Application of Taylor Series in Black-Scholes Model for Option Pricing Problems.

Nijat Hasanli, Daryush Ray

The University of Warsaw, Faculty of Economics, Department of Quantitative Finance.

Abstract

This study compares the Black-Scholes model to the Taylor series approach in option pricing. We ran a series of tests to assess the accuracy of these two techniques in forecasting option prices. Our findings show that the Taylor series technique surpasses the Black-Scholes model in terms of accuracy, particularly for a tiny period of time. Overall, our work adds to the current debate on the best option pricing approach and provides important guidance for investors and financial sector practitioners.

Keywords: Black-Scholes Model, Taylor series, financial market, option pricing.

I Introduction

Option pricing is an important field of study in financial mathematics, and it has been the subject of countless studies throughout the years. The Black-Scholes model (Fischer Black and Myron Scholes, 1973) and the Taylor series approach (Brook Taylor, 1715) are two of the most extensively utilized option pricing strategies. This research compares the Black-Scholes model with the Taylor series method to add to the continuing discussion on the optimal option pricing strategy. We took a series of tests to evaluate the accuracy of these two strategies in projecting option prices in two parts first when the stock price increase second when the time to expiration decreases.

This paper is organized as follows: Section II explains the description of the problem. Section III shows the data that we used for our experiment. Section IV introduces the methodology. Section V lists the results and sensitivity analysis, and Section VI describes the summary and our recommendations.

II Description of the problem

The issue addressed by this paper is the ongoing dispute over the appropriate option pricing strategy. While the Black-Scholes model is widely used and recognized as a standard model in finance, new research reveals that the Taylor series approach may provide more accurate pricing, particularly for small fluctuations in the underlying asset. The purpose of this research is to contrast the Black-Scholes model with the Taylor series approach and give insights into their different strengths and weaknesses. This research seeks to contribute to the current debate over the ideal option pricing strategy and provide information for investors and financial industry practitioners.

III Data

As data, we take call options for Meta Inc from yahoo finance and we chose options that will be expired on 16-06-2023. For reproducible purposes, we choose any random option from the list that contains 106 options for that date. It is shown some of them:

	contractSymbol	lastTradeDate	strike	lastPrice	bid	ask	change	percentChange	volume	openInterest	impliedVolatility	inTheMoney	contr
0	META230616C00005000	2023-04-10 16:48:53+00:00	5.0	208.82	227.45	228.25	0.0	0.0	1.0	32	4.500004	True	RE
1	META230616C00010000	2022-10-27 19:02:35+00:00	10.0	88.00	101.30	102.30	0.0	0.0	3.0	0	0.000010	True	RE
2	META230616C00015000	2022-11-03 13:50:43+00:00	15.0	74.83	108.55	109.45	0.0	0.0	NaN	2	0.000010	True	RE
3	META230616C00020000	2023-05-03 17:04:02+00:00	20.0	218.55	212.50	213.35	0.0	0.0	1.0	2	2.992190	True	RE
4	META230616C00025000	2022-10-28 14:57:55+00:00	25.0	75.00	86.85	87.75	0.0	0.0	2.0	17	0.000010	True	RE
...	...	...	...	...	...	...	...	...	...	...	...	...	
102	META230616C00580000	2022-11-17 20:41:29+00:00	580.0	0.03	0.00	0.04	0.0	0.0	5.0	256	0.851564	False	RE
103	META230616C00590000	2023-02-01 14:42:38+00:00	590.0	0.03	0.00	0.18	0.0	0.0	5.0	573	0.988281	False	RE
104	META230616C00600000	2023-02-28 20:52:02+00:00	600.0	0.01	0.00	0.02	0.0	0.0	5.0	1654	0.835939	False	RE
105	META230616C00650000	2023-04-24 15:29:03+00:00	650.0	0.01	0.00	0.02	0.0	0.0	1.0	1679	0.906251	False	RE
106	META230616C00700000	2023-05-03 14:15:25+00:00	700.0	0.01	0.00	0.01	0.0	0.0	29.0	8185	0.906251	False	RE

IV Methodology

This study's methodology entails comparing the accuracy of the Black-Scholes model with the Taylor series technique in pricing choices with varying degrees of complexity. We ran a series of experiments on two different scenarios: when the stock price rises and when the time to expiry falls.

