.legend(loc='upper left')

# plt.savefig('images/02_06.png', dpi=300)
plt.show()
```



Training the perceptron model

- misclassification errors vs epochs

```
ppn = Perceptron(eta=0.1, n_iter=10)

ppn.fit(X, y)

plt.plot(range(1, len(ppn.errors_) + 1), ppn.errors_, marker='o')
plt.xlabel('Epochs')
plt.ylabel('Number of updates')

# plt.savefig('images/02_07.png', dpi=300)
plt.show()
```

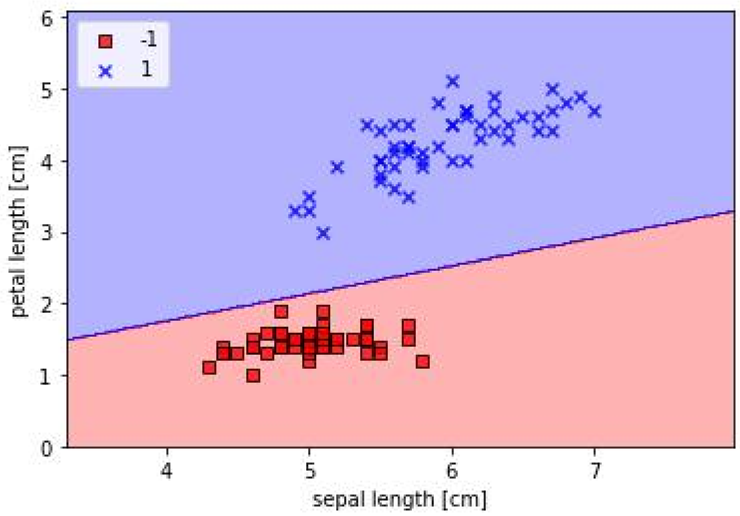


❑ A function for plotting decision regions

```python
from matplotlib.colors import ListedColormap


def plot_decision_regions(X, y, classifier, resolution=0.02):

    # setup marker generator and color map
    markers = ('s', 'x', 'o', '^', 'v')
    colors = ('red', 'blue', 'lightgreen', 'gray', 'cyan')
    cmap = ListedColormap(colors[:len(np.unique(y))])

    # plot the decision surface
    x1_min, x1_max = X[:, 0].min() - 1, X[:, 0].max() + 1
    x2_min, x2_max = X[:, 1].min() - 1, X[:, 1].max() + 1
    xx1, xx2 = np.meshgrid(np.arange(x1_min, x1_max, resolution),
                           np.arange(x2_min, x2_max, resolution))
    Z = classifier.predict(np.array([xx1.ravel(), xx2.ravel()]).T)
    Z = Z.reshape(xx1.shape)
    plt.contourf(xx1, xx2, Z, alpha=0.3, cmap=cmap)
    plt.xlim(xx1.min(), xx1.max())
    plt.ylim(xx2.min(), xx2.max())

    # plot class examples
    for idx, cl in enumerate(np.unique(y)):
        plt.scatter(x=X[y == cl, 0],
                    y=X[y == cl, 1],
                    alpha=0.8,
                    c=colors[idx],
                    marker=markers[idx],
                    label=cl,
                    edgecolor='black')
```

❑ plotting decision regions

```
plot_decision_regions(X, y, classifier=ppn)
plt.xlabel('sepal length [cm]')
plt.ylabel('petal length [cm]')
plt.legend(loc='upper left')


