Project2 Report

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Overview:

In this project, I plot the implied volatility surface and try to find the dependence of volatility with expiry time T and strike price by creating the PDE equation. I create the following equation based on the historical data and some reliable assumptions.

$$\frac{\partial IV}{\partial t} = c1 \times IV \times e^{-(T-t)} \times \left(\frac{k}{s}\right)^2, c < 0$$

$$\frac{\partial IV}{\partial k} = ak + b \qquad a, b \ \ are\ const\ for\ a\ fixed\ time\ T$$

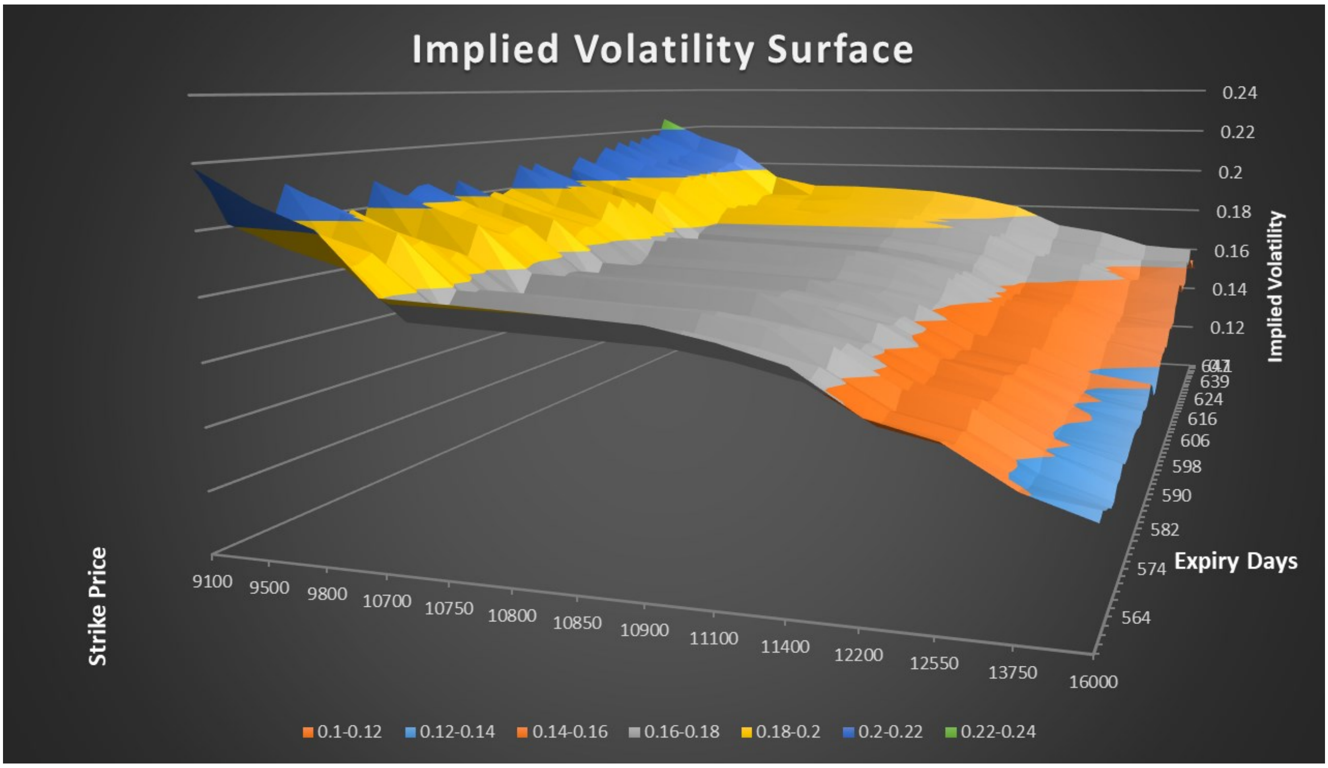
$$\frac{\partial IV}{\partial k^2} = c2 \qquad c\ is\ \ const\ for\ a\ fixed\ time\ T$$

$$\frac{\partial IV}{\partial k} = 0\ (\frac{K}{s} \approx 1).$$

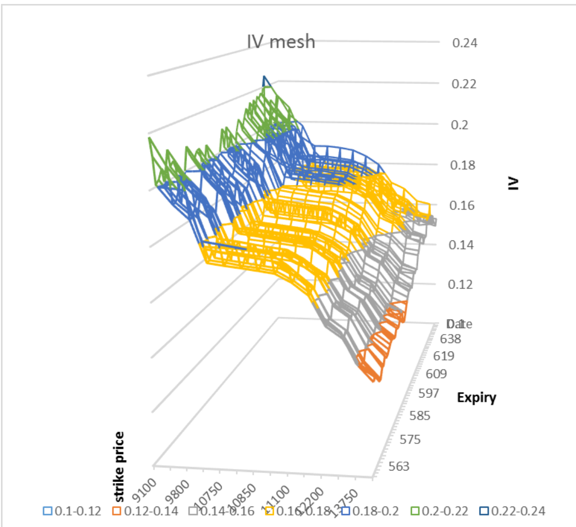
In these formulas, IV means the implied volatility , (T-t) means the expiry time, k is the strike price, s is the stock price.

The Implied Volatility surface

First, I use Newton-iteration method to calculate the implied volatility from the given option price by B-S formula and plot that as following, the stock price is move around 10900. Here we can observe some import features to help us to build the PDE equation.

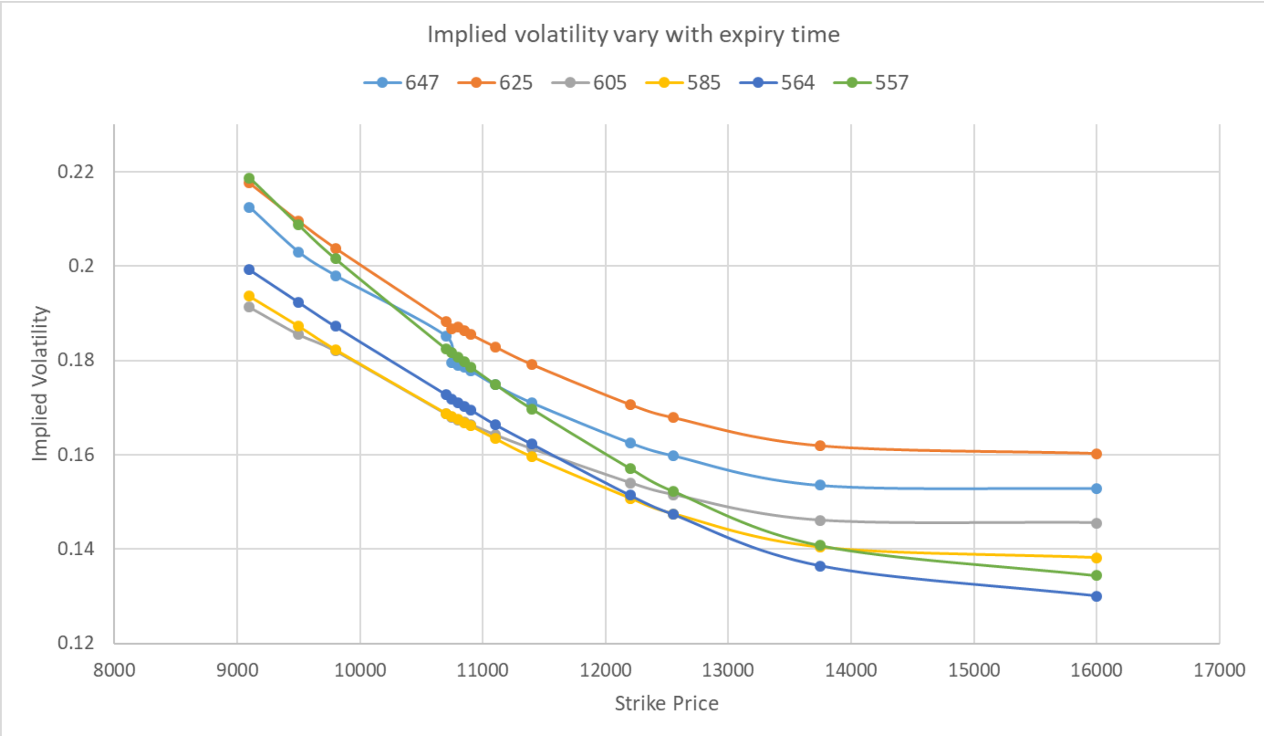


1. The whole volatility surface is a volatility skew but not is a volatility smile, it means for the deep in-the-money call option (k=9100), the implied volatility is highly greater than the deep out-the-money call option.
2. In the surface, the implied volatility around the at-the-money option (10850, 10900, 11100) is very flat, we may can assume one reasonable restriction condition $\frac{\partial IV}{\partial k} = 0$ $(\frac{K}{s} \approx 1)$, which is very significant in the mesh picture.

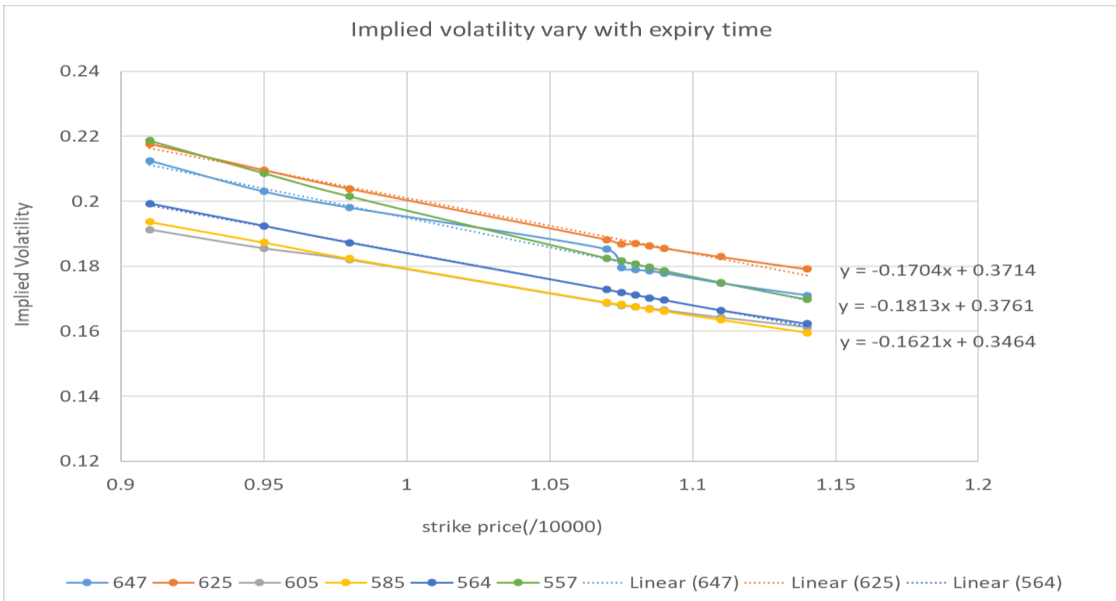


3. For deeply out-the-money (K=16000), the implied volatility is drop swiftly with expiry time.
4. The skew sample looks stable and does not change a lot with expiry time.