To conduct our experiment, we first collect a sample of option pricing with varied strike prices and the same expiration date. Then we choose randomly one option from the data. Next, the option price is then calculated using the Black-Scholes model and compared to the real market values. Following that, we estimated the option prices using the Taylor series approximation method and compared them to both the real market prices and the theoretical values determined using the Black-Scholes model. We compare both methods in two different scenarios. First when the time gets close to the expiration date and second when the price of stock increases.

V Results

Taylor approximation using Time Greeks (Θ, etc.)

As it is mentioned in the IV section first we take random call option from the data and then calculate the Black-Scholes model to compare with the real price of call option from the data.

Black-Scholes Model:

$$C(S_{t,}t) = N(d_1)S_t - N(d_2)Ke^{-r(T-t)}$$

$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[ln(\frac{S_t}{k}) + (r + \frac{\sigma^2}{2})(T-t)\right]$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

C(0,t) = 0 for all t

C(S,t) ⟶ S - K as S ⟶ ∞

C(S,T) = max{S - K,0}

$C(S_{t,}t)$ is the price of a European call option

K is the strike price; t is the current time

S_t is stock price; r is risk-free rate; T is the time option expiration;

```
Option data:
         contractSymbol             lastTradeDate  strike  lastPrice     bid  \
19  META230616C00100000 2023-05-05 19:56:59+00:00   100.0     133.37  133.15

      ask    change  percentChange  volume  openInterest  impliedVolatility  \
19  133.8 -5.160004      -3.724827     3.0          7697           1.313968

    inTheMoney contractSize currency
19        True      REGULAR      USD
Underlying asset price: 232.77999877929688
Strike price 100.0
Expiration in years 0.10684931506849316
IV 1.3139682739257812
Black and Scholes - Call Option Price: 133.47894624814495
```

As a result of the Black-Scholes model, we get 133.48 and the real last price of the call option is 133.37 which is pretty close to our prediction.

Then we use the Taylor series, we take the first, second, third, fourth, fifth, and sixth derivatives of option price with respect to time.

Taylor series:

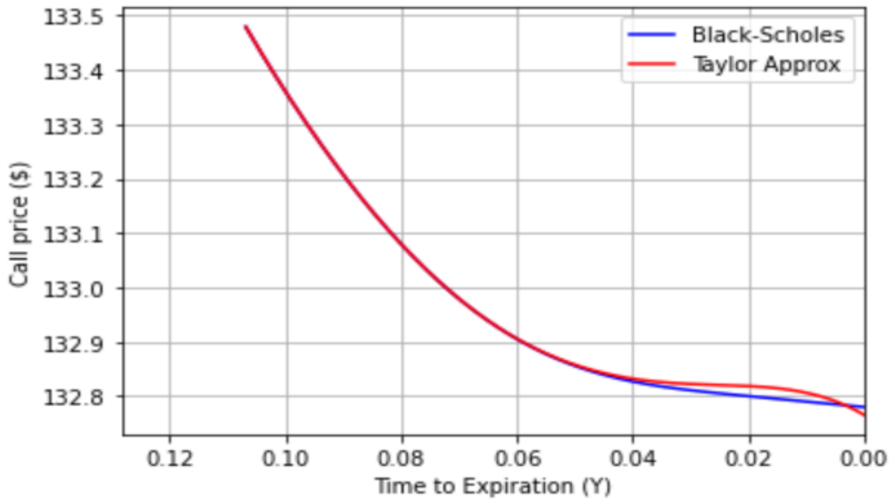
$$theta\ (\Theta) = f'(C_{t-1}) = \frac{\partial C_t}{\partial t}$$

$$f(C_t) = f(C_t) + \frac{f'(C_t)}{1!}(\Delta t)^1 + \frac{f''(C_t)}{2!}(\Delta t)^2 + \frac{f'''(C_t)}{3!}(\Delta t)^3 + \frac{f''''(C_t)}{4!}(\Delta t)^4 + \frac{f'''''(C_t)}{5!}(\Delta t)^5 + \frac{f''''''(C_t)}{6!}(\Delta t)^6$$