# plt.savefig('images/02_08.png', dpi=300)
plt.show()
```

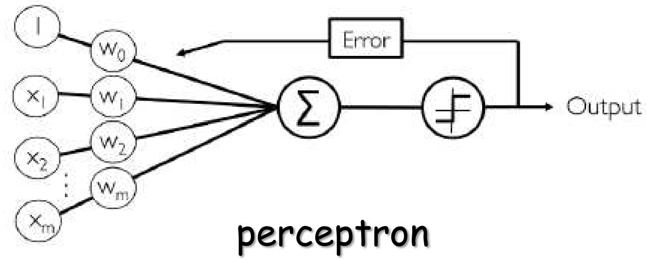


Adaptive linear neurons rule (Adaline rule)

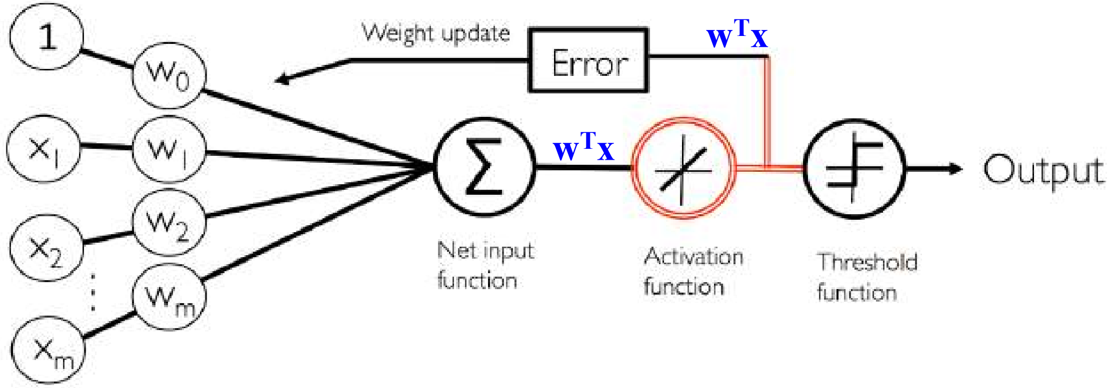
- aka Widrow-Hoff rule
- weights are updated based on a linear activation function
- activation function, $\phi(z)$: identity function, linear

$$\boxed{\phi(w^T x) = w^T x}$$

$$\boxed{\begin{aligned} & w_0 = -\theta \text{ and } x_0 = 1 \\ & z = w_0 x_0 + w_1 x_1 + \cdots + w_m x_m = \boldsymbol{w}^T \boldsymbol{x} \\ & \phi(z) = \begin{cases} 1 & \text{if } z \geq 0, \\ -1 & \text{otherwise.} \end{cases} \end{aligned}}$$



perceptron

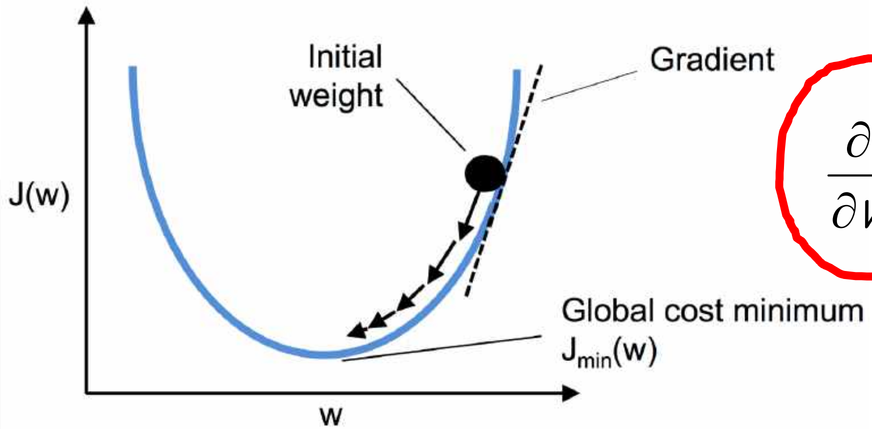


Adaptive linear neuron (Adaline)

Minimizing cost functions

- objective function – sum of squared errors (SSE) between calculated outcome and true class label

- differentiable, convex, gradient descent



$$\frac{\partial J}{\partial w_j}$$

The squared error derivative

$$\frac{\partial J}{\partial w_j} = \frac{\partial}{\partial w_j} \frac{1}{2} \sum_i (y^{(i)} - \phi(z^{(i)}))^2$$

$$=$$

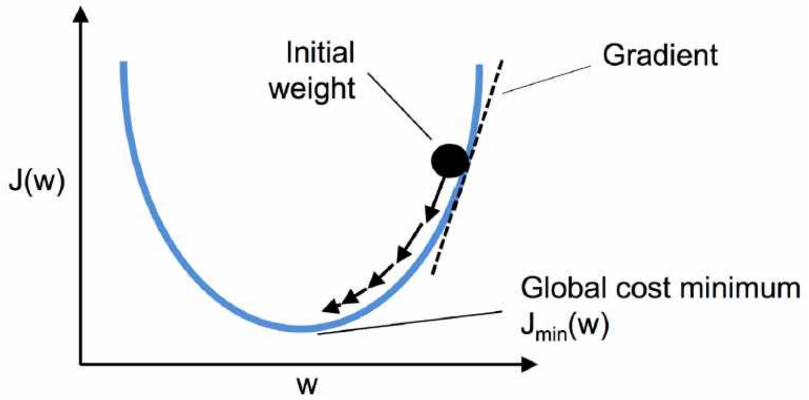
Minimizing cost functions: weight update

$$\frac{\partial J}{\partial w_j} = -\sum_i (y^{(i)} - \phi(z^{(i)}))x_j^{(i)}$$

$$\Delta w_j = -\eta \frac{\partial J}{\partial w_j} = \eta \sum_i (y^{(i)} - \phi(z^{(i)}))x_j^{(i)}$$

$$\Delta w = -\eta \nabla J(w)$$

$$w = w + \Delta w$$



Implementing Adaline in Python

```python
class AdalineGD(object):
    """ADAptive LInear NEuron classifier.