Now plot volatility as a function of strike for several given times to expiry. And we can analyse

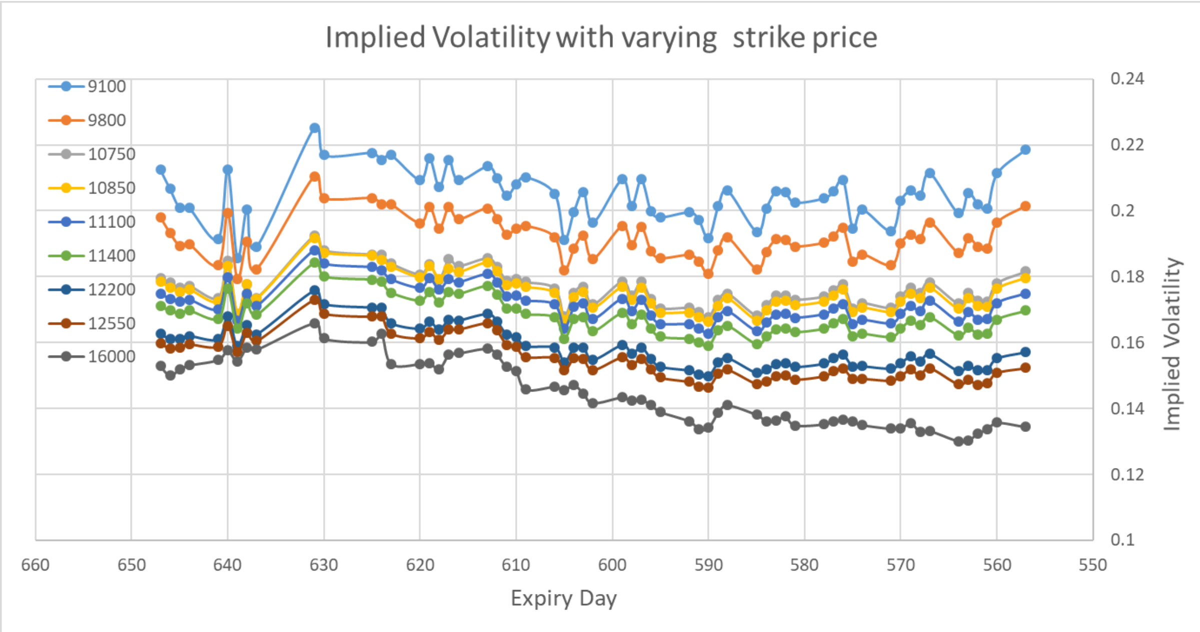


For the whole observation time, from the expiry time 647 to expiry time 557, the whole curve is almost keep the same shape. If we ignore the deeply out-of-the-money call option for which the strike price (k=16000,13750,12550,11400), the in-the-money options may have a linear relation with the at-the-money options, which can be seen clearly in the next picture after moving the deeply out-the-money option.



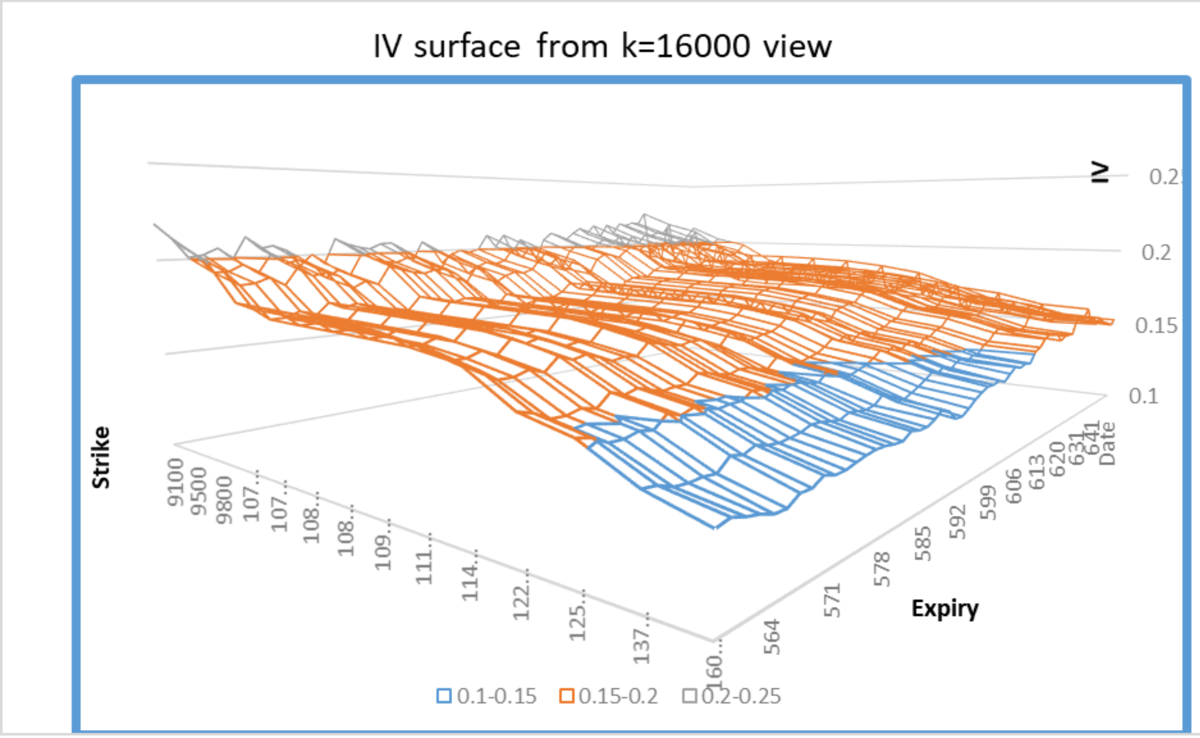
For long time scale from expiry time 647 to expiry time 557, there is highly significant linear ship between the deeply in-the-money call option implied volatility and the at-the-money call option implied volatility. So we may can get the deeply in-the-money call option Implied volatility by linear interpolant after we solving the at the money option.

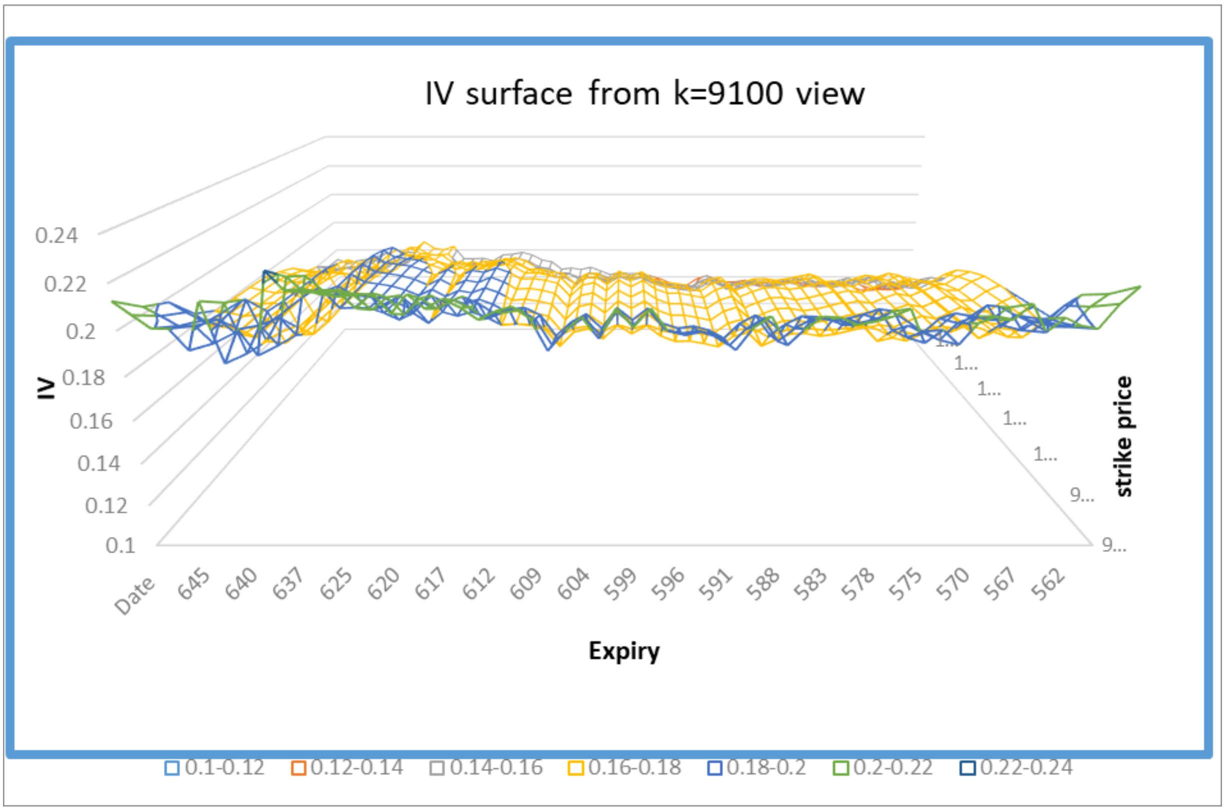
Then I draw the Implied time series with different strike price



We can see all time series experienced a fluctuate on the expiry time 640 to 630, and the lower strike price may have the more fluctuate, which is because the share price drop down soon in the period and cause a huge influence on call option's the price. Apart from this period, All option's IV decreased with time, Except from the deeply in-the-money call option (K=9100,9800). Actually, the two features reflect the sensitivity of the different call options with the change of share price and the time value of call option, which will be illustrated in the PDE model chapter.

By the way, the following picture shows the IV surface from different point of views, which is useful in our real algorithm to simulate the IV motion.

