Negative Shocks, Public Debt, and Interest Rates *

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Abstract

Levels of debt-to-GDP across developed economies are at historically high levels and government borrowing rates have remained persistently low. Blanchard (2019) provides evidence that the fiscal costs are low and welfare effects can be positive of increased government debt in low interest rate environments. This paper provides a replication of some of the Blanchard results and tests the robustness of those results to some key assumptions. This study finds that the replication of Blanchard's stated approach results in no welfare gains from increased government debt and that those welfare losses are exacerbated if some strong risk-reducing assumptions are relaxed to more realistic values. Furthermore, I suggest that the Blanchard calibration strategy also biases the results toward more beneficial government debt.

keywords: Public debt, overlapping generations, fiscal policy, interest rates

JEL classification: [Put JEL codes here.]

*This research benefited from support from the Open Source Economics Laboratory at the University of Chicago. All Python code and documentation for the computational model is available at https://github.com/OpenSourceEcon/PubDebtNegShocks.

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1 Introduction

Outgoing President of the American Economic Association, Olivier Blanchard, gave the AEA Presidential Address at the January 2019 annual meeting on a timely topic on which a consensus has not yet been established in the field and among policy makers. Blanchard (2019) provides evidence that the fiscal and welfare costs of public debt may be very small in economic environments of low interest rates. A significant contribution of his paper shows that the United States is in a prolonged period of low interest rates, calculates a careful measure of average borrowing rate for U.S. government debt, and provides evidence that this low-interest-rate environment is likely to persist. This topic of fiscal and welfare costs of public debt is also timely because debt-to-GDP ratios among developed countries are historically high, and the policy response to increased debt has been varied since the 2008-2009 global recession.

It is mechanically true that the fiscal cost of expanded public debt is low in a low-interest-rate environment. That is, if the borrowing rate for government debt is less than the rate of economic growth $r_t < g_t$ and if new debt from the primary deficit x_t does not outsize the natural reduction in debt-to-GDP from its previous stock d_t, then the future debt-to-GDP ratio d_{t+1} falls.

$$d_{t+1} = \left(\frac{1 + r_{t+1}}{1 + g_{t+1}}\right) d_t + x_t$$

Despite the many interesting questions having to do with the dynamics of fiscal costs on the government budget constraint, this paper only addresses them indirectly. Instead, I focus on the welfare effect of increased debt in a low interest rate environment. The government transfer obligation distorts the agent's intertemporal savings decision.

This paper explores the robustness of the Blanchard (2019) paper's "strong argument for using fiscal policy to sustain demand" in a persistent low-interest-rate environment with respect to two of his paper's main assumptions. Using Blanchard's calibration strategy, I first test whether his positive welfare effects of increased debt survive realistic increases in risk. In his model, Blanchard makes a strong assumption that forces the risk from public debt to be low. He assumes that each agent receives a "manna from heaven" consumption endowment when young that is large enough to preclude any form of government default on its commitment to transfer resources from the young to the old. The size of this assumed transfer is equal to the average wage an individual would expect to earn in a regime in which the government makes no fiscal tax on the young. Furthermore, this endowment does not enter into any government budget constraint or resource constraint and, therefore, provides a costless safety net to both individuals and government. This is a very strong assumption about risk exposure in this model economy.

Using Blanchard's calibration approach, I test the welfare effects of the same increases in transfers from young to old with lower levels of non-governmental endowments to the young. Not surprisingly, Blanchard's simulation results of welfare increases for many parameterizations of the model decline with more risk in the economy, and the sign of all the positive effects become negative. I find the same result, to a much lesser extent when more risk is added by maintaining the Blanchard endowment level to the young and implementing a mean preserving spread to the variance of the total factor productivity aggregate shock.

A more subtle assumption of Blanchard (2019) is his calibration approach. The model is calibrated to match low average risky returns, low average riskless interest rates, and small average spreads between the two. The interaction with this calibration approach and the endowment assumption previously discussed bias Blanchard's results toward positive welfare effects of increased debt.