    Parameters
    ------------
    eta : float
      Learning rate (between 0.0 and 1.0)
    n_iter : int
      Passes over the training dataset.
    random_state : int
      Random number generator seed for random weight
      initialization.


    Attributes
    -----------
    w_ : 1d-array
      Weights after fitting.
    cost_ : list
      Sum-of-squares cost function value in each epoch.

    """
    def __init__(self, eta=0.01, n_iter=50, random_state=1):
        self.eta = eta
        self.n_iter = n_iter
        self.random_state = random_state
```

```python
class Perceptron(object):
    """Perceptron classifier.
```

```python
    errors_ : list
      Number of misclassifications (updates) in each epoch.
```

```python
    def fit(self, X, y):
        """ Fit training data.

        Parameters
        ----------
        X : {array-like}, shape = [n_examples, n_features]
          Training vectors, where n_examples is the number of examples and
          n_features is the number of features.
        y : array-like, shape = [n_examples]
          Target values.

        Returns
        -------
        self : object

        """
        rgen = np.random.RandomState(self.random_state)
        self.w_ = rgen.normal(loc=0.0, scale=0.01, size=1 + X.shape[1])
        self.cost_ = []

        for i in range(self.n_iter):
            net_input = self.net_input(X)
            # Please note that the "activation" method has no effect
            # in the code since it is simply an identity function. We
            # could write `output = self.net_input(X)` directly instead.
            # The purpose of the activation is more conceptual, i.e.,
            # in the case of logistic regression (as we will see later),
            # we could change it to
            # a sigmoid function to implement a logistic regression classifier.
            output = self.activation(net_input)
            errors = (y - output)
            self.w_[1:] += self.eta * X.T.dot(errors)
            self.w_[0] += self.eta * errors.sum()
            cost = (errors**2).sum() / 2.0
            self.cost_.append(cost)
        return self

    def net_input(self, X):
        """Calculate net input"""
        return np.dot(X, self.w_[1:]) + self.w_[0]

    def activation(self, X):
        """Compute linear activation"""
        return X

    def predict(self, X):
        """Return class label after unit step"""
        return np.where(self.activation(self.net_input(X)) >= 0.0, 1, -1)
```

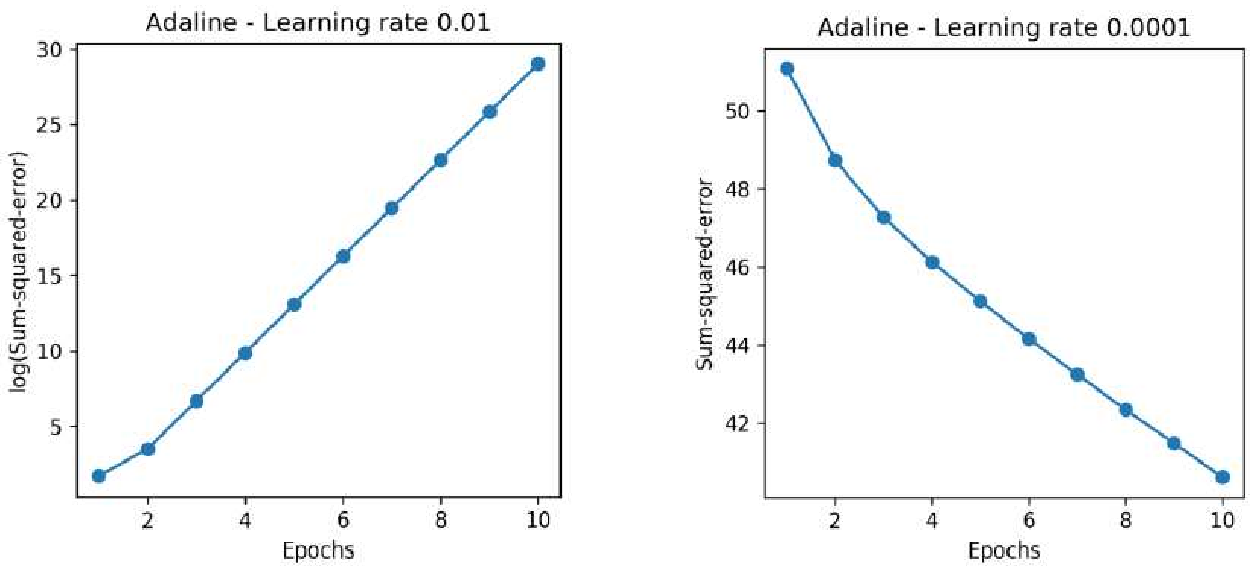
Adaline in Python

