

# TECHNICAL APPENDIX

## T-1 Blanchard (2019) model and calibration

This section shows how the [Blanchard \(2019\)](#) model is a nested case of the model described in the body of this paper, and it derives and computes some of the results from [Blanchard \(2019\)](#).

### T-1.1 Households

[Blanchard \(2019\)](#) assumes that a unit measure of identical households is born each period and live for two periods. A household supplies a unit of labor inelastically when young  $n_{1,t} = 1$  for all  $t$  and does not work when old  $n_{2,t} = 0$  for all  $t$ . Young households have lump sum amount  $\bar{H}$  taken from them and given to the current period old each period. Young households also receive an endowment  $x_1$  each period. This endowment comes exogenously from some other economy and does not figure into this economy's government budget constraint. Households choose how much to consume each period  $c_{1,t}$  and  $c_{2,t+1}$  and how much to save in terms of risky savings  $k_{2,t+1}$  and riskless bonds  $b_{2,t+1}$ . The household maximization problem is the following,

$$\max_{k_{2,t+1}, b_{2,t+1}} (1 - \beta) \ln(c_{1,t}) + \beta \frac{1}{1 - \gamma} \ln\left(E_t[(c_{2,t+1})^{1-\gamma}]\right) \quad \forall t \quad (??)$$

$$\text{such that } c_{1,t} + k_{2,t+1} + p_t b_{2,t+1} = w_t + x_1 - \bar{H} \quad (T.1.1)$$

$$\text{and } c_{2,t+1} = R_{t+1} k_{2,t+1} + b_{2,t+1} + \bar{H} \quad (T.1.2)$$

$$\text{and } c_{1,t}, c_{2,t+1}, k_{2,t+1} \geq 0 \quad (T.1.3)$$

where  $R_t$  is the gross return on risky savings,  $w_t$  is the wage on the unit of inelastically supplied labor by the young, and  $p_t$  is the price per unit of the riskless bond. The resulting Euler equation for optimal risky savings  $k_{2,t+1}$  is the following.

$$\frac{1 - \beta}{c_{1,t}} = \beta \frac{E_t \left[ R_{t+1} (c_{2,t+1})^{-\gamma} \right]}{E_t \left[ (c_{2,t+1})^{1-\gamma} \right]} \quad \forall t \quad (T.1.4)$$

And the resulting Euler equation for optimal riskless savings  $b_{2,t+1}$  is the following.

$$\begin{aligned} \frac{1}{\bar{R}_t} \equiv p_t &= \left( \frac{\beta}{1 - \beta} \right) \frac{(c_{1,t}) E_t \left[ (c_{2,t+1})^{-\gamma} \right]}{E_t \left[ (c_{2,t+1})^{1-\gamma} \right]} \quad \forall t \\ \Rightarrow \bar{R}_t &= \left( \frac{1 - \beta}{\beta} \right) \frac{E_t \left[ (c_{2,t+1})^{1-\gamma} \right]}{(c_{1,t}) E_t \left[ (c_{2,t+1})^{-\gamma} \right]} \quad \forall t \end{aligned} \quad (T.1.5)$$

Substituting the period budget constraints (T.1.1) and (T.1.2) into the two Euler equations (T.1.4) and (T.1.5), we can show that optimal risky savings  $k_{2,t+1}$  and

riskless savings  $b_{2,t+1}$  are functions  $\psi(\cdot)$  and  $\phi(\cdot)$ , respectively, of the time paths of transfers and prices over the lifetime of the household,

$$k_{2,t+1} = \psi(\bar{H}, w_t, R_{t+1}) \quad \forall t \quad (\text{T.1.6})$$

$$b_{2,t+1} = \phi(\bar{H}, w_t, R_{t+1}) \quad \forall t \quad (\text{T.1.7})$$

where  $R_{t+1}$  is in the expectations operator.

Implicit in [Blanchard \(2019\)](#) is the assumption of, generally, an exogenous supply of riskless bonds that is nonnegative  $B_t \geq 0$  for all  $t$ . However, this model specifically assumes a zero supply of riskless bonds  $B_t = 0$ . So the general version of our riskless bond market clearing condition is the following.

$$b_{2,t} = B_t \quad \forall t \quad (\text{T.1.8})$$

With the zero supply assumption  $B_t = 0$ , the household demand for riskless bonds is zero in equilibrium through the market clearing condition,

$$b_{2,t} = 0 \quad \forall t \quad (\text{T.1.9})$$

all the other endogenous variables are determined by the equilibrium described without the riskless bonds, and the riskless return  $\bar{R}_t$  is characterized by Euler equation [\(T.1.5\)](#).