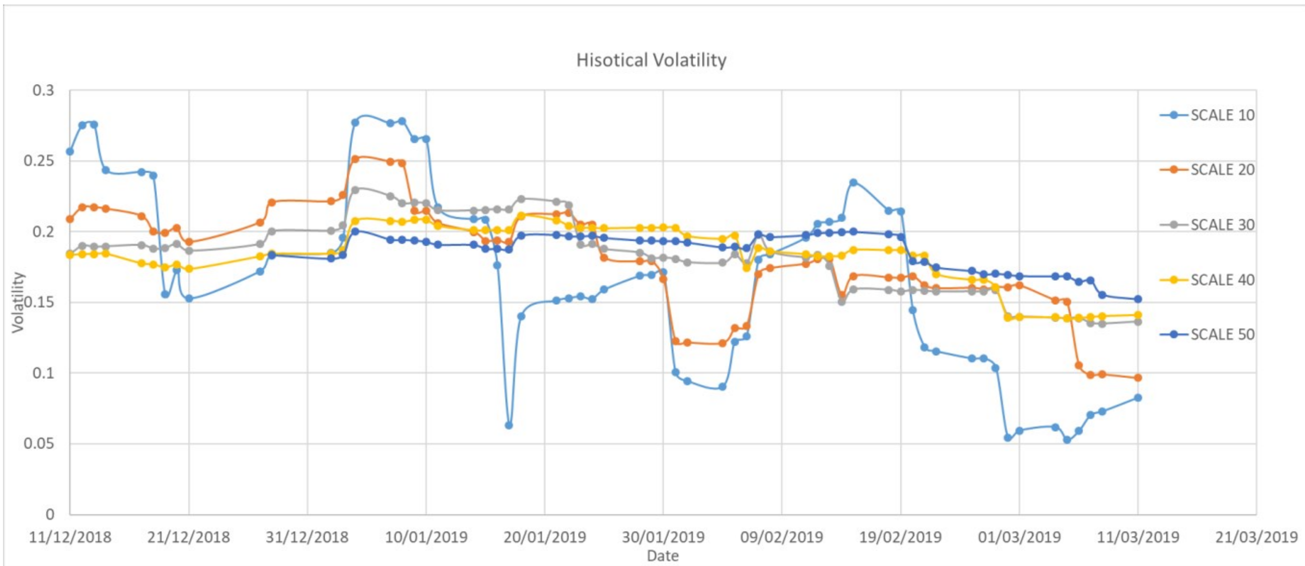




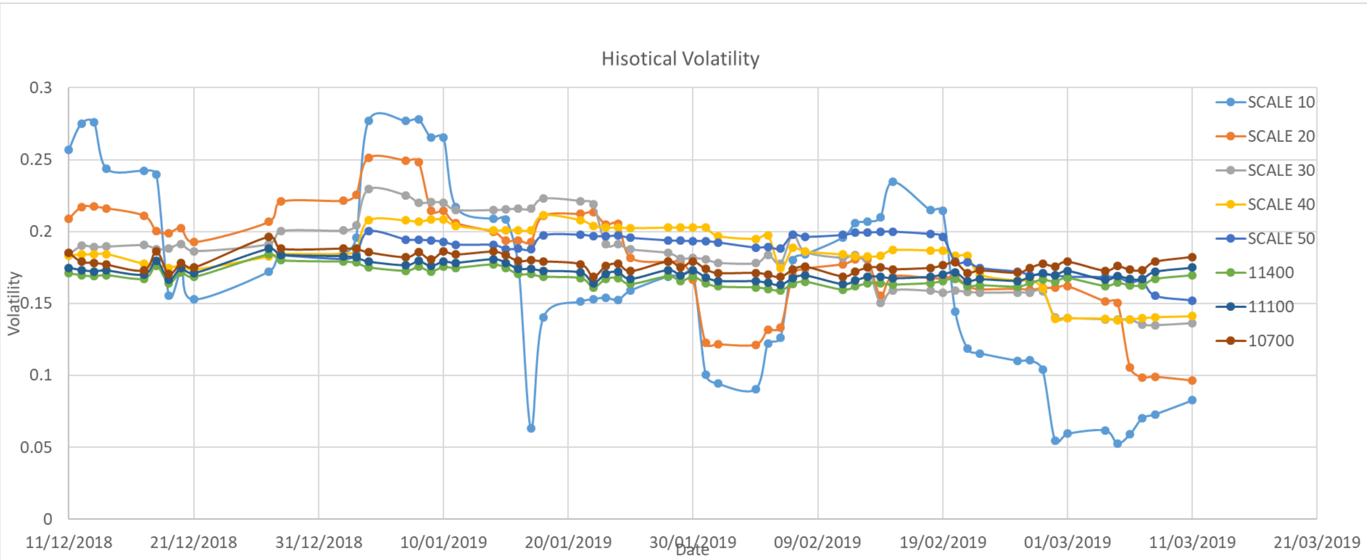
Historical Volatility

Here I calculate the long-term historical volatility from DAX, It is 0.171399, then draw the historical volatility using different scale, the scale 10 means we just calculate the 10-day historical volatility and fix the time scale size and move it in whole time series.

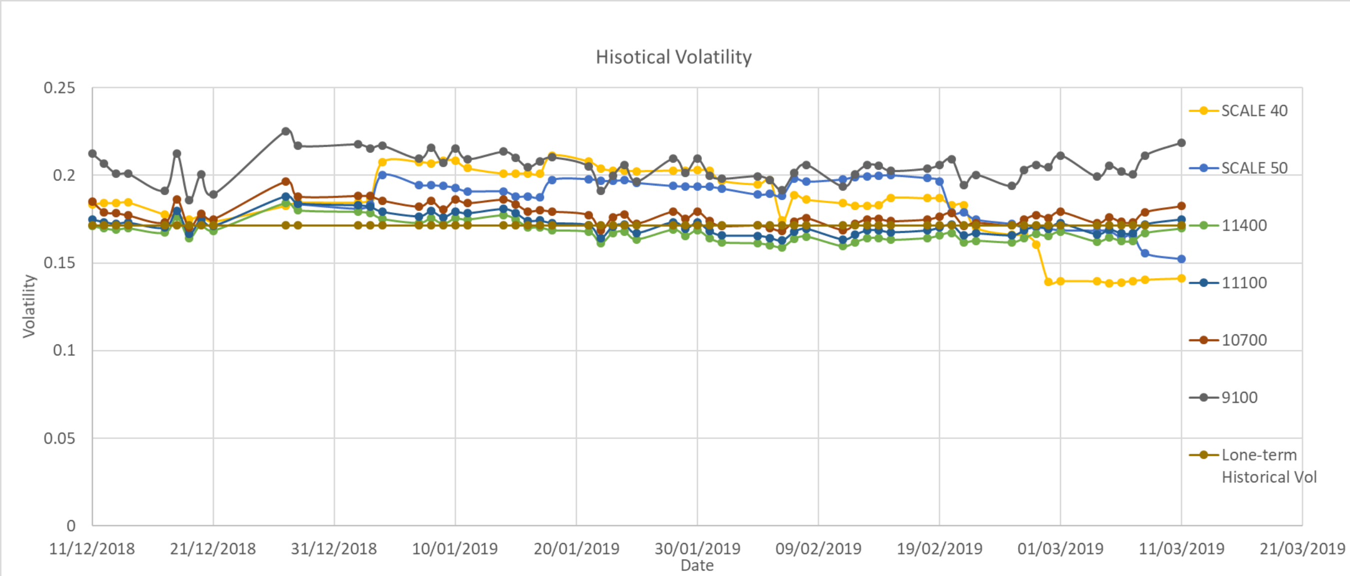
We can observe that with the time scale increasing, the historical volatility will be more smooth and the jump and fall situation will be also missed, which is very significant for comparing scale 10 and scale 30 historical volatility. And we can see all time series share the same trend.



Then we will add some at-the-money option implied volatility to comparison, we can see the historical volatility is far away from the implied volatility even for a large scale. Here we add the call option with strike price 11400, 11100, 10700. All of them are at-the-money call option at different time, which can be show in the next page sheet.



But for long-term historical volatility, it is obvious that the IV fluctuate with it and have the mean-reversion trend. The reason is the implied volatility can be seen as the future volatility in a long time (almost 2 years in this example),which make them hard to be influence by the recently historical volatility.



The yellow cell in the next sheet is the at-the-money call option at that expiry time. Because the stock price is stochastic the at-the-money option can be out-the-money option. We can see from that from the tail of the sheet(for expriy days in reverse order), there is a huge drop in the stock market, many in-the-money options turn to be out-of-the-money option, so the price of option changed dramatically, that is why on 21/12/2018 , we can observe the all options have different-level fluctuate.

Date	9100	9500	9800	10700	10750	10800	10850	10900	11100	11400	12200	12550	13750	16000
592	0.814203	0.849992	0.876834	0.957359	0.961833	0.966306	0.97078	0.975254	0.993148	1.01999	1.09157	1.12288	1.23025	1.43156
IV	0.199557	0.192077	0.186606	0.171354	0.170614	0.169807	0.169115	0.168355	0.165523	0.161207	0.151557	0.14811	0.139927	0.13619
595	0.813905	0.849681	0.876514	0.95701	0.961482	0.965954	0.970426	0.974898	0.992786	1.01962	1.09117	1.12247	1.2298	1.43104
IV	0.198041	0.190991	0.185616	0.17115	0.170417	0.1698	0.169113	0.168359	0.16573	0.161808	0.152718	0.149434	0.141754	0.139031
596	0.814456	0.850256	0.877107	0.957657	0.962132	0.966607	0.971082	0.975557	0.993458	1.02031	1.09191	1.12323	1.23063	1.43201
IV	0.199885	0.192762	0.187846	0.173926	0.173163	0.172517	0.171804	0.171023	0.168303	0.16428	0.155136	0.151932	0.144018	0.141194
597	0.813833	0.849606	0.876435	0.956924	0.961396	0.965867	0.970339	0.974811	0.992697	1.01953	1.09107	1.12237	1.22969	1.43091
IV	0.209511	0.201082	0.195118	0.179258	0.178472	0.177622	0.176709	0.176095	0.17298	0.168621	0.158624	0.155155	0.146636	0.142745
598	0.811136	0.846791	0.873531	0.953754	0.95821	0.962667	0.967124	0.971581	0.989408	1.01615	1.08746	1.11865	1.22562	1.42617
IV	0.201442	0.194542	0.189586	0.175093	0.174386	0.173614	0.17296	0.172238	0.169581	0.165599	0.156598	0.153243	0.145618	0.142385
599	0.811753	0.847434	0.874195	0.954479	0.958939	0.963399	0.967859	0.972319	0.99016	1.01692	1.08828	1.11951	1.22655	1.42726
IV	0.209495	0.201067	0.195296	0.179279	0.178532	0.177721	0.176847	0.176273	0.173317	0.169029	0.159241	0.15557	0.147544	0.143436
602	0.80661	0.842065	0.868656	0.948431	0.952863	0.957295	0.961727	0.966159	0.983886	1.01048	1.08139	1.11241	1.21878	1.41821
IV	0.196533	0.190065	0.18548	0.172519	0.171734	0.171069	0.170519	0.169901	0.167119	0.163559	0.15484	0.151744	0.144362	0.141693
603	0.817597	0.853535	0.880489	0.96135	0.965842	0.970335	0.974827	0.979319	0.997288	1.02424	1.09612	1.12756	1.23538	1.43753
IV	0.205756	0.198149	0.192424	0.177614	0.176949	0.176218	0.175422	0.174742	0.171892	0.167814	0.158438	0.155044	0.14769	0.144559
604	0.821927	0.858056	0.885152	0.966442	0.970958	0.975474	0.97999	0.984506	1.00257	1.02967	1.10192	1.13354	1.24192	1.44515
IV	0.199714	0.193175	0.188473	0.176047	0.175125	0.174502	0.173813	0.173235	0.170782	0.167082	0.15844	0.155396	0.148829	0.147209
605	0.820551	0.856619	0.88367	0.964824	0.969332	0.973841	0.978349	0.982858	1.00089	1.02794	1.10008	1.13164	1.23984	1.44273
IV	0.191279	0.185483	0.181988	0.16864	0.167971	0.167416	0.16697	0.166453	0.164207	0.161258	0.154054	0.151533	0.1461	0.145575
606	0.817155	0.853074	0.880013	0.96083	0.96532	0.96981	0.9743	0.97879	0.996749	1.02369	1.09553	1.12696	1.23471	1.43676
IV	0.205202	0.19744	0.19201	0.177368	0.176529	0.175809	0.175023	0.174354	0.171546	0.167707	0.158525	0.155433	0.14844	0.146678
609	0.812098	0.847795	0.874567	0.954885	0.959347	0.963809	0.968271	0.972733	0.990581	1.01735	1.08875	1.11998	1.22707	1.42787
IV	0.210224	0.201549	0.195413	0.179343	0.178404	0.177586	0.176887	0.176124	0.172794	0.168669	0.158908	0.155661	0.148122	0.145775
610	0.833439	0.870073	0.897549	0.979977	0.984557	0.989136	0.993715	0.998295	1.01661	1.04409	1.11736	1.14941	1.25932	1.46539
IV	0.208144	0.200249	0.194666	0.180094	0.179297	0.178618	0.177872	0.177059	0.174217	0.170409	0.161673	0.15866	0.152378	0.151323
611	0.832476	0.869069	0.896513	0.978846	0.98342	0.987994	0.992568	0.997142	1.01544	1.04288	1.11607	1.14809	1.25786	1.46369
IV	0.20464	0.197681	0.192788	0.179484	0.17889	0.178049	0.177503	0.17689	0.174123	0.17043	0.162415	0.159401	0.15348	0.152612
612	0.835492	0.872217	0.89976	0.982391	0.986982	0.991573	0.996163	1.00075	1.01912	1.04666	1.12011	1.15224	1.26242	1.469
IV	0.209864	0.202633	0.197511	0.18357	0.182904	0.182353	0.181555	0.180873	0.178382	0.174701	0.16644	0.163706	0.157542	0.156364
613	0.838253	0.875099	0.902734	0.985638	0.990244	0.99485	0.999456	1.00406	1.02248	1.05012	1.12381	1.15605	1.26659	1.47385
IV	0.21348	0.205974	0.200773	0.186139	0.185594	0.184803	0.184308	0.183388	0.180855	0.177229	0.168695	0.165955	0.159366	0.158247
616	0.835824	0.872563	0.900118	0.982782	0.987374	0.991967	0.996559	1.00115	1.01952	1.04708	1.12056	1.1527	1.26292	1.46958
IV	0.209221	0.202192	0.197457	0.18412	0.183264	0.182165	0.181364	0.180856	0.178346	0.17482	0.166733	0.164023	0.15809	0.156816
617	0.833212	0.869837	0.897305	0.979711	0.984289	0.988867	0.993445	0.998023	1.01634	1.0438	1.11705	1.1491	1.25897	1.46499
IV	0.215439	0.206766	0.201147	0.186114	0.185476	0.183338	0.18258	0.181937	0.179255	0.175491	0.166964	0.164029	0.157822	0.156383
618	0.835374	0.872094	0.899634	0.982253	0.986843	0.991433	0.996023	1.00061	1.01897	1.04651	1.11995	1.15208	1.26224	1.46879
IV	0.207193	0.199516	0.194576	0.180457	0.179083	0.180168	0.179382	0.17871	0.176076	0.172254	0.163961	0.160942	0.154303	0.152037
619	0.842282	0.879306	0.907073	0.990376	0.995004	0.999632	1.00426	1.00889	1.0274	1.05517	1.12921	1.16161	1.27268	1.48094
IV	0.215916	0.207187	0.201282	0.185547	0.183847	0.184072	0.183148	0.182519	0.179531	0.175481	0.166435	0.163271	0.156108	0.153842
620	0.846684	0.883901	0.911814	0.995552	1.0002	1.00486	1.00951	1.01416	1.03277	1.06068	1.13512	1.16768	1.27933	1.48868
IV	0.209427	0.201636	0.196254	0.182131	0.180725	0.1807	0.179885	0.179184	0.176607	0.172772	0.164251	0.161384	0.154716	0.153566
623	0.845121	0.882269	0.91013	0.993714	0.998357	1.003	1.00764	1.01229	1.03086	1.05872	1.13302	1.16552	1.27697	1.48593
IV	0.216889	0.207914	0.201881	0.185592	0.184025	0.183657	0.183045	0.182188	0.179182	0.175193	0.165897	0.162778	0.155667	0.153536
624	0.873601	0.912001	0.940801	1.0272	1.032	1.0368	1.0416	1.0464	1.0656	1.0944	1.1712	1.2048	1.32	1.536
IV	0.21539	0.207558	0.20205	0.188108	0.186566	0.185879	0.185124	0.184485	0.18199	0.178435	0.170509	0.168025	0.162754	0.162634
625	0.860098	0.897904	0.926259	1.01132	1.01605	1.02078	1.0255	1.03023	1.04913	1.07749	1.1531	1.18618	1.2996	1.51226
IV	0.217617	0.209548	0.203819	0.188212	0.186733	0.187005	0.186304	0.185536	0.1829	0.179139	0.170642	0.167875	0.161905	0.160273
630	0.861827	0.89971	0.928122	1.01336	1.01809	1.02283	1.02756	1.0323	1.05124	1.07965	1.15542	1.18856	1.30221	1.5153
IV	0.216916	0.209291	0.203792	0.188047	0.187993	0.188052	0.187322	0.186527	0.183966	0.180065	0.171683	0.168851	0.162851	0.161268
631	0.876558	0.915088	0.943986	1.03068	1.03549	1.04031	1.04513	1.04994	1.06921	1.09811	1.17517	1.20888	1.32447	1.5412
IV	0.225244	0.21627	0.210459	0.196382	0.192404	0.192405	0.191789	0.190741	0.188106	0.18423	0.175761	0.173022	0.167289	0.165952
637	0.85576	0.893376	0.921588	1.00622	1.01093	1.01563	1.02033	1.02503	1.04384	1.07205	1.14728	1.1802	1.29304	1.50463
IV	0.189088	0.185451	0.182305	0.174941	0.17358	0.173764	0.173341	0.172848	0.171081	0.168425	0.162554	0.160507	0.156623	0.157942
638	0.857593	0.895289	0.923561	1.00838	1.01309	1.0178	1.02251	1.02723	1.04607	1.07435	1.14974	1.18272	1.29581	1.50786
IV	0.200481	0.194572	0.190664	0.178101	0.177777	0.1781	0.177639	0.176931	0.174857	0.171875	0.165254	0.162989	0.158485	0.158545
639	0.845237	0.88239	0.910255	0.99385	0.998494	1.00314	1.00778	1.01243	1.031	1.05887	1.13318	1.16568	1.27714	1.48613
IV	0.185678	0.181689	0.179303	0.170385	0.168881	0.169265	0.168866	0.168398	0.166534	0.164298	0.158937	0.157155	0.153695	0.154366
640	0.84723	0.88447	0.912401	0.996193	1.00085	1.0055	1.01016	1.01481	1.03343	1.06136	1.13585	1.16843	1.28015	1.48963
IV	0.212497	0.204813	0.199291	0.186377	0.184785	0.184024	0.183378	0.182668	0.179902	0.1763	0.167996	0.165187	0.158985	0.157663
641	0.844767	0.8819	0.909749	0.993298	0.997939	1.00258	1.00722	1.01186	1.03043	1.05828	1.13254	1.16504	1.27643	1.4853
IV	0.191348	0.186312	0.183424	0.173058	0.173509	0.173001	0.172426	0.171959	0.170118	0.167259	0.161017	0.158855	0.154413	0.154811
644	0.837492	0.874305	0.901915	0.984744	0.989345	0.993947	0.998549	1.00315	1.02156	1.04917	1.12279	1.155	1.26544	1.47251
IV	0.200793	0.194338	0.189786	0.177071	0.177282	0.176716	0.176086	0.175391	0.173015	0.169752	0.162044	0.159528	0.153728	0.153148
645	0.832975	0.869589	0.89705	0.979432	0.984009	0.988585	0.993162	0.997739	1.01605	1.04351	1.11674	1.14877	1.25862	1.46457
IV	0.200859	0.194296	0.189366	0.178398	0.176737	0.175903	0.175357	0.174921	0.172355	0.168898	0.161239	0.15849	0.152669	0.151957
646	0.832614	0.869213	0.896662	0.979008	0.983583	0.988158	0.992732	0.997307	1.01561	1.04306	1.11625	1.14828	1.25807	1.46394
IV	0.206614	0.198828	0.19321	0.178993	0.178219	0.177386	0.176665	0.176057	0.173335	0.169746	0.161167	0.158276	0.151764	0.150053
647	0.844116	0.88122	0.909048	0.992532	0.99717	1.00181	1.00645	1.01108	1.02964	1.05746	1.13167	1.16414	1.27545	1.48416
IV	0.212457	0.202959	0.198005	0.185199	0.179601	0.178911	0.178509	0.177866	0.174829	0.171037	0.162584	0.159872	0.153603	0.152928