```python
def fit(self, X, y):
    """ Fit training data.

    Parameters
    ----------
    X : {array-like}, shape = [n_examples, n_features]
      Training vectors, where n_examples is the number of examples and
      n_features is the number of features.
    y : array-like, shape = [n_examples]
      Target values.

    Returns
    -------
    self : object

    """
    rgen = np.random.RandomState(self.random_state)
    self.w_ = rgen.normal(loc=0.0, scale=0.01, size=1 + X.shape[1])
    self.cost_ = []

    for i in range(self.n_iter):
        net_input = self.net_input(X)
        # Please note that the "activation" method has no effect
        # in the code since it is simply an identity function. We
        # could write `output = self.net_input(X)` directly instead.
        # The purpose of the activation is more conceptual, i.e.,
        # in the case of logistic regression (as we will see later),
        # we could change it to
        # a sigmoid function to implement a logistic regression classifier.
        output = self.activation(net_input)
        errors = (y - output)
        self.w_[1:] += self.eta * X.T.dot(errors)
        self.w_[0] += self.eta * errors.sum()
        cost = (errors**2).sum() / 2.0
        self.cost_.append(cost)
    return self
```

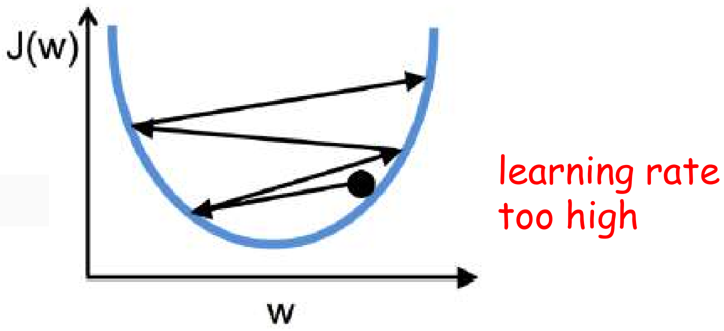
```python
def net_input(self, X):
    """Calculate net input"""
    return np.dot(X, self.w_[1:]) + self.w_[0]

def activation(self, X):
    """Compute linear activation"""
    return X

def predict(self, X):
    """Return class label after unit step"""
    return np.where(self.activation(self.net_input(X)) >= 0.0, 1, -1)
```

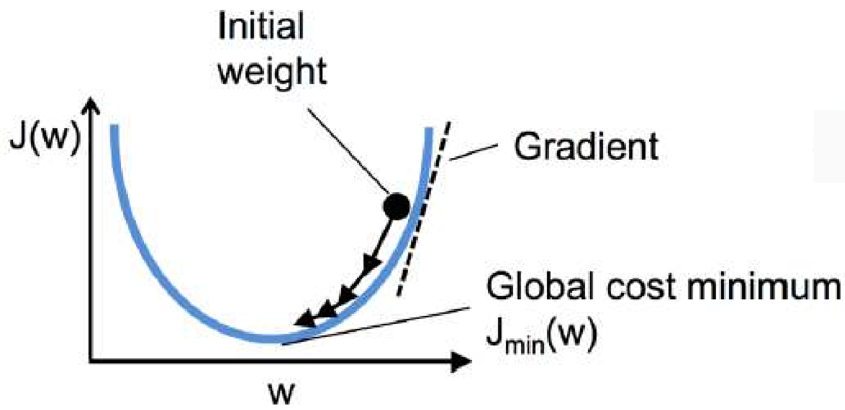
Convergence of Adaline rule