A large literature connects fiscal stress to increasing equity premia or spreads between the risky return and riskless return. The Blanchard (2019) modeling approach is nearly identical to the approach of Evans et al. (2013), who show that increased government debt leads to more frequent default which in turn increases the interest rate spread. In particular, they find that the equity premium increases as the economy gets closer to a default event.

Rebelo et al. (2019) study a model in which rare disasters generate increased hedging and savings behavior and increased credit spreads. Tsai and Wachter (2015) provide a broad survey of the rare disaster literature and its effect on asset prices, especially building off of the work by Gourio (2012) and Barro (2009). All of these papers find that rare negative events generate higher equity premia, more insurance and hedging behavior, and lower overall utility, even when the economy is mostly in a more moderate range.

The modeling assumptions of Blanchard (2019) doubly bias the results toward welfare improvements from increased debt in low interest rate environment. First, the assumption of an endowment that precludes government default gets rid of any catastrophic rare events. Furthermore, the calibration of the model to an assumed low interest rate spread implements a parameterization that is associated with low fiscal stress. The quantitative results of this paper provide evidence counter to the findings of Blanchard (2019).

2 Economic Model

A detailed specification and derivation of the model is available in the online technical appendix.[1] The economic environment is an overlapping generations model with two-period-lived agents for which age s is indexed by $s = 1$ for young and $s = 2$ for old. Agents supply a unit of labor inelastically for the market wage w_t when young and are retired and supply no labor when old. Agents choose how much to consume when they are young $c_{s=1,t}$ and old $c_{s=2,t+1}$, and they choose how much to save when young $k_{s=2,t+1}$ which comes back to them at the risky interest rate when old. The household optimization problem is the following,

$$\max_{k_{2,t+1}} \,(1-\beta)\ln(c_{1,t}) + \beta\frac{1}{1-\gamma}\ln\Big(E_t\big[(c_{2,t+1})^{1-\gamma}\big]\Big) \quad \forall t \tag{1}$$

$$\text{such that} \quad c_{1,t} + k_{2,t+1} = w_t + x_1 - H_t \tag{2}$$

$$\text{and} \quad c_{2,t+1} = R_{t+1}k_{2,t+1} + H_{t+1} \tag{3}$$

$$\text{and} \quad c_{1,t}, c_{2,t+1}, k_{2,t+1} > 0 \tag{4}$$

[1] See online technical appendix here https://github.com/OpenSourceEcon/PubDebtNegShocks.

where R_t is the gross return on risky savings and w_t is the wage on the unit of inelastically supplied labor by the young.

The functional form for lifetime utility in (1) is the Epstein-Zin-Weil used in Blanchard (2019).[2] The value x_1 in the young age $s = 1$ budget constraint (2) is the endowment that the young receive, and H_t is the lump sum government transfer taken from the young and given to the old each period. In general, H_t equals the promised amount $\bar{H}$. In Blanchard (2019), the endowment x_1 guarantees that this is always the case. But I will allow x_1 to be small enough that the government might not always be able to collect $\bar{H}$ in every period, as in the case in Evans et al. (2013). I will specify H_t in more detail in Equation (12). The resulting Euler equation for optimal risky savings $k_{2,t+1}$ is the following.

$$\frac{1-\beta}{c_{1,t}} = \beta \frac{E_t\left[R_{t+1}\left(c_{2,t+1}\right)^{-\gamma}\right]}{E_t\left[\left(c_{2,t+1}\right)^{1-\gamma}\right]} \quad \forall t \tag{5}$$

We can independently derive the equilibrium price of a riskless bond, the exogenous supply of which is arbitrarily set to zero, as is shown in the technical appendix. Let $\bar{R}_t$ be the return on the riskless bond (the inverse of the price). The derived Euler equation characterizing the equilibrium riskless bond return in each period is the following.