Date	9100	9500	9800	10700	10750	10800	10850	10900	11100	11400	12200	12550	13750	16000
557	0.788324	0.822975	0.848964	0.92693	0.931262	0.935593	0.939924	0.944256	0.961582	0.98757	1.05687	1.08719	1.19115	1.38606
IV	0.218683	0.208663	0.201506	0.182437	0.181574	0.180649	0.179663	0.178617	0.174946	0.169712	0.157127	0.15231	0.140802	0.134441
560	0.794216	0.829127	0.85531	0.933858	0.938222	0.942586	0.94695	0.951314	0.968769	0.994952	1.06477	1.09532	1.20005	1.39642
IV	0.211341	0.203	0.196422	0.178982	0.178029	0.177206	0.176319	0.175369	0.172039	0.167043	0.155213	0.150844	0.140042	0.135762
561	0.790081	0.82481	0.850857	0.928997	0.933338	0.937679	0.94202	0.946361	0.963726	0.989772	1.05923	1.08962	1.1938	1.38915
IV	0.200783	0.193856	0.188534	0.173191	0.172376	0.171689	0.170934	0.170112	0.16709	0.162593	0.151753	0.147571	0.137575	0.133769
562	0.78532	0.81984	0.845729	0.923398	0.927713	0.932028	0.936343	0.940658	0.957918	0.983808	1.05285	1.08305	1.18661	1.38078
IV	0.20205	0.193925	0.18911	0.173397	0.172678	0.1717	0.171045	0.170131	0.166957	0.16255	0.15154	0.147257	0.136967	0.132448
563	0.783083	0.817504	0.84332	0.920768	0.92507	0.929373	0.933675	0.937978	0.955189	0.981005	1.04985	1.07997	1.18323	1.37685
IV	0.20549	0.197458	0.191778	0.175999	0.175103	0.174341	0.173515	0.172626	0.16919	0.164455	0.153023	0.148641	0.13727	0.130338
564	0.784979	0.819484	0.845362	0.922998	0.927311	0.931624	0.935937	0.94025	0.957502	0.983381	1.05239	1.08258	1.1861	1.38018
IV	0.199239	0.192361	0.187209	0.172765	0.171863	0.171092	0.170256	0.169547	0.166417	0.16225	0.151391	0.147363	0.13647	0.130093
567	0.784369	0.818847	0.844705	0.92228	0.92659	0.9309	0.935209	0.939519	0.956758	0.982616	1.05157	1.08174	1.18517	1.37911
IV	0.211417	0.202643	0.196555	0.179112	0.178353	0.177531	0.176647	0.175892	0.172612	0.167772	0.156561	0.152304	0.141188	0.133255
568	0.79023	0.824965	0.851017	0.929171	0.933513	0.937855	0.942197	0.946539	0.963907	0.989958	1.05943	1.08982	1.19403	1.38941
IV	0.204667	0.19706	0.191345	0.17583	0.175176	0.174264	0.173479	0.17263	0.169515	0.16528	0.154288	0.150078	0.139683	0.132983
569	0.792177	0.826998	0.853114	0.931461	0.935814	0.940166	0.944519	0.948871	0.966282	0.992398	1.06204	1.09251	1.19697	1.39284
IV	0.206082	0.198542	0.19272	0.177459	0.17675	0.175784	0.174946	0.174233	0.171094	0.166567	0.155771	0.151865	0.141704	0.135669
570	0.788508	0.823167	0.849162	0.927146	0.931479	0.935811	0.940144	0.944476	0.961806	0.987801	1.05712	1.08745	1.19143	1.38639
IV	0.203125	0.195655	0.190232	0.17487	0.174065	0.173389	0.172457	0.171839	0.168703	0.164358	0.153838	0.149917	0.140014	0.133877
571	0.790934	0.8257	0.851775	0.929999	0.934345	0.938691	0.943036	0.947382	0.964765	0.99084	1.06037	1.09079	1.19509	1.39065
IV	0.19387	0.187903	0.183448	0.17143	0.170607	0.169911	0.169338	0.168507	0.165813	0.161779	0.152161	0.148513	0.139271	0.133912
574	0.794226	0.829137	0.85532	0.93387	0.938234	0.942598	0.946961	0.951325	0.968781	0.994964	1.06479	1.09533	1.20007	1.39644
IV	0.20027	0.191807	0.186815	0.17295	0.172044	0.171455	0.170609	0.169885	0.166873	0.16274	0.152848	0.149084	0.139591	0.135029
575	0.796619	0.831635	0.857897	0.936684	0.941061	0.945438	0.949815	0.954192	0.9717	0.997962	1.06799	1.09863	1.20368	1.40065
IV	0.194652	0.189025	0.184507	0.171271	0.170331	0.169896	0.169201	0.168438	0.165627	0.161928	0.152598	0.148998	0.140298	0.135977
576	0.798108	0.833189	0.859501	0.938434	0.94282	0.947205	0.95159	0.955975	0.973516	0.999827	1.06999	1.10069	1.20593	1.40327
IV	0.209297	0.200971	0.194732	0.178446	0.177605	0.176889	0.176109	0.175264	0.171794	0.167221	0.156366	0.152152	0.14199	0.136518
577	0.804654	0.840023	0.86655	0.946132	0.950553	0.954974	0.959395	0.963816	0.981501	1.00803	1.07877	1.10972	1.21582	1.41478
IV	0.205932	0.197657	0.192175	0.176565	0.175797	0.174964	0.174251	0.173472	0.170238	0.16577	0.15527	0.151271	0.141555	0.136191
578	0.805367	0.840767	0.867318	0.94697	0.951395	0.95582	0.960245	0.96467	0.98237	1.00892	1.07972	1.1107	1.2169	1.41603
IV	0.203827	0.195703	0.190303	0.174788	0.174019	0.173184	0.172469	0.171687	0.168613	0.164267	0.153662	0.149853	0.14047	0.135279
581	0.805324	0.840723	0.867272	0.946919	0.951344	0.955769	0.960194	0.964619	0.982318	1.00887	1.07967	1.11064	1.21684	1.41595
IV	0.202509	0.194765	0.18919	0.173846	0.173084	0.172256	0.171362	0.170587	0.167541	0.163226	0.152652	0.148828	0.13949	0.1349
582	0.820575	0.856644	0.883696	0.964851	0.96936	0.973869	0.978377	0.982886	1.00092	1.02797	1.10011	1.13167	1.23988	1.44277
IV	0.20554	0.197363	0.191108	0.175243	0.174352	0.173579	0.172737	0.171828	0.16877	0.164299	0.153829	0.150181	0.141356	0.137565
583	0.814885	0.850704	0.877568	0.958161	0.962639	0.967116	0.971594	0.976071	0.993981	1.02084	1.09248	1.12382	1.23128	1.43276
IV	0.205958	0.197497	0.191365	0.175016	0.17424	0.173397	0.172487	0.171694	0.168564	0.164098	0.153443	0.149696	0.140558	0.136355
584	0.817898	0.85385	0.880813	0.961704	0.966198	0.970692	0.975186	0.97968	0.997656	1.02462	1.09652	1.12798	1.23584	1.43806
IV	0.200778	0.193253	0.187586	0.172195	0.171377	0.170676	0.169904	0.169247	0.166091	0.16181	0.151821	0.148286	0.139794	0.136229
585	0.826177	0.862492	0.889729	0.971439	0.975978	0.980518	0.985057	0.989597	1.00775	1.03499	1.10762	1.1394	1.24834	1.45262
IV	0.193661	0.187311	0.18234	0.168802	0.168206	0.167536	0.166795	0.166162	0.163444	0.15956	0.150783	0.147478	0.140381	0.138148
588	0.834343	0.871018	0.898524	0.981041	0.985625	0.99021	0.994794	0.999378	1.01772	1.04522	1.11857	1.15066	1.26068	1.46698
IV	0.206079	0.197613	0.191836	0.17568	0.174884	0.174203	0.173451	0.172628	0.169536	0.165166	0.155357	0.15181	0.143726	0.141034
589	0.82562	0.861911	0.889129	0.970784	0.97532	0.979857	0.984393	0.988929	1.00707	1.03429	1.10688	1.13863	1.2475	1.45164
IV	0.201333	0.193124	0.187938	0.173663	0.172866	0.172001	0.171249	0.170427	0.167706	0.163708	0.154008	0.150484	0.142209	0.138764
590	0.803552	0.838873	0.865364	0.944836	0.949251	0.953666	0.958081	0.962496	0.980157	1.00665	1.07729	1.1082	1.21416	1.41284
IV	0.191609	0.185411	0.180777	0.168392	0.167705	0.166951	0.166314	0.165608	0.162818	0.159014	0.149815	0.146415	0.13808	0.134165
591	0.800494	0.835681	0.86207	0.94124	0.945639	0.950037	0.954435	0.958833	0.976427	1.00282	1.07319	1.10398	1.20954	1.40746
IV	0.197152	0.189831	0.184644	0.170028	0.16939	0.168499	0.167728	0.167075	0.164323	0.160175	0.150334	0.146719	0.138182	0.133776
592	0.814203	0.849992	0.876834	0.957359	0.961833	0.966306	0.97078	0.975254	0.993148	1.01999	1.09157	1.12288	1.23025	1.43156
IV	0.199557	0.192077	0.186606	0.171354	0.170614	0.169807	0.169115	0.168355	0.165523	0.161207	0.151557	0.14811	0.139927	0.13619
595	0.813905	0.849681	0.876514	0.95701	0.961482	0.965954	0.970426	0.974898	0.992786	1.01962	1.09117	1.12247	1.2298	1.43104
IV	0.198041	0.190991	0.185616	0.17115	0.170417	0.1698	0.169113	0.168359	0.16573	0.161808	0.152718	0.149434	0.141754	0.139031
596	0.814456	0.850256	0.877107	0.957657	0.962132	0.966607	0.971082	0.975557	0.993458	1.02031	1.09191	1.12323	1.23063	1.43201
IV	0.199885	0.192762	0.187846	0.173926	0.173163	0.172517	0.171804	0.171023	0.168303	0.16428	0.155136	0.151932	0.144018	0.141194
597	0.813833	0.849606	0.876435	0.956924	0.961396	0.965867	0.970339	0.974811	0.992697	1.01953	1.09107	1.12237	1.22969	1.43091
IV	0.209511	0.201082	0.195118	0.179258	0.178472	0.177622	0.176709	0.176095	0.17298	0.168621	0.158624	0.155155	0.146636	0.142745
598	0.811136	0.846791	0.873531	0.953754	0.95821	0.962667	0.967124	0.971581	0.989408	1.01615	1.08746	1.11865	1.22562	1.42617