```
fig, ax = plt.subplots(nrows=1, ncols=2, figsize=(10, 4))

ada1 = AdalineGD(n_iter=10, eta=0.01).fit(X, y)
ax[0].plot(range(1, len(ada1.cost_) + 1), np.log10(ada1.cost_), marker='o')
ax[0].set_xlabel('Epochs')
ax[0].set_ylabel('log(Sum-squared-error)')
ax[0].set_title('Adaline - Learning rate 0.01')

ada2 = AdalineGD(n_iter=10, eta=0.0001).fit(X, y)
ax[1].plot(range(1, len(ada2.cost_) + 1), ada2.cost_, marker='o')
ax[1].set_xlabel('Epochs')
ax[1].set_ylabel('Sum-squared-error')
ax[1].set_title('Adaline - Learning rate 0.0001')

# plt.savefig('images/02_11.png', dpi=300)
plt.show()
```



Convergence of Adaline rule





Feature scaling

- helps gradient descent learning to converge more quickly
- standardization
  - gives the data the properties of standard normal distribution, N(0, 1)

$$
\begin{bmatrix}
x_1^{(1)} & x_2^{(1)} & x_3^{(1)} & x_4^{(1)} \\
x_1^{(2)} & x_2^{(2)} & x_3^{(2)} & x_4^{(2)} \\
\vdots & \vdots & \vdots & \vdots \\
x_1^{(150)} & x_2^{(150)} & x_3^{(150)} & x_4^{(150)}
\end{bmatrix}
$$

$$
\boldsymbol{x}_j = \begin{bmatrix} x_j^{(1)} \\ x_j^{(2)} \\ \cdots \\ x_j^{(150)} \end{bmatrix}
$$

sample mean: μ_j
standard deviation: σ_j

normalization of jth feature values of training examples

$$x'_j = \frac{x_j - \mu_j}{\sigma_j}$$

Feature scaling

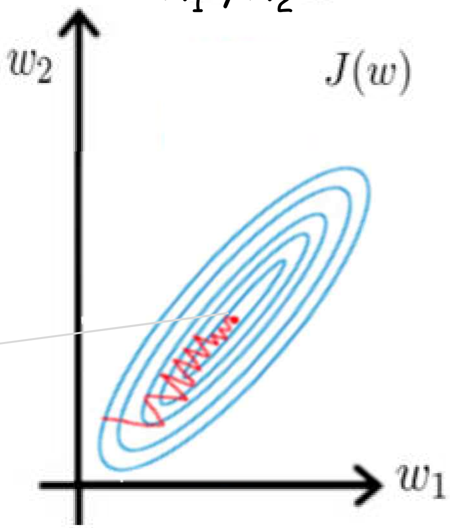
$$\boldsymbol{x}_j = \begin{bmatrix} x_j^{(1)} \\ x_j^{(2)} \\ \cdots \\ x_j^{(150)} \end{bmatrix}$$

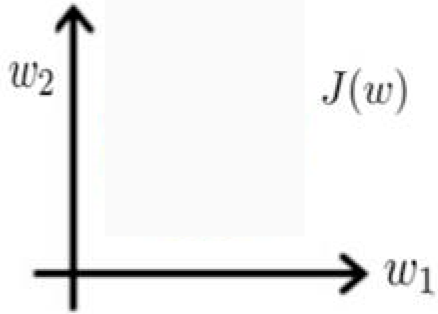
$$x'_j = \frac{x_j - \mu_j}{\sigma_j}$$

sample mean: μ_j
standard deviation: σ_j

normalization of jth feature values of training examples

x_1 , x_2





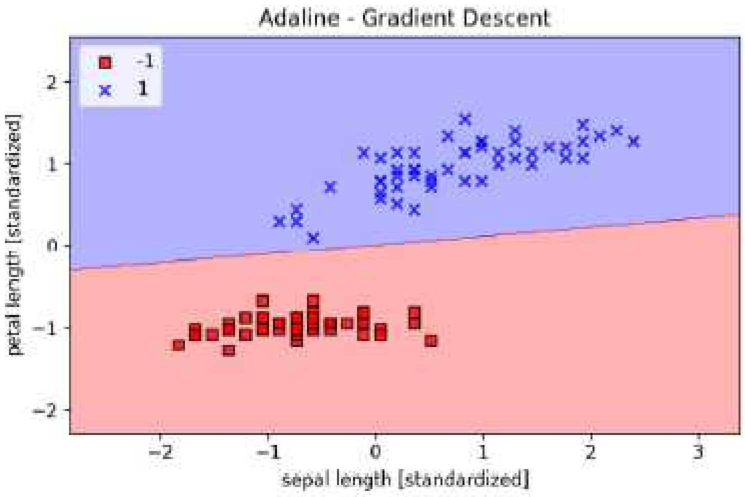
$x'_j = \frac{x_j - \mu_j}{\sigma_j}$

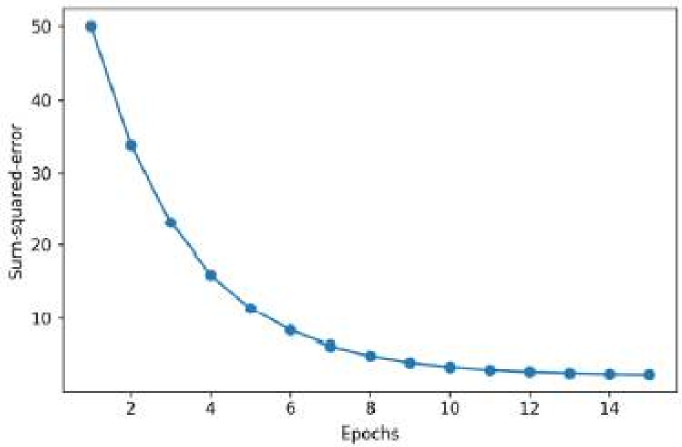
i

feature scaling