$\Delta t\ is\ difference\ in\ time$

$C_t\ is\ option\ price\ at\ time\ t$

Figure 1:

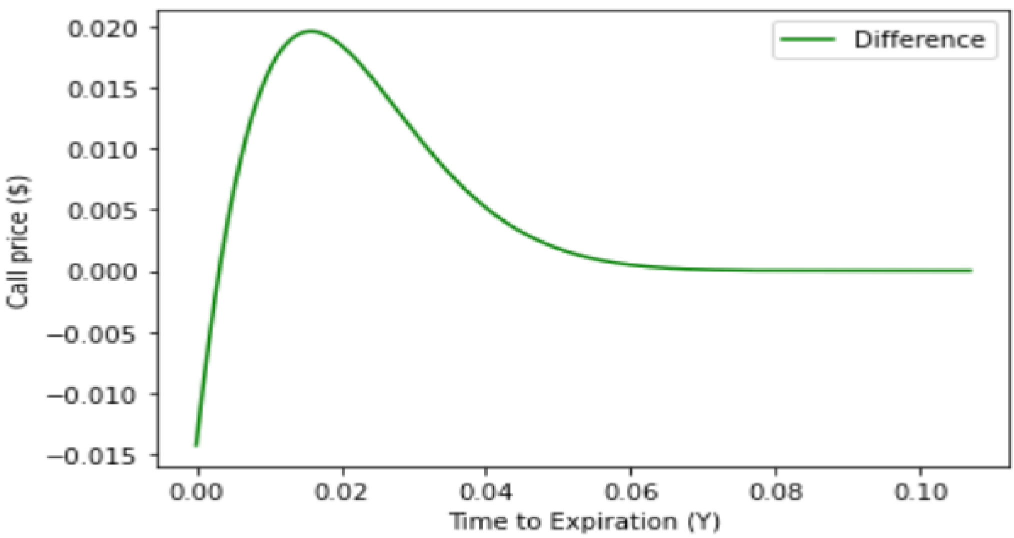



```
The Taylor approximation at t = 0.10684931506849316 is 132.7656952170691
Black and Scholes at t = 0.10684931506849316 is 132.7656952170691
0.10684931506849316
```

Here we can see that as time gets close to the expiration date the price of the call option in both methods decreases and the results, at time t = 0.107, are same 132.77

But we can see that after time t = 0.06 we have different results. The Taylor series expansion only approximates the option's value at a particular moment by employing a small number of terms in the expansion. As it goes away from the point of expansion, the accuracy of the approximation declines. In other words, the Taylor series approximation works best around the point of expansion, but its accuracy declines as one moves away.

Figure 2:



In the Figure 2, we can closely see the difference between the two approaches. At the end of the period the difference increased to 0.020 ($)

Taylor approximation using Price Greeks (Δ, Γ, etc.)

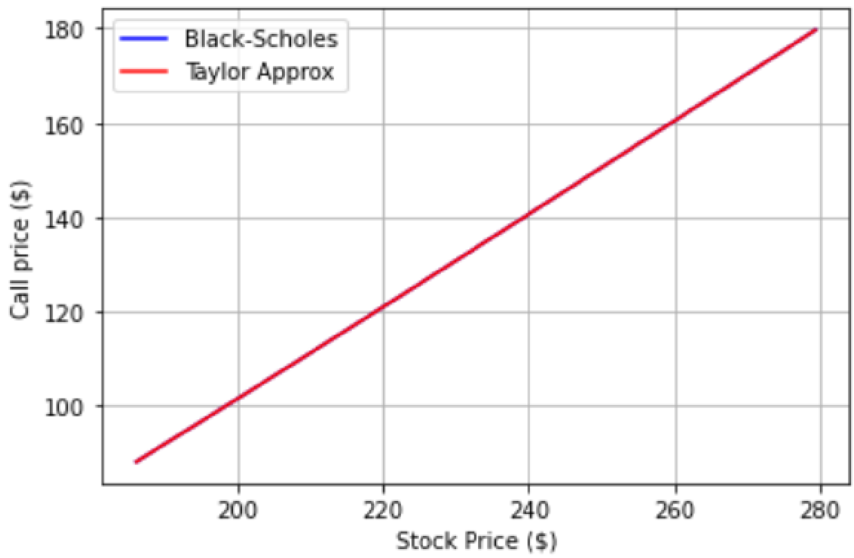
In this part the idea is same but the difference is that we take the derivative of option with respect to stock price. Additionally, in the calculation of Taylor series instead of time difference Δt we use stock price difference ΔS.