IV	0.201442	0.194542	0.189586	0.175093	0.174386	0.173614	0.17296	0.172238	0.169581	0.165599	0.156598	0.153243	0.145618	0.142385
599	0.811753	0.847434	0.874195	0.954479	0.958939	0.963399	0.967859	0.972319	0.99016	1.01692	1.08828	1.11951	1.22655	1.42726
IV	0.209495	0.201067	0.195296	0.179279	0.178532	0.177721	0.176847	0.176273	0.173317	0.169029	0.159241	0.15557	0.147544	0.143436
602	0.80661	0.842065	0.868656	0.948431	0.952863	0.957295	0.961727	0.966159	0.983886	1.01048	1.08139	1.11241	1.21878	1.41821
IV	0.196533	0.190065	0.18548	0.172519	0.171734	0.171069	0.170519	0.169901	0.167119	0.163559	0.15484	0.151744	0.144362	0.141693
603	0.817597	0.853535	0.880489	0.96135	0.965842	0.970335	0.974827	0.979319	0.997288	1.02424	1.09612	1.12756	1.23538	1.43753
IV	0.205756	0.198149	0.192424	0.177614	0.176949	0.176218	0.175422	0.174742	0.171892	0.167814	0.158438	0.155044	0.14769	0.144559
604	0.821927	0.858056	0.885152	0.966442	0.970958	0.975474	0.97999	0.984506	1.00257	1.02967	1.10192	1.13354	1.24192	1.44515
IV	0.199714	0.193175	0.188473	0.176047	0.175125	0.174502	0.173813	0.173235	0.170782	0.167082	0.15844	0.155396	0.148829	0.147209
605	0.820551	0.856619	0.88367	0.964824	0.969332	0.973841	0.978349	0.982858	1.00089	1.02794	1.10008	1.13164	1.23984	1.44273
IV	0.191279	0.185483	0.181988	0.16864	0.167971	0.167416	0.16697	0.166453	0.164207	0.161258	0.154054	0.151533	0.1461	0.145575
606	0.817155	0.853074	0.880013	0.96083	0.96532	0.96981	0.9743	0.97879	0.996749	1.02369	1.09553	1.12696	1.23471	1.43676
IV	0.205202	0.19744	0.19201	0.177368	0.176529	0.175809	0.175023	0.174354	0.171546	0.167707	0.158525	0.155433	0.14844	0.146678
609	0.812098	0.847795	0.874567	0.954885	0.959347	0.963809	0.968271	0.972733	0.990581	1.01735	1.08875	1.11998	1.22707	1.42787
IV	0.210224	0.201549	0.195413	0.179343	0.178404	0.177586	0.176887	0.176124	0.172794	0.168669	0.158908	0.155661	0.148122	0.145775
610	0.833439	0.870073	0.897549	0.979977	0.984557	0.989136	0.993715	0.998295	1.01661	1.04409	1.11736	1.14941	1.25932	1.46539
IV	0.208144	0.200249	0.194666	0.180094	0.179297	0.178618	0.177872	0.177059	0.174217	0.170409	0.161673	0.15866	0.152378	0.151323
611	0.832476	0.869069	0.896513	0.978846	0.98342	0.987994	0.992568	0.997142	1.01544	1.04288	1.11607	1.14809	1.25786	1.46369
IV	0.20464	0.197681	0.192788	0.179484	0.17889	0.178049	0.177503	0.17689	0.174123	0.17043	0.162415	0.159401	0.15348	0.152612
612	0.835492	0.872217	0.89976	0.982391	0.986982	0.991573	0.996163	1.00075	1.01912	1.04666	1.12011	1.15224	1.26242	1.469
IV	0.209864	0.202633	0.197511	0.18357	0.182904	0.182353	0.181555	0.180873	0.178382	0.174701	0.16644	0.163706	0.157542	0.156364
613	0.838253	0.875099	0.902734	0.985638	0.990244	0.99485	0.999456	1.00406	1.02248	1.05012	1.12381	1.15605	1.26659	1.47385
IV	0.21348	0.205974	0.200773	0.186139	0.185594	0.184803	0.184308	0.183388	0.180855	0.177229	0.168695	0.165955	0.159366	0.158247
616	0.835824	0.872563	0.900118	0.982782	0.987374	0.991967	0.996559	1.00115	1.01952	1.04708	1.12056	1.1527	1.26292	1.46958
IV	0.209221	0.202192	0.197457	0.18412	0.183264	0.182165	0.181364	0.180856	0.178346	0.17482	0.166733	0.164023	0.15809	0.156816
617	0.833212	0.869837	0.897305	0.979711	0.984289	0.988867	0.993445	0.998023	1.01634	1.0438	1.11705	1.1491	1.25897	1.46499
IV	0.215439	0.206766	0.201147	0.186114	0.185476	0.183338	0.18258	0.181937	0.179255	0.175491	0.166964	0.164029	0.157822	0.156383
618	0.835374	0.872094	0.899634	0.982253	0.986843	0.991433	0.996023	1.00061	1.01897	1.04651	1.11995	1.15208	1.26224	1.46879
IV	0.207193	0.199516	0.194576	0.180457	0.179083	0.180168	0.179382	0.17871	0.176076	0.172254	0.163961	0.160942	0.154303	0.152037
619	0.842282	0.879306	0.907073	0.990376	0.995004	0.999632	1.00426	1.00889	1.0274	1.05517	1.12921	1.16161	1.27268	1.48094
IV	0.215916	0.207187	0.201282	0.185547	0.183847	0.184072	0.183148	0.182519	0.179531	0.175481	0.166435	0.163271	0.156108	0.153842
620	0.846684	0.883901	0.911814	0.995552	1.0002	1.00486	1.00951	1.01416	1.03277	1.06068	1.13512	1.16768	1.27933	1.48868
IV	0.209427	0.201636	0.196254	0.182131	0.180725	0.1807	0.179885	0.179184	0.176607	0.172772	0.164251	0.161384	0.154716	0.153566
623	0.845121	0.882269	0.91013	0.993714	0.998357	1.003	1.00764	1.01229	1.03086	1.05872	1.13302	1.16552	1.27697	1.48593
IV	0.216889	0.207914	0.201881	0.185592	0.184025	0.183657	0.183045	0.182188	0.179182	0.175193	0.165897	0.162778	0.155667	0.153536
624	0.873601	0.912001	0.940801	1.0272	1.032	1.0368	1.0416	1.0464	1.0656	1.0944	1.1712	1.2048	1.32	1.536
IV	0.21539	0.207558	0.20205	0.188108	0.186566	0.185879	0.185124	0.184485	0.18199	0.178435	0.170509	0.168025	0.162754	0.162634
625	0.860098	0.897904	0.926259	1.01132	1.01605	1.02078	1.0255	1.03023	1.04913	1.07749	1.1531	1.18618	1.2996	1.51226
IV	0.217617	0.209548	0.203819	0.188212	0.186733	0.187005	0.186304	0.185536	0.1829	0.179139	0.170642	0.167875	0.161905	0.160273
630	0.861827	0.89971	0.928122	1.01336	1.01809	1.02283	1.02756	1.0323	1.05124	1.07965	1.15542	1.18856	1.30221	1.5153
IV	0.216916	0.209291	0.203792	0.188047	0.187993	0.188052	0.187322	0.186527	0.183966	0.180065	0.171683	0.168851	0.162851	0.161268
631	0.876558	0.915088	0.943986	1.03068	1.03549	1.04031	1.04513	1.04994	1.06921	1.09811	1.17517	1.20888	1.32447	1.5412
IV	0.225244	0.21627	0.210459	0.196382	0.192404	0.192405	0.191789	0.190741	0.188106	0.18423	0.175761	0.173022	0.167289	0.165952
637	0.85576	0.893376	0.921588	1.00622	1.01093	1.01563	1.02033	1.02503	1.04384	1.07205	1.14728	1.1802	1.29304	1.50463
IV	0.189088	0.185451	0.182305	0.174941	0.17358	0.173764	0.173341	0.172848	0.171081	0.168425	0.162554	0.160507	0.156623	0.157942
638	0.857593	0.895289	0.923561	1.00838	1.01309	1.0178	1.02251	1.02723	1.04607	1.07435	1.14974	1.18272	1.29581	1.50786
IV	0.200481	0.194572	0.190664	0.178101	0.177777	0.1781	0.177639	0.176931	0.174857	0.171875	0.165254	0.162989	0.158485	0.158545
639	0.845237	0.88239	0.910255	0.99385	0.998494	1.00314	1.00778	1.01243	1.031	1.05887	1.13318	1.16568	1.27714	1.48613
IV	0.185678	0.181689	0.179303	0.170385	0.168881	0.169265	0.168866	0.168398	0.166534	0.164298	0.158937	0.157155	0.153695	0.154366
640	0.84723	0.88447	0.912401	0.996193	1.00085	1.0055	1.01016	1.01481	1.03343	1.06136	1.13585	1.16843	1.28015	1.48963
IV	0.212497	0.204813	0.199291	0.186377	0.184785	0.184024	0.183378	0.182668	0.179902	0.1763	0.167996	0.165187	0.158985	0.157663
641	0.844767	0.8819	0.909749	0.993298	0.997939	1.00258	1.00722	1.01186	1.03043	1.05828	1.13254	1.16504	1.27643	1.4853
IV	0.191348	0.186312	0.183424	0.173058	0.173509	0.173001	0.172426	0.171959	0.170118	0.167259	0.161017	0.158855	0.154413	0.154811
644	0.837492	0.874305	0.901915	0.984744	0.989345	0.993947	0.998549	1.00315	1.02156	1.04917	1.12279	1.155	1.26544	1.47251
IV	0.200793	0.194338	0.189786	0.177071	0.177282	0.176716	0.176086	0.175391	0.173015	0.169752	0.162044	0.159528	0.153728	0.153148
645	0.832975	0.869589	0.89705	0.979432	0.984009	0.988585	0.993162	0.997739	1.01605	1.04351	1.11674	1.14877	1.25862	1.46457
IV	0.200859	0.194296	0.189366	0.178398	0.176737	0.175903	0.175357	0.174921	0.172355	0.168898	0.161239	0.15849	0.152669	0.151957
646	0.832614	0.869213	0.896662	0.979008	0.983583	0.988158	0.992732	0.997307	1.01561	1.04306	1.11625	1.14828	1.25807	1.46394
IV	0.206614	0.198828	0.19321	0.178993	0.178219	0.177386	0.176665	0.176057	0.173335	0.169746	0.161167	0.158276	0.151764	0.150053
647	0.844116	0.88122	0.909048	0.992532	0.99717	1.00181	1.00645	1.01108	1.02964	1.05746	1.13167	1.16414	1.27545	1.48416
IV	0.212457	0.202959	0.198005	0.185199	0.179601	0.178911	0.178509	0.177866	0.174829	0.171037	0.162584	0.159872	0.153603	0.152928