```
# standardize features
X_std = np.copy(X)
X_std[: , 0] = (X[: , 0] – X[: , 0].mean( ) ) / X[: , 0].std( )
X_std[: , 1] = (X[: , 1] – X[: , 1].mean( ) ) / X[: , 1].std( )
```

```
ada_gd = AdalineGD(n_iter=15, eta=0.01)
ada_gd.fit(X_std, y)

plot_decision_regions(X_std, y, classifier=ada_gd)
plt.title('Adaline - Gradient Descent')
plt.xlabel('sepal length [standardized]')
plt.ylabel('petal length [standardized]')
plt.legend(loc='upper left')
plt.tight_layout()
# plt.savefig('images/02_14_1.png', dpi=300)
plt.show()

plt.plot(range(1, len(ada_gd.cost_) + 1), ada_gd.cost_, marker='o')
plt.xlabel('Epochs')
plt.ylabel('Sum-squared-error')

plt.tight_layout()
# plt.savefig('images/02_14_2.png', dpi=300)
plt.show()
```



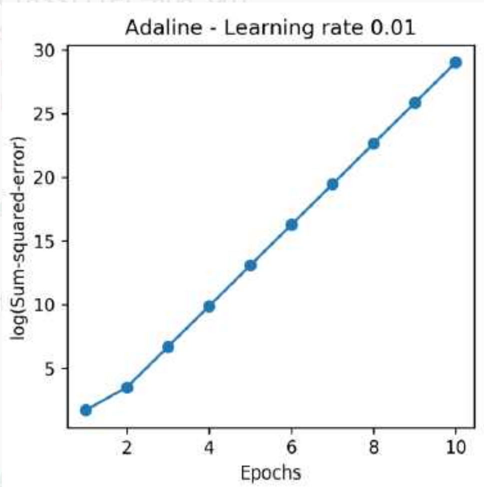


feature scaling

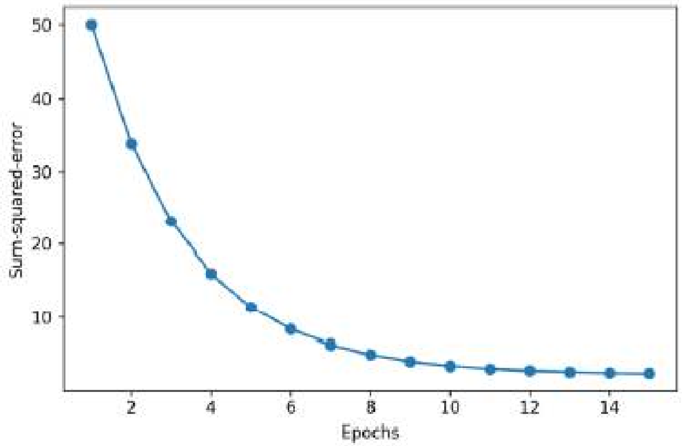
```
# standardize features
X_std = np.copy(X)
X_std[: , 0] = (X[: , 0] – X[: , 0].mean( ) ) / X[: , 0].std( )
X_std[: , 1] = (X[: , 1] – X[: , 1].mean( ) ) / X[: , 1].std( )
```