$$\bar{R}_t = \left(\frac{1-\beta}{\beta}\right) \frac{E_t\Big[\left(c_{2,t+1}\right)^{1-\gamma}\Big]}{(c_{1,t}) E_t\Big[\left(c_{2,t+1}\right)^{-\gamma}\Big]} \quad \forall t \tag{6}$$

I assume a unit measure of identical perfectly competitive firms that rent capital K_t at rental rate r_t and hire labor L_t at wage w_t to produce consumption good output Y_t and maximize profits according to a constant elasticity of substitution production function with stochastic total factor productivity,

$$Y_t = F(K_t, L_t, z_t) = A_t\Big[\alpha(K_t)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\alpha)(L_t)^{\frac{\varepsilon-1}{\varepsilon}}\Big]^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall t \quad \text{where} \quad A_t \equiv e^{z_t} \tag{7}$$

where the capital share of income is given by $\alpha \in (0, 1)$ and $\varepsilon \geq 1$ is the constant

[2] See Epstein and Zin (2013) and Weil (1990).

elasticity of substitution between capital and labor in the production process. Total factor productivity $A_t \equiv e^{z_t}$ is distributed log normally, and z_t follows a normally distributed $AR(1)$ process.

$$z_t = \rho z_{t-1} + (1-\rho)\mu + \epsilon_t \quad \text{where} \quad \rho \in [0,1) \quad \text{and} \quad \epsilon_t \sim N(0,\sigma) \tag{8}$$

Two important special parameterizations of the production function (7) are the unit elasticity case $\varepsilon = 1$ in which the limit is the Cobb-Douglas production function and the perfectly elastic case $\varepsilon = \infty$ in which the production function is linear in K_t and L_t (perfect substitutes).

The firm's problem each period is to choose how much capital K_t to rent and how much labor L_t to hire in order to maximize profits,

$$\max_{K_t,L_t} Pr_t = F(K_t,L_t,z_t) - w_t L_t - R_t K_t \quad \forall t \tag{9}$$

where the marginal cost of capital is the gross interest rate R_t because the depreciation rate is assumed to be 100 percent. Profit maximization implies that the wage and interest rate are determined by the standard first order conditions for the firm.

$$R_t = \alpha (A_t)^{\frac{\varepsilon-1}{\varepsilon}} \left[\frac{Y_t}{K_t}\right]^{\frac{1}{\varepsilon}} \quad \forall t \tag{10}$$

$$w_t = (1-\alpha)(A_t)^{\frac{\varepsilon-1}{\varepsilon}} \left[\frac{Y_t}{L_t}\right]^{\frac{1}{\varepsilon}} \quad \forall t \tag{11}$$

As can be seen from first order conditions (10) and (11), in the case of perfect substitutes (linear production, $\varepsilon = \infty$), the first order conditions are independent of capital and labor.

Because the interest rate R_t in (10) is not defined when the capital stock is zero $K_t = 0$, the wage w_t in (11) is not defined when aggregate labor is zero $L_t = 0$, and output Y_t is not defined when capital or labor are less-than-or-equal-to zero, we know that both values must be strictly positive $K_t, L_t > 0$ in equilibrium.

The government has committed to a balanced-budget lump-sum transfer each period $\bar{H} \geq 0$ from the young to the old subject to feasibility of the transfer. Let $c_{min} > 0$ and $K_{min} > 0$ be minimum positive levels of consumption and aggregate capital. Then the government transfer rule characterizing H_t is that it equals $\bar{H}$ except in periods when the promised transfer is greater than the total income minus minimum values of consumption and aggregate capital.[3]

$$\begin{aligned} H_t &\equiv \begin{cases} \bar{H} & \text{if} \quad w_t \geq \bar{H} - x_1 + c_{min} + K_{min} \\ w_t + x_1 - c_{min} - K_{min} & \text{if} \quad w_t < \bar{H} - x_1 + c_{min} + K_{min} \end{cases} \quad \forall t \\ &= \min\left(\bar{H}, w_t + x_1 - c_{min} - K_{min}\right) \quad \forall t \end{aligned} \tag{12}$$