The logic of PDE formula

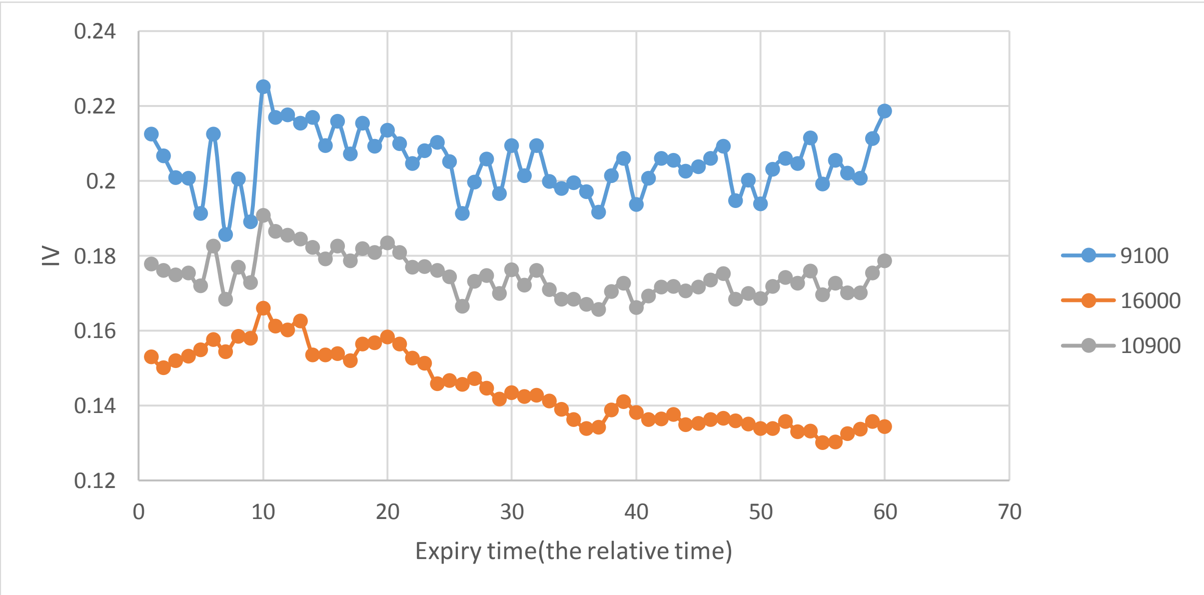
When conclude the B-S model, we use the Ito lemma

$$\frac{\partial f}{\partial t} + rs\frac{\partial f}{\partial s} + \frac{1}{2}s\sigma^2\frac{\partial^2 f}{\partial s} = rf$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial \tau}$$

It is obvious that the implied volatility also affected by the stock price's stochastic process. So here I want to create some PDE equations to simulate the IV surface and using diffusion equation to solve it further.

First, we can observe that the option with the low strike price (9100),



Now for comparing the implied volatility with time t, I draw the picture with the time series order(expiry days from 645 to 562 days), just use number 0 data point to represent the expiry time 645 and number 60 to represent the 562 expiry time.

Here we can see for 3 different trends for 3 different kinds of options, for the out-of-money option (strike price=16000), the implied volatility is almost decreasing all the time with a swift velocity relatively. But for the in-the-money option (strike price=9100), we almost cannot find a significant time trend. As for the at-the-money option (strike price=10900), the trend is not very significant but still can see this trend.

If we plot more data, we can see this phenomenon more clearly. Because for every option, the value is derived from two parts, one is the intrinsic value which is the payoff that the option is exercised at the moment, so for in-the money call option the intrinsic value is absolutely positive, the other part is time value which is the value for in the future that the option may be more expensive (people are willing to pay for the future probability).

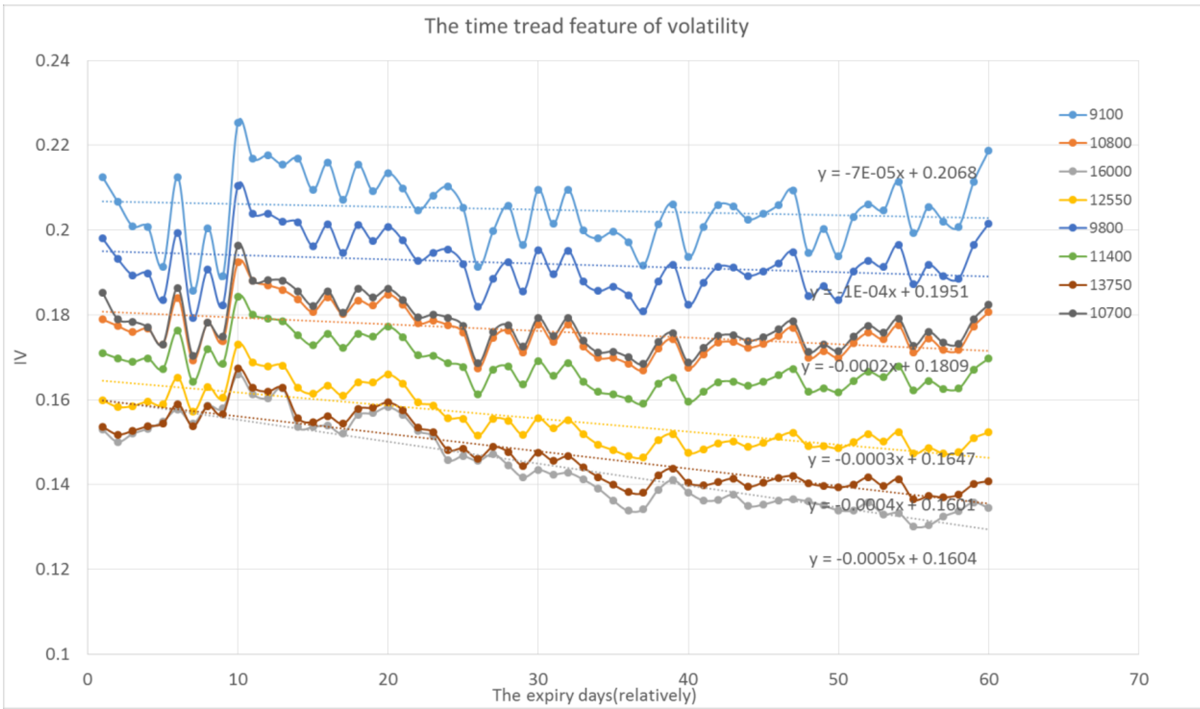
$$Option\ value = time\ value + intrinsic\ value$$

So it is easy to understand that a deeply out-the-money call option (K=16000) whose intrinsic value is zero every time during the observation period, So it is not sensitive to the change of moneyness (strike price/ stock price), but it is sensitive to the expiry time, because with a short expiry time, the out-of-money option is harder to be exercised on the expiry date. And with the time closing to the expiry date, the price decreasing rate will be faster, for the probability that the option can be exercised acceleratedly, and after drawing the implied volatility from given option price, we can observe the volatility experienced a slow but continuously drop for out-the-money call option (K=16000).

Meanwhile, for deeply in-the-money call option (k=9100), the time value is actually not very meaningful, the intrinsic value will almost always be positive (except extreme situation, the crisis) so for that option price is decided by intrinsic value mainly

$$intrinsic\ value = \max(st - k, 0)$$

That is the reason why this kind of option is so sensitive to the change of the stock price, we can see at the x=10 in the picture(means expiry time=635), there is big fluctuate on all options, the option(k=9100) fluctuated the most dramatically.