## T-1.2 Firms

A unit measure of identical perfectly competitive firms exist in this economy that hire aggregate labor  $L_t$  at wage  $w_t$  and rent aggregate capital  $K_t$  at rental rate  $r_t$  every period in order to produce consumption good  $Y_t$  according to a Cobb-Douglas production function,

$$Y_t = F(K_t, L_t, z_t) = A_t \left[ \alpha (K_t)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\alpha)(L_t)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall t \quad (??)$$

where the capital share of income is given by  $\alpha \in (0, 1)$  and  $\varepsilon > 0$  is the constant elasticity of substitution between capital and labor in the production process. Total factor productivity  $A_t \equiv e^{z_t} > 0$  is distributed log normally, and  $z_t$  follows a normally distributed  $AR(1)$  process. Two important special parameterizations are the unit elasticity case  $\varepsilon = 1$  in which the limit of [\(??\)](#) is the Cobb-Douglas production function and the perfectly elastic case  $\varepsilon = \infty$  in which the production function is linear in  $K_t$  and  $L_t$  (perfect substitutes).

$$z_t = \rho z_{t-1} + (1-\rho)\mu + \epsilon_t \quad (\text{T.1.10})$$

where  $\rho \in [0, 1)$ ,  $\mu \geq 0$ , and  $\epsilon_t \sim N(0, \sigma)$

The firm's problem each period is to choose how much capital  $K_t$  to rent and how much labor  $L_t$  to hire in order to maximize profits,

$$\max_{K_t, L_t} Pr_t = F(K_t, L_t, z_t) - w_t L_t - R_t K_t \quad \forall t \quad (\text{T.1.11})$$

where this equation implies full depreciation of capital each period  $\delta = 1$ . Profit maximization implies that the real wage and real rental rate are determined by the standard first order conditions for the firm.

$$R_t = \alpha(A_t)^{\frac{\varepsilon-1}{\varepsilon}} \left[ \frac{Y_t}{K_t} \right]^{\frac{1}{\varepsilon}} \quad \forall t \quad (\text{T.1.12})$$

$$w_t = (1 - \alpha)(A_t)^{\frac{\varepsilon-1}{\varepsilon}} \left[ \frac{Y_t}{L_t} \right]^{\frac{1}{\varepsilon}} \quad \forall t \quad (\text{T.1.13})$$

Because the risky interest rate  $R_t$  in (T.1.12) is not defined when the capital stock is zero  $K_t = 0$  and because the wage  $w_t$  in (T.1.13) is not defined when aggregate labor is zero  $L_t = 0$ , we know that both values must be strictly positive  $K_t, L_t > 0$ .

Blanchard (2019) looks at two cases of the production function. Perfect substitutes ( $\varepsilon = \infty$ ) is the simplest case in which the production function simplifies to the following linear function of  $K_t$  and  $L_t$  and the first order conditions become independent of  $K_t$  and  $L_t$  and simply functions of  $\alpha$  and  $A_t$ .

$$Y_t = A_t[\alpha K_t + (1 - \alpha)L_t] \quad \forall t \quad (\text{T.1.14})$$

$$R_t = \alpha A_t \quad \forall t \quad (\text{T.1.15})$$

$$w_t = (1 - \alpha)(A_t) \quad \forall t \quad (\text{T.1.16})$$

The second case is that of unit elasticity ( $\varepsilon = 1$ ), which results in a Cobb-Douglas production function of  $K_t$  and  $L_t$  with the corresponding first order conditions.

$$Y_t = A_t(K_t)^\alpha(L_t)^{1-\alpha} \quad \forall t \quad (\text{T.1.17})$$

$$R_t = \alpha A_t \left( \frac{L_t}{K_t} \right)^{1-\alpha} \quad \forall t \quad (\text{T.1.18})$$

$$w_t = (1 - \alpha)(A_t) \left( \frac{K_t}{L_t} \right)^\alpha \quad \forall t \quad (\text{T.1.19})$$

### T-1.3 Government budget constraint and market clearing

The government budget constraint in Blanchard (2019) is a simple balanced one in which revenues taken lump sum from the young in every period  $\bar{H}$  equal transfer expenditures given to the old each period  $\bar{H}$ . This does not cause any default or inability for the young to pay because the Blanchard analysis assumes the endowment  $x_1$  is big enough so that no adverse shock will make  $\bar{H} \geq w_t + x_1$ .