In equilibrium, the aggregate capital, labor, riskless assets, and goods markets must clear. The goods market clearing condition (16) is redundant by Walras' Law.

$$K_t = k_{2,t} \quad \forall t \tag{13}$$

$$L_t = 1 \quad \forall t \tag{14}$$

$$0 = b_{2,t} \quad \forall t \tag{15}$$

$$\begin{aligned} Y_t &= C_t + K_{t+1} - (1-\delta)K_t \quad \forall t \\ &\text{where} \quad C_t \equiv c_{1,t} + c_{2,t} \end{aligned} \tag{16}$$

Equilibrium is defined as stationary allocation functions a and price functions of the state for which household optimality conditions hold (5) and (6), firm optimality conditions hold (10) and (11), markets clear (13) and (14), and government transfers follow the feasible transfer rule (12).

[3] I remain agnostic about what happens after the government defaults on its promised transfer $\bar{H}$ in any period in which $w_t < \bar{H} - x_1 + c_{min} + K_{min}$ as shown in the second case in (12). This case forces the consumption of young agents to be the minimum value $c_{1,t} = c_{min}$. Technically, that household can survive beyond the default period because consumption is positive. Evans et al. (2013) study cases in which the government default causes either a complete economic shut down and reversion to autarky or cases in which it causes a regime shift to a new tax regime.

3 Blanchard Calibration

The online technical appendix provides a detailed description and derivation of the calibration.[4] Table 1 shows the values of variables in the Blanchard (2019) calibration. Blanchard calibrates the capital share of income parameter $\alpha = 1/3$. He calibrates the annual standard deviation of the normally distributed component of z_t the total factor productivity process to be $\sigma_{an} = 0.2$, consistent with U.S. stock market returns historical average, which implies a model 25-year standard deviation of $\sigma \approx 0.615$.

Table 1: Blanchard (2019) calibration values

Variable	Value(s)	Variable	Value(s)	Variable	Value(s)
α	0.33	$E[R_{t+1,an}]$	[0.00, 0.04]	β	func. of $E[R_{t+1}]$
ε	1.0 or ∞	avg. $\bar{R}_{t,an}$	[-0.02, 0.01]	x_1	func. of $E[R_{t+1}]$
ρ_{an}	0.95	μ	func. of $E[R_{t+1}]$	avg. $k_{2,t}$	func. of $E[R_{t+1}]$
ρ	0.21	γ	func. of $E[R_{t+1}]$ and avg. $\bar{R}_t$	$\bar{H}$	[0, 0.05(avg. $k_{2,t}$)]
z_0	μ				

Given a calibrated value for σ, Blanchard (2019, p. 1213) identifies the value of μ independently of β using the linear production ($\varepsilon = \infty$) expression for the average value of the risky return, derived from marginal product of capital (10),

$$E_t\big[R_{t+1}\big] = \alpha e^{\rho z_t + (1-\rho)\mu + \frac{\sigma^2}{2}} \quad \forall t \tag{17}$$

and calibrates the value for γ given σ from equilibrium expression for the spread between the log average risky return and the log riskless return derived from (10) and (6)

$$\ln\Big(E_t\big[R_{t+1}\big]\Big) - \ln\big(\bar{R}_t\big) = \gamma\sigma^2 \quad \forall t \tag{18}$$

For higher values of average risky returns $E[R_{t+1}]$ the calibrated value of μ is higher, which reduces risk and counterbalances the higher risky returns. And for larger average interest rate spreads, agents have higher risk aversion γ. Despite using these two specifications of the production function to calibrate μ and γ, Blanchard analyses

[4] See online technical appendix here https://github.com/OpenSourceEcon/PubDebtNegShocks.

the cases of both the Cobb-Douglas production function ($\varepsilon = 1$) and the perfect substitutes production function ($\varepsilon = \infty$), separately.