```
ada_gd = AdalineGD(n_iter=15, eta=0.01)
ada_gd.fit(X_std, y)
```

```
plot_decision_regions(X_std, y, classifier=ada_gd)
plt.title('Adaline - Gradient De
plt.xlabel('sepal length [standa
plt.ylabel('petal length [standa
plt.legend(loc='upper left')
plt.tight_layout()
# plt.savefig('images/02_14_1.pn
plt.show()

plt.plot(range(1, len(ada_gd.cos
plt.xlabel('Epochs')
plt.ylabel('Sum-squared-error')

plt.tight_layout()
# plt.savefig('images/02_14_2.pn
plt.show()
```



Learning rate = 0.01



Stochastic gradient descent (SGD)

- also called iterative or online gradient descent
- update the weights incrementally for *each* training example, $\mathbf{x}^{(i)}$:

$$\begin{bmatrix} x_1^{(1)} & x_2^{(1)} & x_3^{(1)} & x_4^{(1)} \\ x_1^{(2)} & x_2^{(2)} & x_3^{(2)} & x_4^{(2)} \\ \vdots & \vdots & \vdots & \vdots \\ x_1^{(150)} & x_2^{(150)} & x_3^{(150)} & x_4^{(150)} \end{bmatrix}$$

- instead of updating weights based on sum of accumulated errors over all training examples, $\mathbf{x}^{(i)}$, i = 1, 2, …, 150.
  - when minimizes cost function with gradient descent

$$\boldsymbol{x}_j = \begin{bmatrix} x_j^{(1)} \\ x_j^{(2)} \\ \cdots \\ x_j^{(150)} \end{bmatrix}$$

$$\Delta w_j = -\eta \frac{\partial J}{\partial w_j} = \eta \sum_i (y^{(i)} - \phi(z^{(i)})) x_j^{(i)}$$

Stochastic gradient descent (SGD)

- update the weights incrementally for each training example, $\mathbf{x}^{(i)}$:

$$\Delta w_j = \eta(y^{(i)} - \phi(z^{(i)}))x_j^{(i)}$$

- can be considered as an approximation of gradient descent.
- it typically reaches convergence much faster because of more frequent weight updates.
  - for each iteration, 150 updates using SGD but only one update using GD.

$$\begin{bmatrix} x_1^{(1)} & x_2^{(1)} & x_3^{(1)} & x_4^{(1)} \\ x_1^{(2)} & x_2^{(2)} & x_3^{(2)} & x_4^{(2)} \\ \vdots & \vdots & \vdots & \vdots \\ x_1^{(150)} & x_2^{(150)} & x_3^{(150)} & x_4^{(150)} \end{bmatrix}$$

- to obtain satisfying results, important to present traning data in a random order.
- to prevent cycles, shuffle training data for every epoch.
- can be used for online learning

```python
class AdalineSGD(object):
    """ADAptive LInear NEuron classifier.

    Parameters
    ------------
    eta : float
      Learning rate (between 0.0 and 1.0)
    n_iter : int
      Passes over the training dataset.
    shuffle : bool (default: True)
      Shuffles training data every epoch if True to prevent cycles.
    random_state : int
      Random number generator seed for random weight
      initialization.


    Attributes
    -----------
    w_ : 1d-array
      Weights after fitting.
    cost_ : list
      Sum-of-squares cost function value averaged over all
      training examples in each epoch.


    """
```

```python
def __init__(self, eta=0.01, n_iter=10, shuffle=True, random_state=None):
    self.eta = eta
    self.n_iter = n_iter
    self.w_initialized = False
    self.shuffle = shuffle
    self.random_state = random_state

def fit(self, X, y):
    """ Fit training data.

    Parameters
    ----------
    X : {array-like}, shape = [n_examples, n_features]
      Training vectors, where n_examples is the number of examples and
      n_features is the number of features.
    y : array-like, shape = [n_examples]
      Target values.