The model includes four market clearing conditions, only three of which are necessary for the solution—the labor market (T.1.20), risky capital market (??), riskless bond market (T.1.21), and the goods market (T.1.22). We will leave the goods market clearing condition (T.1.22) out of the solution method due to its redundancy by

Walras' Law.

$$L_t = 1 \quad \forall t \quad (\text{T.1.20})$$

$$K_t = k_{2,t} \quad \forall t \quad (??)$$

$$B_t = 0 \quad \forall t \quad (\text{T.1.21})$$

$$Y_t = C_t + K_{t+1} \quad \forall t \quad (\text{T.1.22})$$

$$\text{where } C_t \equiv c_{1,t} + c_{2,t} \quad \text{and} \quad K_{t+1} = I_t$$

## T-1.4 Equilibrium

The equilibrium in [Blanchard \(2019\)](#) is similar to Definition ?? from Section ??.

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**Definition 1 (Blanchard (2019) functional stationary equilibrium).** A non-autarkic functional stationary equilibrium in the two-period-lived overlapping generations model with exogenous labor supply and aggregate shocks in [Blanchard \(2019\)](#) is defined by stationary price functions  $R(k, z)$ ,  $w(k, z)$ , and  $\bar{R}(k, z)$  and a stationary risky savings function  $k' = \psi(k, z)$  for all current state wealth  $k$  and total factor productivity component  $z$  such that:

- i. households optimize according to (T.1.1) and (T.1.2), (T.1.4), and (T.1.5)
  - ii. firms optimize according to (T.1.12) and (T.1.13),
  - iii. markets clear according to (??) and (??).
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### T-1.4.1 Zero transfers and perfect substitutes

When transfers are zero  $\bar{H} = 0$  and capital and labor are perfect substitutes in production  $\varepsilon = \infty$ , the equilibrium has an analytical solution.

$$R_t = \alpha e^{z_t} \quad \forall z_t \quad (\text{T.1.23})$$

$$w_t = (1 - \alpha) e^{z_t} \quad \forall z_t \quad (\text{T.1.24})$$

$$\bar{R}_t = \alpha e^{\rho z_t + (1-\rho)\mu + \frac{\sigma^2(1-2\gamma)}{2}} \quad \forall z_t \quad (\text{T.1.25})$$

$$c_{1,t} = (1 - \beta) \left( [1 - \alpha] e^{z_t} + x_1 \right) \quad \forall z_t \quad (\text{T.1.26})$$

$$k_{2,t+1} = \beta \left( [1 - \alpha] e^{z_t} + x_1 \right) \quad \forall z_t \quad (\text{T.1.27})$$

$$c_{2,t} = \alpha e^{z_t} k_{2,t} \quad \forall k_{2,t}, z_t \quad (\text{T.1.28})$$

An important relationship that comes out of the equilibrium solution described above is the percent spread between the expected risky gross return next period and the current riskless return.

$$E_t[R_{t+1}] = \alpha e^{\rho z_t + (1-\rho)\mu + \frac{\sigma^2}{2}} \quad \forall t \quad (\text{T.1.29})$$

$$\ln(E_t[R_{t+1}]) - \ln(\bar{R}_t) = \gamma \sigma^2 \quad \forall t \quad (\text{T.1.30})$$

### T-1.4.2 Zero transfers and unit elasticity

When transfers are zero  $\bar{H} = 0$  and capital and labor have unit elasticity in the production function  $\varepsilon = 1$ , the equilibrium also has an analytical solution.

$$R_t = \alpha e^{z_t} (k_{2,t})^{\alpha-1} \quad \forall k_{2,t}, z_t \quad (\text{T.1.31})$$

$$w_t = (1 - \alpha) e^{z_t} (k_{2,t})^\alpha \quad \forall k_{2,t}, z_t \quad (\text{T.1.32})$$

$$\bar{R}_t = \frac{\alpha e^{\rho z_t + (1-\rho)\mu + \frac{\sigma^2(1-2\gamma)}{2}}}{\left(\beta[(1-\alpha)e^{z_t}(k_{2,t})^\alpha + x_1]\right)^{1-\alpha}} \quad \forall k_{2,t}, z_t \quad (\text{T.1.33})$$

$$c_{1,t} = (1 - \beta) \left( [1 - \alpha] e^{z_t} (k_{2,t})^\alpha + x_1 \right) \quad \forall k_{2,t}, z_t \quad (\text{T.1.34})$$

$$k_{2,t+1} = \beta \left( [1 - \alpha] e^{z_t} (k_{2,t})^\alpha + x_1 \right) \quad \forall k_{2,t}, z_t \quad (\text{T.1.35})$$

$$c_{2,t} = \alpha e^{z_t} (k_{2,t})^\alpha \quad \forall k_{2,t}, z_t \quad (\text{T.1.36})$$