Blanchard (2019) uses the Cobb-Douglas specification of the model ($\varepsilon = 1$) to identify β independent of μ and as a function of the average risky return.

$$\beta = \left(\frac{\alpha}{1-\alpha}\right)\frac{1}{2E[R_{t+1}]} \tag{19}$$

One of the main focuses of this paper is Blanchard's inclusion and calibration of the endowment to all young individuals x_1. He calibrates this value to be 100 percent of the average wage in the model in which the transfer is set to zero $\bar{H} = 0$.

$$x_1 = \left[(1-\alpha)e^{\mu+\frac{\sigma^2}{2}}(2\beta)^{\alpha}\right]^{\frac{1}{1-\alpha}} \tag{20}$$

This value constitutes a large safety net, and guarantees that the promised transfer never induces a default $w_t \geq \bar{H} - x_1 + c_{min} + K_{min}$. It is the effect of reducing this value x_1 that will be the main experiment of this paper.

4 Simulations

The primary experiment of Blanchard (2019) is to measure the average change in realized lifetime utility of agents across simulations of the model from a baseline version of the model in which there is no government transfer program $\bar{H} = 0$ to an economy in which the government transfer equals 5 percent of average savings $\bar{H} = 0.05(\text{avg. } k_{2,t})$. I simulate 15 independent time series of 25 periods each and take averages.

Table 2 shows the percent change in average lifetime welfare across simulations for nine different calibrations of the model based on all permutations of three values of average risky interest rates and average riskless interest rates and their implied spreads. The left-side panel of 3-by-3 results in Table 2 is a replication of Figure 7 in Blanchard (2019), and the right-side panel of 3-by-3 results is the replication of

Table 2: Percent change in average lifetime utility from increased transfer $\bar{H}$: constant $\mu = 1.0786$

		Linear production $\varepsilon = \infty$			Cobb-Douglas $\varepsilon = 1$		
		average $\bar{R}$ (annual)			average $\bar{R}$ (annual)		
		-0.020	-0.005	0.010	-0.020	-0.005	0.010
average	0.00	-0.59%	-0.59%	n/a	-0.78%	-0.77%	n/a
R_t	0.02	-0.73%	-0.73%	-0.73%	-1.62%	-1.58%	-1.54%
(annual)	0.04	-0.86%	-0.86%	-0.86%	-3.35%	-3.23%	-3.10%

* NOTE: The upper left element of each 3-by-3 set of percent changes in welfare is labeled "n/a" because that combination of average risky rate and average riskless rate implies a negative spread avg. $R_t <$ avg. $\bar{R}_t$, which is not possible in equilibrium given equation (18). Averages calculated as average over 15 simulated time series of 25 periods each.

Figure 9 in Blanchard (2019).

Notable is that the percent change in average welfare from an increase in the promised transfer $\bar{H}$ is nowhere positive. Another notable difference in these results from Blanchard's is that, although the qualitative relationship between welfare changes and respective risky and riskless interest rate changes are the same, the percent change in welfare is most sensitive to different average risky returns and is relatively non responsive to different average riskless returns. This is opposite of Blanchard's findings and is almost certainly a result of the calibrated parameter values shown in Table 1 being mostly functions of average risky returns and only γ being a function of average riskless returns.

It is unclear why Blanchard (2019, Figure 7) keeps μ constant at 1.0786 in the simulations, which we replicated in Table 2, given that the calibration strategy in Equation (17) suggests that μ should be a function of the average risky rate $E[R_{t+1}]$. The difference in μ values is striking with calibrated values in the range $\mu \in [2.76, 3.74]$ for average risky asset values in the range $E[R_{t+1,an}] \in [0.00, 0.04]$. Table 3 shows the percent change in average lifetime welfare when the calibrated value of μ adjusts with the assumed average risky rate indicated in the different rows of the table.