Now we can define the first PDE equation by this phenomenon .

$$\frac{\partial IV}{\partial t} = f((T - t), \frac{k}{s})$$

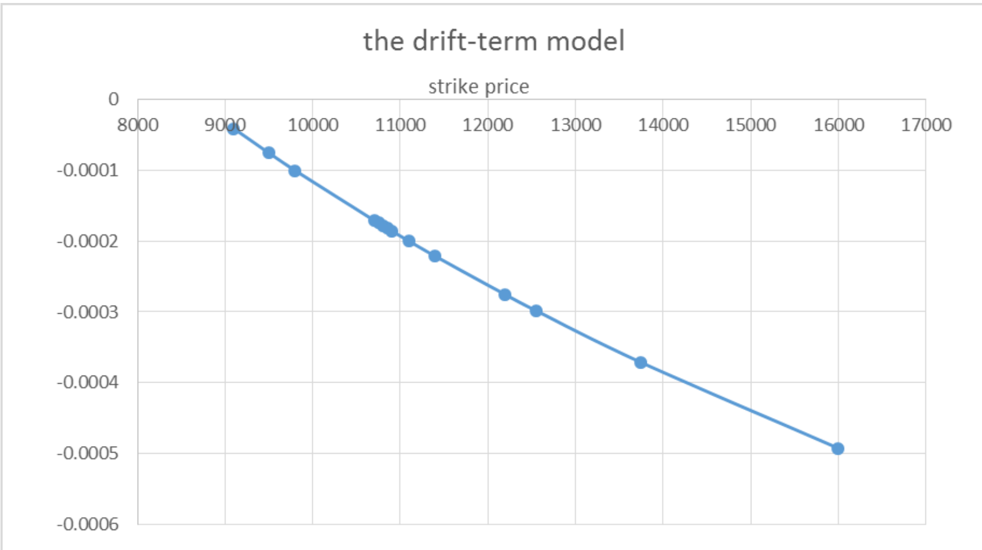
Here we use this kind of formula to simulate the time trend item. The (T-t) means the expiry date, $\frac{k}{s}$ means the moneyness, which measures how far from the stock price to the strike price. And k is strike price, s is stock price, the IV means the implied volatility, I propose a possible formula for this item:

$$f((T-t),\frac{k}{s}) = ce^{-(T-t)}IV\left(\frac{k}{s}\right)^2, c < 0$$

We can see if (T-t) is close to zero the $e^{-(T-t)}$ will increase to 1, so the $\frac{\partial IV}{\partial t} for$ will increase swiftly, which satisfies the fact that time value will lose faster and faster when expiry day is coming as (T-t) is closing to zero. And the item $\left(\frac{k}{s}\right)^2$, it can make in-the-money call option IV change slower for whose $\left(\frac{k}{s}\right)$<1, so the equation has little influence about value, but for the out-the-money option, this term can make it change more.

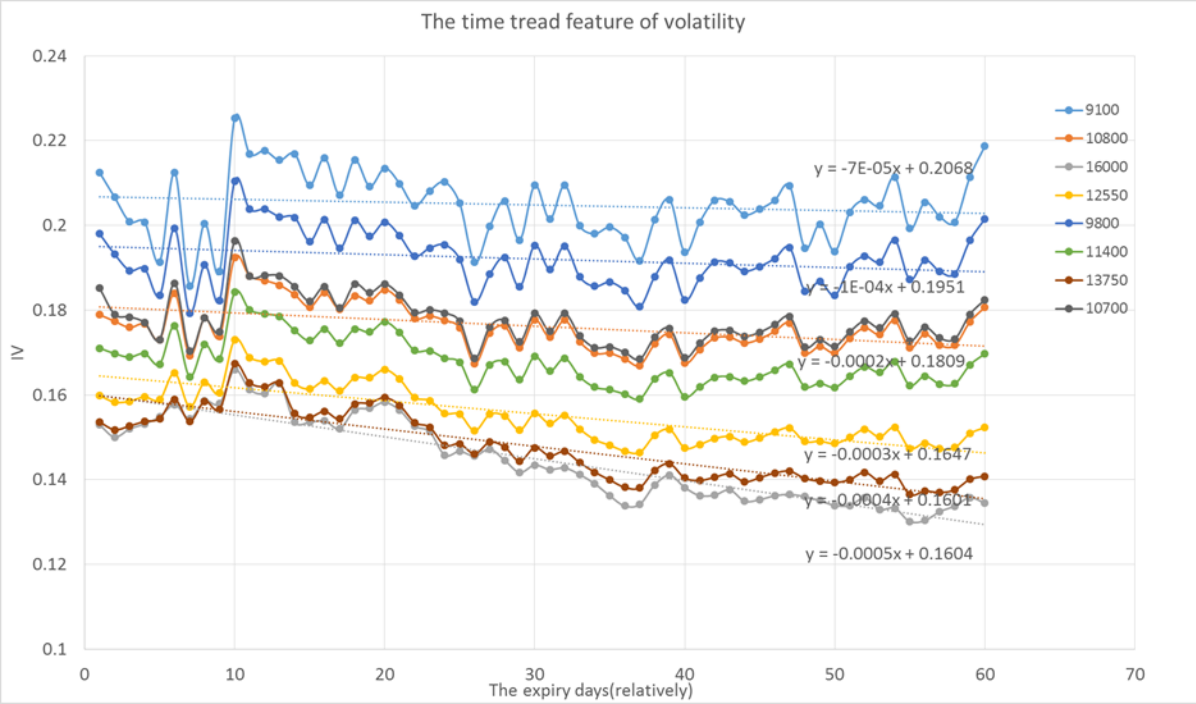
After several times experiments, I think we can omit the time term $e^{-(T-t)}$, because this is a short-period observation, the decreasing trend does not accelerate with time significantly, and we can also change the whole formula in a linear form. For simplifying the model, we can just use this equation to replace PDE form, and which can be easily calculated. Draw the picture/

$$\frac{\partial IV}{\partial t} = -0.0002 \times [4 \times log\left(\frac{k}{s}\right) + 1]$$



We can see this equation can fit the IV motion linear trend perfectly, for the option (k=16000), the time trend term will be -0.0005 and for at-the-money option (k around

11000) , the time trend will be -0.0002, and for in-the-money option, term is very small



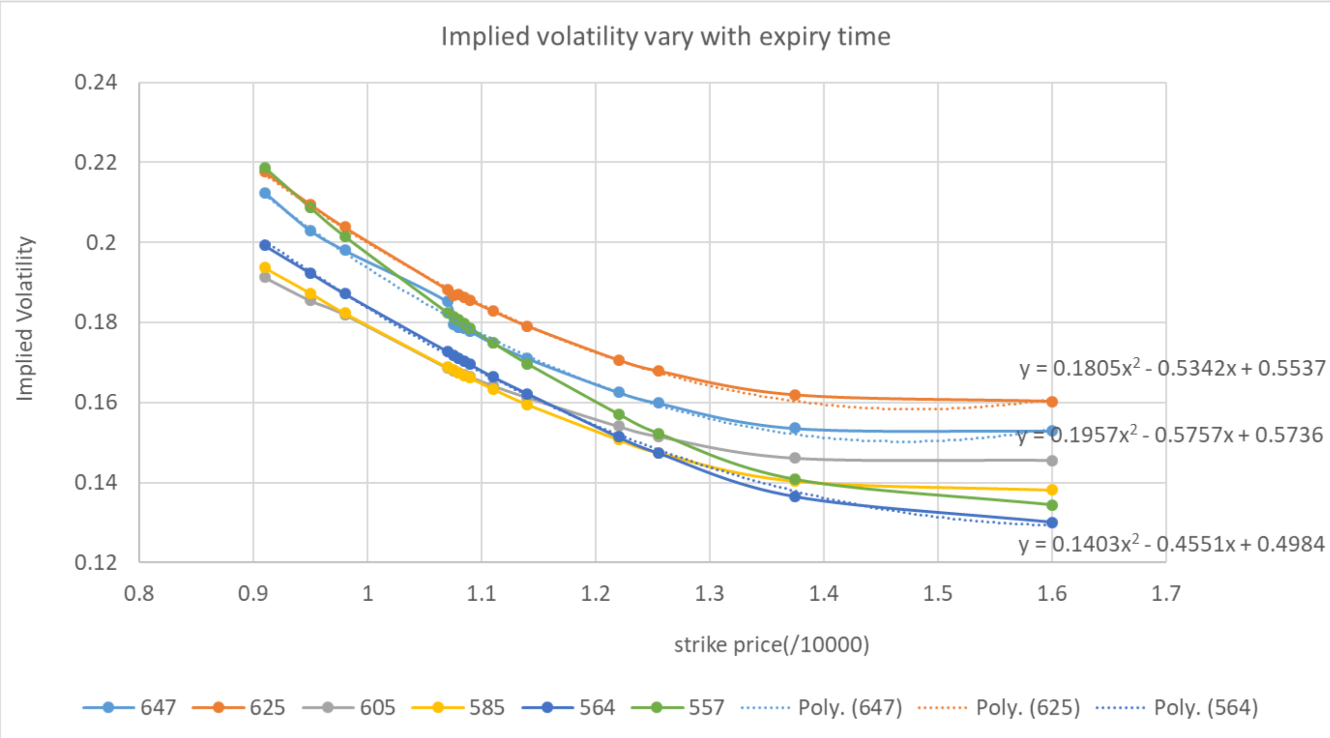
Here we use moneyness $\left(\frac{k}{s}\right)$ here to give different option different drop velocity. And the moneyness value can handle the situation that some previous in-the-money options turn to be out-of-the-money, after a crisis.

Now we could consider another factor

$$\frac{\partial IV}{\partial k} = f(k, t, s \dots)$$

$$\frac{\partial^2 IV}{\partial k^2} = f(k, t, s \dots.)$$

Plot the picture by cross-sectional view with some time



We can observe the cross-sectional curve is stable and highly similar with each other even for different periods (for example expiry 647 and 564). The possible function form of $\frac{\partial IV}{\partial k}$ and $\frac{\partial IV}{\partial k^2}$ seem to be simple quadratic function and the constant item can be generated by $\frac{\partial IV}{\partial t}$ and historical data.

$$\frac{\partial IV}{\partial k} = ax + b$$

$$\frac{\partial^2 IV}{\partial k^2} = a$$

The parameter a and b can be got by fitting the skewness surface with quadratic function, we can assume this function relation $ax + b$ does not change a lot in a close time interval.(which is not true, but I need to simplify the model, the further research may be later)

$$\frac{\partial^2 IV}{\partial k \partial t} = 0$$

So In conclude, we create the PDE as following:

$$\frac{\partial IV}{\partial t} = -0.0002 \times [4 \times log\left(\frac{k}{s}\right) + 1]$$

$$\frac{\partial IV}{\partial k} = ax + b$$

$$\frac{\partial^2 IV}{\partial k^2} = a$$

$$\frac{\partial IV}{\partial k} = 0 \; (\frac{K}{s} \approx 1)$$

$$\frac{\partial^2 IV}{\partial k^2} = c\frac{\partial IV}{\partial t}$$

By adding some the boundary conditions and use historical data as initial condition, we can use PDE solver to solve these PDE equations, we may find the solution by that.

Here in this project, I will not implement the model, one reason is the strike price is from 9100 to 16000, I need to construct a very big PDE-mesh to solve that, and I test some experiments on PDE solver, the convergence rate is very slow. On the other hand, the diffusion equation coefficient c, I just guess it may be close to 1 for at-the-money options. But for the out-of-the-money option and in-the-money option, I am not sure, because the skew can keep the basic shape in such a long time, which is a stable condition, so I need to do more investigation on that further.