The analogous relationship to (T.1.30) that comes out of the equilibrium solution described above is the percent spread between the expected risky gross return next period and the current riskless return.

$$E_t[R_{t+1}] = \alpha e^{\rho z_t + (1-\rho)\mu + \frac{\sigma^2}{2}} (k_{2,t+1})^{\alpha-1} \quad \forall t \quad (\text{T.1.37})$$

$$\ln(E_t[R_{t+1}]) - \ln(\bar{R}_t) = \gamma \sigma^2 \quad \forall t \quad (\text{T.1.30})$$

### T-1.5 Calibration

Blanchard assumes that households inelastically supply a unit of labor when young  $n_{1,t} = 1$  and supply no labor when old  $n_{2,t} = 0$  for all  $t$ . He calibrates the capital share of income parameter  $\alpha = 1/3$ . He calibrates the annual standard deviation of the normally distributed component of  $z_t$  the total factor productivity process to be  $\sigma_{an} = 0.2$ , which implies a model 25-year standard deviation of  $\sigma \approx 0.615$ . Blanchard assumes full depreciation of capital each period  $\delta = 1$ .

Given a calibrated value for  $\sigma$ , Blanchard (2019, p. 1213) identifies the value of  $\mu$  independently of  $\beta$  using the linear production expression for the expected value of the marginal product of capital (T.1.29),

$$E_t[R_{t+1}] = \alpha e^{\rho z_t + (1-\rho)\mu + \frac{\sigma^2}{2}} \quad \forall t \quad (\text{T.1.29})$$

and calibrates  $\gamma$  from the difference in the expected marginal product from the riskless rate (T.1.30), which expression holds in both the linear and Cobb-Douglas production cases.

$$\ln(E_t[R_{t+1}]) - \ln(\bar{R}_t) = \gamma \sigma^2 \quad \forall t \quad (\text{T.1.30})$$

He then identifies  $\beta$  independent of  $\mu$  using the Cobb-Douglas expression for the expected value of the marginal product of capital, the derivation of which is given below in the lead up to (T.1.45). This use of two separate models to identify two

respective parameters to be used in the same model is justified given the independence of the identifying equations on the other parameter.

To derive the independent expression for  $\beta$  from the Cobb-Douglas specification of the model, we must solve for the long run average of  $R_{t+1}$ ,  $k_{2,t}$  and  $w_t$ , of which  $x_1$  is a function. The long-run expected value version of the expected marginal product of capital from (T.1.37) is the following.

$$E[R_{t+1}] = \alpha e^{\mu + \frac{\sigma^2}{2}} \beta^{\alpha-1} [(1-\alpha)e^{\mu}(\bar{k}_2)^{\alpha} + x_1]^{\alpha-1} \quad (\text{T.1.38})$$

We solve for the average capital stock as the expected value of savings tomorrow  $E_t[k_{2,t+2}]$ .

$$E_t[k_{2,t+2}] = \beta \left[ (1-\alpha)e^{\rho z_t + (1-\rho)\mu + \frac{\sigma^2}{2}} (k_{2,t+1})^{\alpha} + x_1 \right] \quad \forall t \quad (\text{T.1.39})$$

Then let  $\bar{k}_2$  be the average  $k_{2,t}$  across a simulation by setting  $k_{2,t} = \bar{k}_2$  for all  $t$  in (T.1.39) and set  $z_t$  to its average value  $z_t = \mu$  for all  $t$ .

$$\bar{k}_2 = \beta \left[ (1-\alpha)e^{\mu + \frac{\sigma^2}{2}} (\bar{k}_2)^{\alpha} + x_1 \right] \quad (\text{T.1.40})$$

We solve for the average wage as the expected value of the wage tomorrow  $E_t[w_{t+1}]$ .

$$E_t[w_{t+1}] = (1-\alpha)e^{\rho z_t + (1-\rho)\mu + \frac{\sigma^2}{2}} (k_{2,t+1})^{\alpha} \quad \forall t \quad (\text{T.1.41})$$

Then set  $k_{2,t} = \bar{k}_2$  and  $z_t = \mu$  for all  $t$ , and the average wage  $\bar{w}$  is the following.