As with Table 2, all of the percent changes in average welfare from the increased transfer are negative. However, the direction of the relationship changes between percent changes in welfare and the calibrated average risky return. At higher average

Table 3: Percent change in average lifetime utility from increased transfer $\bar{H}$: variable μ as a function of $E[R_{t+1}]$

		Linear production $\varepsilon = \infty$			Cobb-Douglas $\varepsilon = 1$		
		average $\bar{R}$ (annual)			average $\bar{R}$ (annual)		
		-0.020	-0.005	0.010	-0.020	-0.005	0.010
average	0.00	-0.42%	-0.42%	n/a	-0.25%	-0.25%	n/a
R_t	0.02	-0.24%	-0.24%	-0.24%	-0.18%	-0.18%	-0.17%
(annual)	0.04	-0.14%	-0.14%	-0.14%	-0.14%	-0.13%	-0.13%

* NOTE: The upper left element of each 3-by-3 set of percent changes in welfare is labeled "n/a" because that combination of average risky rate and average riskless rate implies a negative spread avg. $R_t <$ avg. $\bar{R}_t$, which is not possible in equilibrium given equation (18). Averages calculated as average over 15 simulated time series of 25 periods each.

risky returns, the loss in welfare becomes smaller. It seems likely that, under this calibration strategy, there exists a higher risky return that would result in an increase in welfare from the increased transfer. But it is likely that this calibration strategy is not ideal.

I now proceed to test how the results of Table 3 change when more riskiness is added to the model. I first study the effect of reducing the endowment x_1. Table 4 shows the percent change in average lifetime utility across simulations from an increase in the transfer given the same calibrations of the model from Table 3 but with an endowment to the young that is equal to 50 percent of the average wage from the model in which there is no transfer—half the size of the endowment x_1 in the Blanchard calibration. In this setting, the government can default on its promised transfer, which default implies minimal consumption for the young in the default period. And some simulations default before the maximal 25 periods.

Table 5 shows the results for the highest risk environment in which the young agent endowment is completely removed $x_1 = 0$. In this setting, the government can default on its promised transfers, which default happens more often than in the simulation from Table 4.

The direction of welfare effects in Tables 4 and 5 with respect to different average risky and riskless asset calibrations remains the same as in Table 3. And the losses in welfare from the increased transfer become larger as the young agent endowment

Table 4: Percent change in average lifetime utility from increased transfer $\bar{H}$: variable μ as a function of $E[R_{t+1}]$, $x_1 = 0.5x_{1,orig}$

		Linear production $\varepsilon = \infty$			Cobb-Douglas $\varepsilon = 1$		
		average $\bar{R}$ (annual)			average $\bar{R}$ (annual)		
		-0.020	-0.005	0.010	-0.020	-0.005	0.010
average	0.00	-0.78%	-0.78%	n/a	-0.52%	-0.51%	n/a
R_t	0.02	-0.44%	-0.44%	-0.44%	-0.37%	-0.36%	-0.35%
(annual)	0.04	-0.25%	-0.25%	-0.25%	-0.27%	-0.26%	-0.25%

* NOTE: The upper left element of each 3-by-3 set of percent changes in welfare is labeled "n/a" because that combination of average risky rate and average riskless rate implies a negative spread avg. $R_t <$ avg. $\bar{R}_t$, which is not possible in equilibrium given equation (18). Averages calculated as average over 15 simulated time series of 25 periods each.

Table 5: Percent change in average lifetime utility from increased transfer $\bar{H}$: variable μ as a function of $E[R_{t+1}]$, $x_1 = 0$

		Linear production $\varepsilon = \infty$			Cobb-Douglas $\varepsilon = 1$		
		average $\bar{R}$ (annual)			average $\bar{R}$ (annual)		
		-0.020	-0.005	0.010	-0.020	-0.005	0.010
average	0.00	-3.61%	-3.61%	n/a	-3.42%	-3.28%	n/a
R_t	0.02	-1.89%	-1.88%	-1.88%	-2.14%	-2.03%	-1.91%
(annual)	0.04	-1.01%	-1.01%	-1.01%	-1.43%	-1.33%	-1.23%

* NOTE: The upper left element of each 3-by-3 set of percent changes in welfare is labeled "n/a" because that combination of average risky rate and average riskless rate implies a negative spread avg. $R_t <$ avg. $\bar{R}_t$, which is not possible in equilibrium given equation (18). Averages calculated as average over 15 simulated time series of 25 periods each.

is reduced.