$$f(C_t) = f(C_{t-1}) + \frac{f'(C_t)}{1!}(\Delta S)^1 + \frac{f''(C_t)}{2!}(\Delta S)^2 + \frac{f'''(C_t)}{3!}(\Delta S)^3 + \frac{f''''(C_t)}{4!}(\Delta S)^4 + \frac{f'''''(C_t)}{5!}(\Delta S)^5 + \frac{f''''''(C_t)}{6!}(\Delta S)^6$$

$$\text{Delta } (\Delta) = f'(C_t) = \frac{\partial C_t}{\partial S}$$

$$\text{Gamma } (\Gamma) = f''(C_t) = \frac{\partial\, delta(\Delta)}{\partial S}$$

Graph 3:




```
The Taylor approximation at S = 232.77999877929688 is 133.478947846209
Black and Scholes at S = 232.77999877929688 is 133.47894784601726
232.77999877929688
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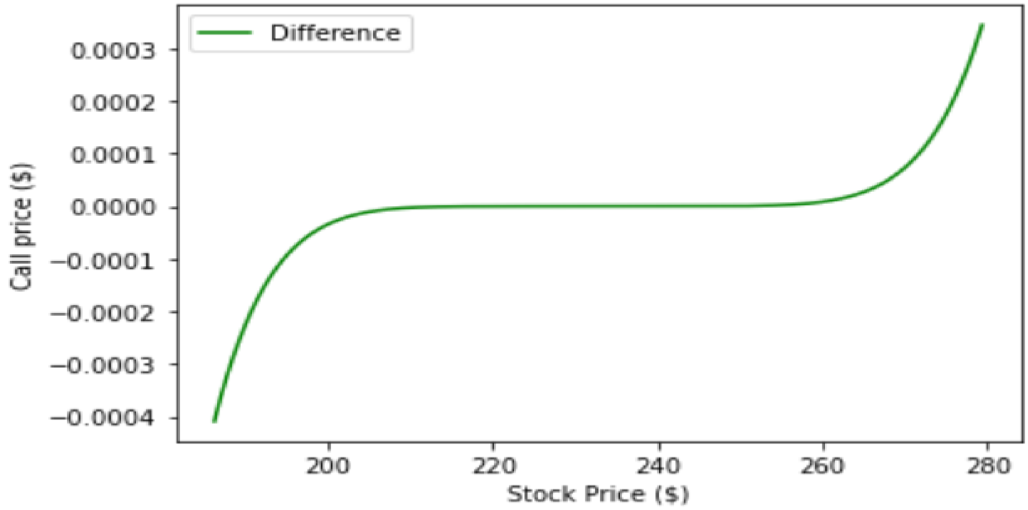
In this part also we can see that the outputs of both methods, when the stock price

S = 232.8 , are same 133.48.

Figure 4:



Even though with in the Figure 3 we can not see the difference between methods but in the Figure 4 they are visible but it is very small values like 0.0003.

VI Summary and Recommendation

According to our findings, using the Taylor series expansion approach with Greeks offers a snapshot of the option's value, demonstrating how it will respond to tiny changes in the underlying asset and time. It should be noted, however, that this approach is not ideal for large-scale forecasts since its accuracy decreases as one advances away from the site of expansion. In such instances, it is best to employ the entire model rather than depending simply on the Taylor series approximation. As a result, we recommend that financial analysts and practitioners use caution when using the Taylor series approach for option pricing, especially when making long-term projections or dealing with complicated options with varied features.

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GitHub: https://github.com/FinNijatTech/Taylor-Series-Approximation