    Returns
    -------
    self : object

    """
    self._initialize_weights(X.shape[1])
    self.cost_ = []
    for i in range(self.n_iter):
        if self.shuffle:
            X, y = self._shuffle(X, y)
        cost = []
        for xi, target in zip(X, y):
            cost.append(self._update_weights(xi, target))
        avg_cost = sum(cost) / len(y)
        self.cost_.append(avg_cost)
    return self
```

```python
def partial_fit(self, X, y):
    """Fit training data without reinitializing the weights"""
    if not self.w_initialized:
        self._initialize_weights(X.shape[1])
    if y.ravel().shape[0] > 1:
        for xi, target in zip(X, y):
            self._update_weights(xi, target)
    else:
        self._update_weights(X, y)
    return self

def _shuffle(self, X, y):
    """Shuffle training data"""
    r = self.rgen.permutation(len(y))
    return X[r], y[r]

def _initialize_weights(self, m):
    """Initialize weights to small random numbers"""
    self.rgen = np.random.RandomState(self.random_state)
    self.w_ = self.rgen.normal(loc=0.0, scale=0.01, size=1 + m)
    self.w_initialized = True

def _update_weights(self, xi, target):
    """Apply Adaline learning rule to update the weights"""
    output = self.activation(self.net_input(xi))
    error = (target - output)
    self.w_[1:] += self.eta * xi.dot(error)
    self.w_[0] += self.eta * error
    cost = 0.5 * error**2
    return cost
```

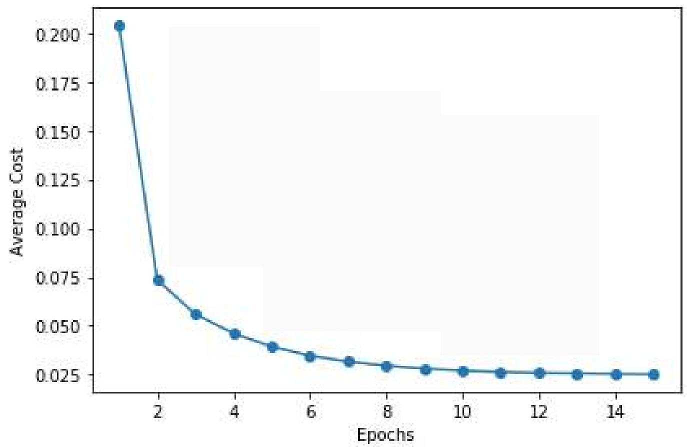
```python
def _update_weights(self, xi, target):
    """Apply Adaline learning rule to update the weights"""
    output = self.activation(self.net_input(xi))
    error = (target - output)
    self.w_[1:] += self.eta * xi.dot(error)
    self.w_[0] += self.eta * error
    cost = 0.5 * error**2
    return cost

def net_input(self, X):
    """Calculate net input"""
    return np.dot(X, self.w_[1:]) + self.w_[0]

def activation(self, X):
    """Compute linear activation"""
    return X

def predict(self, X):
    """Return class label after unit step"""
    return np.where(self.activation(self.net_input(X)) >= 0.0, 1, -1)
```

```python
ada_sgd = AdalineSGD(n_iter=15, eta=0.01, random_state=1)
ada_sgd.fit(X_std, y)

plot_decision_regions(X_std, y, classifier=ada_sgd)
plt.title('Adaline - Stochastic Gradient Descent')
plt.xlabel('sepal length [standardized]')
plt.ylabel('petal length [standardized]')
plt.legend(loc='upper left')

plt.tight_layout()
# plt.savefig('images/02_15_1.png', dpi=300)
plt.show()

plt.plot(range(1, len(ada_sgd.cost_) + 1), ada_sgd.cost_, marker='o')
plt.xlabel('Epochs')
plt.ylabel('Average Cost')

plt.tight_layout()
# plt.savefig('images/02_15_2.png', dpi=300)
plt.show()
```

Stochastic gradient descent (SGD)

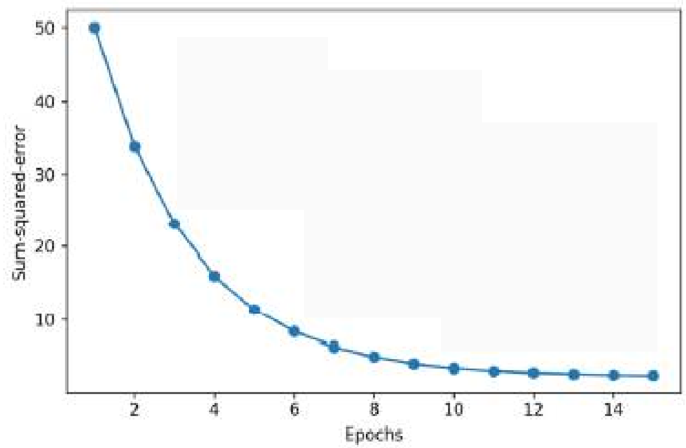
- comparison

Adaline – Stochastic Gradient Descent
Learning rate = 0.01



with feature scaling

Adaline – Gradient Descent
Learning rate = 0.01



with feature scaling