In addition to the reduced endowment comparisons of Tables 4 and 5 to the Blanchard replication results in Table 3, I also show how a different type of increased risk affects the percent welfare increase results from Table 3. Tables 6 and 7 hold the endowment x_1 constant at the original level and increase the standard deviation of the TFP shock by 5 percent and 10 percent respectively. In each case, we reduce the mean of the shock μ by the corresponding amount that keeps the expected value of the shock $E[e^{z_{t+1}}]$ constant. This mean-preserving spread increases risk in the economy while holding average values relatively constant.

Table 6: Percent change in average lifetime utility from increased transfer $\bar{H}$: variable μ as a function of $E[R_{t+1}]$, $\sigma = 1.05\sigma_{orig}$

		Linear production $\varepsilon = \infty$			Cobb-Douglas $\varepsilon = 1$		
		average $\bar{R}$ (annual)			average $\bar{R}$ (annual)		
		-0.020	-0.005	0.010	-0.020	-0.005	0.010
average	0.00	-0.42%	-0.42%	n/a	-0.25%	-0.25%	n/a
R_t	0.02	-0.24%	-0.24%	-0.24%	-0.18%	-0.18%	-0.17%
(annual)	0.04	-0.14%	-0.14%	-0.14%	-0.14%	-0.13%	-0.13%

* NOTE: The upper left element of each 3-by-3 set of percent changes in welfare is labeled "n/a" because that combination of average risky rate and average riskless rate implies a negative spread avg. $R_t <$ avg. $\bar{R}_t$, which is not possible in equilibrium given equation (18). Averages calculated as average over 15 simulated time series of 25 periods each.

Table 7: Percent change in average lifetime utility from increased transfer $\bar{H}$: variable μ as a function of $E[R_{t+1}]$, $\sigma = 1.10\sigma_{orig}$

		Linear production $\varepsilon = \infty$			Cobb-Douglas $\varepsilon = 1$		
		average $\bar{R}$ (annual)			average $\bar{R}$ (annual)		
		-0.020	-0.005	0.010	-0.020	-0.005	0.010
average	0.00	-0.42%	-0.42%	n/a	-0.25%	-0.25%	n/a
R_t	0.02	-0.24%	-0.24%	-0.24%	-0.19%	-0.18%	-0.18%
(annual)	0.04	-0.14%	-0.14%	-0.14%	-0.14%	-0.14%	-0.13%

* NOTE: The upper left element of each 3-by-3 set of percent changes in welfare is labeled "n/a" because that combination of average risky rate and average riskless rate implies a negative spread avg. $R_t <$ avg. $\bar{R}_t$, which is not possible in equilibrium given equation (18). Averages calculated as average over 15 simulated time series of 25 periods each.

As a check that the model's theory is correctly coded and solved, the results in the right-side 3-by-3 panels of Tables 6 and 7 are identical to the right-side 3-by-3 panel of Table 3. The mean preserving spread of the TFP shock does not change the equilibrium results when production is linear. In the Cobb-Douglas cases, the mean preserving increase in risk of either a 5-percent increase in the TFP standard deviation in Table 6 or a 10-percent increase in standard deviation in Table 7 have almost no effect on the welfare changes. The results in the left-side 3-by-3 panels of Tables 6 and 7 are different from Table 3, and the welfare changes decline monotonically as σ increases. But the changes are very small.

5 Conclusion

- Questions
  - What are effects of increased deficits in expansion?

- 80-period lived model.
  - Include potential for default
  - How does government debt affect young/middle aged/old vs poor/rich?
  - Allow gap for gov't rate vs. MPK (multiple rates)
  - Allow for multiple maturities, calibrate percent in each by gov't

- Is this calibration approach appropriate?
  - Blanchard (2019) calibrates off assumed low interest rates
  - Calibrate other macro targets, let interest rates be endogenous

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