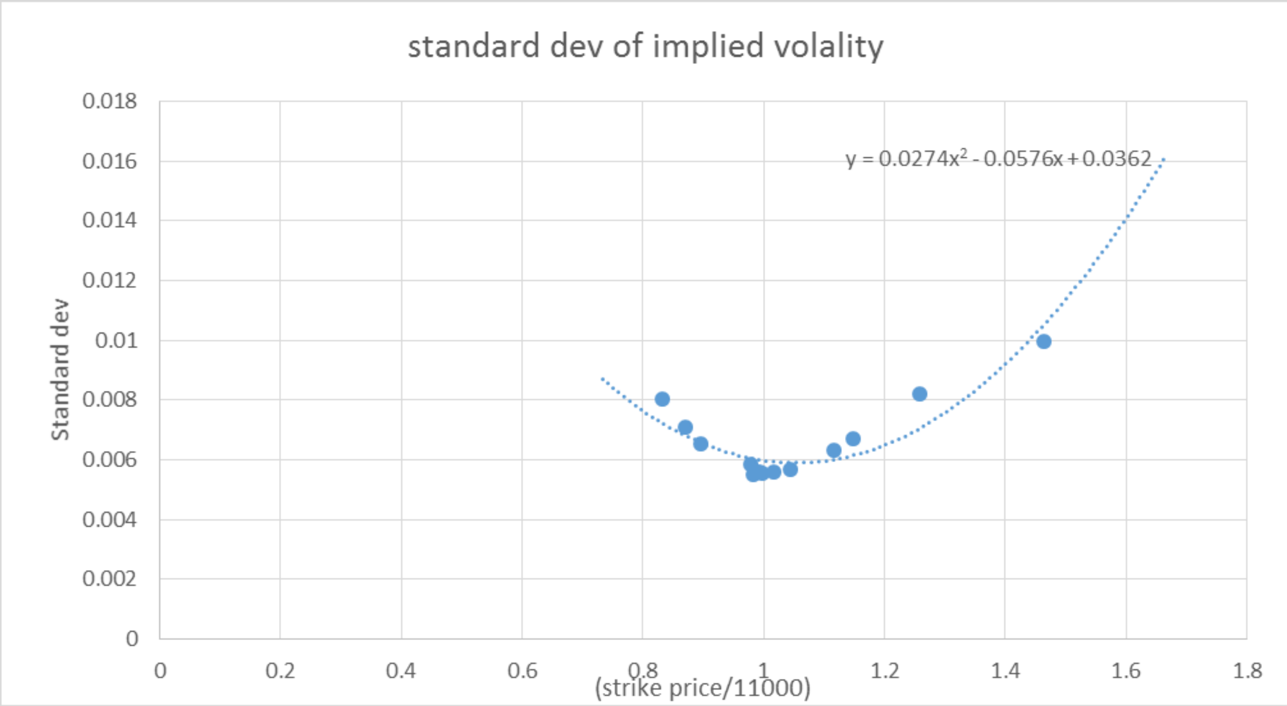
And I will transform the PDE to a much simple ODE form and adding some reliable assumptions to replace it by a much simple model and do the delta hedge with that.

Now implement model,

We can use the formula

$$\frac{dIV}{dt} = -0.0002 \times [4 \times log\left(\frac{k}{s}\right) + 1]$$

Then we can assume the volatility of volatility is a normal distribution for each option.



Here I draw the Volatility stand deviation for all options, we can see the implied volatility changed a lot for in-the-money and out-the-money call options, when we observe the volatility surface in the beginning, I ignore this, because the out-the-money option has smaller implied volatility relatively.

So we should give every option IV different volatility, and we can use moneyness to replace the Independent variable (strike price/11000) to make formula more general.

Now I assume for each option, the volatility of implied volatility is normal distribution, and satisfy this relation

$$V_{t+\Delta t} = V_t + \frac{dIV}{dt}\Delta t + S(\frac{k}{s})\varepsilon$$

$$\varepsilon \sim \mathrm{N}(0,1)$$

$$\mathrm{S}\left(\frac{k}{s}\right) = 0.0274\left(\frac{K}{S}\right) - 0.0576\left(\frac{K}{S}\right) + 0.0362$$

Because I simplify the model in the last chapter, so the skew shape will not change with time.

$$\frac{\partial^2 IV}{\partial k \partial t} = 0$$

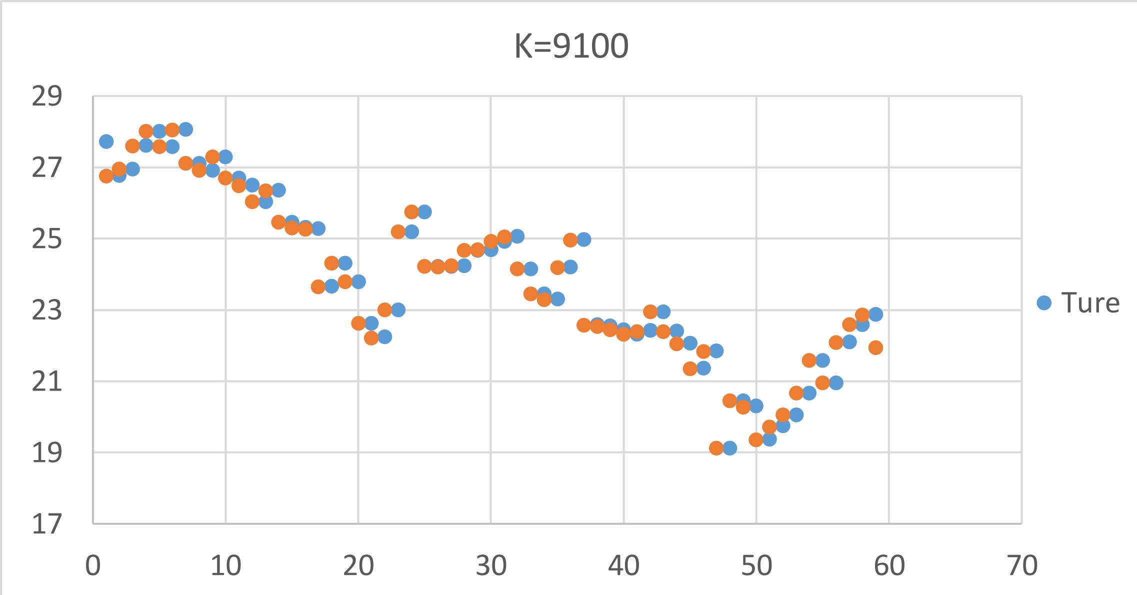
So I can just discuss the Implied volatility only with time t. Everytime when I need to guess next trading day implied volatility, I just use the latest historical data to represent the skew and use the simple model to get the next trading day volatility.

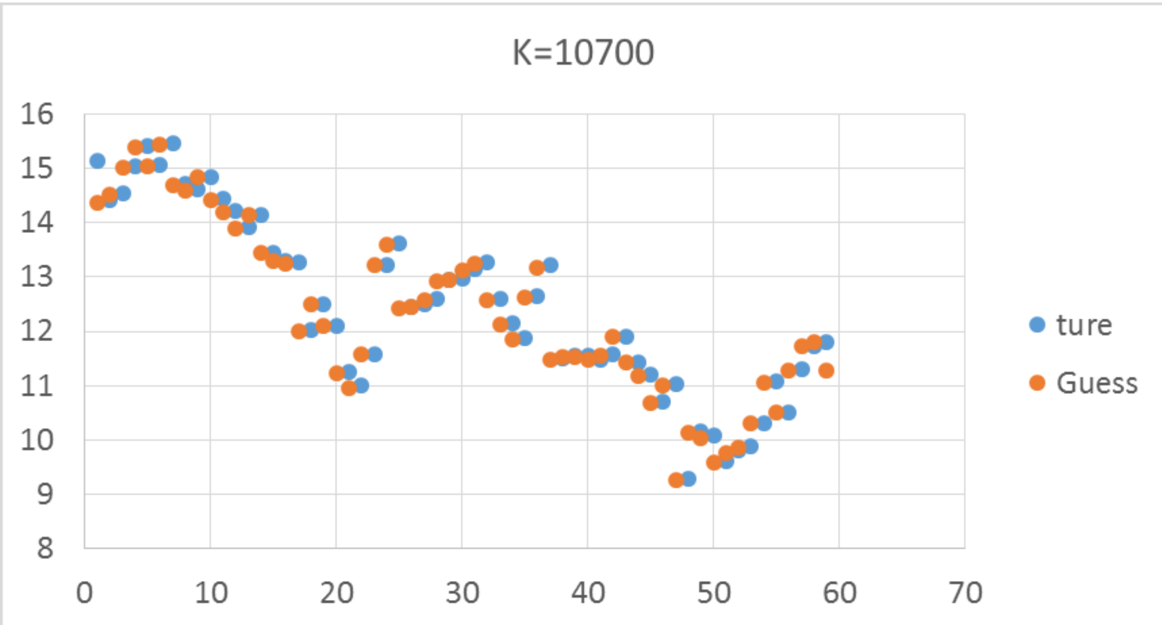
Calculate the delta and hedge it with the implied volatility get from the simple model.

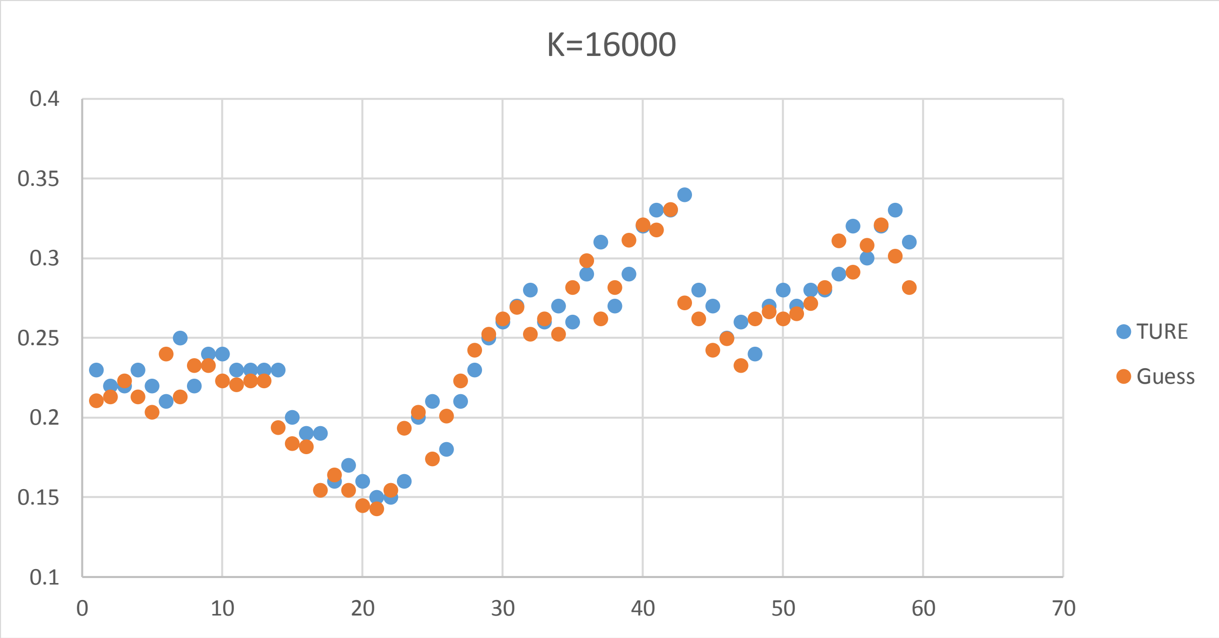
Here are the comparison. I normalise my portfolio in this logic, on first day closing time calculate the next day implied volatility can get the delta at the same time, then open a position to make the delta neutral, in the next day closing time ,calculate the return rate of my whole position and make the delta hedge normalisation.

K	standerror_origan_delta_hedge	standerror_newmodel_delta_hedge
9100	0.00599653	0.00599562
9500	0.00585749	0.00585667
9800	0.0057498	0.0057491
10700	0.0058853	0.0058853
10750	0.0055339	0.0055338
10800	0.00541686	0.00541681
10850	0.00541605	0.00541606
10900	0.00537715	0.00537716
11100	0.00537486	0.00537496
11400	0.00535479	0.00535507
12200	0.00538773	0.00538719
12550	0.00551355	0.0055121
13750	0.00619784	0.00619007
16000	0.0084018	0.00840263

We can see the new delta hedge strategy only change the standard error a little, part of the reason is that the $\frac{dIV}{dt} = -0.0002 \times [4 \times log\left(\frac{k}{s}\right) + 1]$ is so small. But most of options delta hedging can get a better result, it shows the trend item is correct.


K=9100
Ture
29
27
25
23
21
19
17
0
10
20
30
40
50
60
70


K=10700
ture
Guess
16
15
14
13
12
11
10
9
8
0
10
20
30
40
50
60
70



Analyzing the new model price, I choose (k=9100,10700,16000) 3 pictures to show the pricing option result , because I choose the time term is very small, the guess result is almost as same as the ture data. And in the stochastic volatility, I design the standard error term by real data result, so the model price is close to the real one and has the similar distribution. But for K=16000, the result shows the trend term may be too large for this option. Many real data points appear up the Guess ones.