$$\bar{w} = (1-\alpha)e^{\mu + \frac{\sigma^2}{2}} (\bar{k}_2)^{\alpha} \quad (\text{T.1.42})$$

If we calibrate  $x_1$  to be 100 percent of the average wage, then we can rewrite (T.1.42).

$$x_1 = (1-\alpha)e^{\mu + \frac{\sigma^2}{2}} (\bar{k}_2)^{\alpha} \quad (\text{T.1.43})$$

Substituting (T.1.43) into (T.1.40) gives the following equation.

$$\bar{k}_2 = 2\beta x_1 \quad (\text{T.1.44})$$

Then dividing (T.1.43) by (T.1.38) and substituting in (T.1.44) gives an expression for  $\beta$  independent of  $\mu$ ,  $x_1$ , and  $\bar{k}_2$ .

$$\beta = \left( \frac{\alpha}{1-\alpha} \right) \frac{1}{2E[R_{t+1}]} \quad (\text{T.1.45})$$

Substituting (T.1.44) into (T.1.43), we can solve for  $x_1$  as a function of  $\mu$  and  $\beta$ .

$$x_1 = \left[ (1-\alpha)e^{\mu + \frac{\sigma^2}{2}} (2\beta)^{\alpha} \right]^{\frac{1}{1-\alpha}} \quad (\text{T.1.46})$$

Finally, we solve for the long-run value of wealth  $\bar{k}_2$  by substituting the expression for  $x_1$  from (T.1.46) into (T.1.44).

$$\bar{k}_2 = 2\beta \left[ (1-\alpha)e^{\mu + \frac{\sigma^2}{2}} (2\beta)^{\alpha} \right]^{\frac{1}{1-\alpha}} \quad (\text{T.1.47})$$

## T-1.6 Problems with Blanchard (2019) approach

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## T-2 Description of calibration

This section details our calibration of the parameter values listed in Table ?? . In our two-period-lived agent OG model, we assume that each period represents 30 years or, equivalently, a lifetime is 60 years. The model-period (30-year) discount factor  $\beta$  is set to match the annual discount factor common in the RBC literature of 0.96.

$$\beta = (0.96)^{30} \approx 0.2939 \quad (\text{T.2.1})$$

We set the coefficient of relative risk aversion at a midrange value of  $\gamma = 2$ . This value lies in the midrange of values that have been used in the literature.<sup>8</sup> The capital share of income parameter is set to match the U.S. average  $\alpha = 0.35$ , and the model-period (30-year) depreciation rate  $\delta$  is set to match an annual depreciation rate of 5 percent.

$$\delta = 1 - (1 - 0.05)^{30} \approx 0.7854 \quad (\text{T.2.2})$$

The firms' production function in our model is the following,

$$Y_t = e^{z_t} K_t^\alpha L_t^{1-\alpha} \quad \forall t \quad (??)$$

where labor  $L_t$  is supplied inelastically and  $z_t$  is current-period normally distributed component of total factor productivity. We assume that  $z_t$  is an AR(1) process with normally distributed errors.

$$\begin{aligned} z_t &= \rho z_{t-1} + (1 - \rho)\mu + \varepsilon_t \\ \text{where } \rho &\in [0, 1), \quad \mu \geq 0, \quad \text{and } \varepsilon_t \sim N(0, \sigma) \end{aligned} \quad (\text{T.1.10})$$

This implies that the shock process  $e^{z_t}$  is lognormally distributed  $LN(\rho z_t + (1 - \rho)\mu, \sigma)$ . The RBC literature calibrates the parameters on the shock process (T.1.10) to  $\rho = 0.95$  and  $\sigma = 0.4946$  for annual data.

For data in which one period is 30 years, we have to recalculate the analogous  $\tilde{\rho}$  and  $\tilde{\sigma}$ .

$$\begin{aligned} z_{t+1} &= \rho z_t + (1 - \rho)\mu + \varepsilon_{t+1} \\ z_{t+2} &= \rho z_{t+1} + (1 - \rho)\mu + \varepsilon_{t+2} \\ &= \rho^2 z_t + \rho(1 - \rho)\mu + \rho \varepsilon_{t+1} + (1 - \rho)\mu + \varepsilon_{t+2} \\ z_{t+3} &= \rho z_{t+2} + (1 - \rho)\mu + \varepsilon_{t+3} \\ &= \rho^3 z_t + \rho^2(1 - \rho)\mu + \rho^2 \varepsilon_{t+1} + \rho(1 - \rho)\mu + \rho \varepsilon_{t+2} + (1 - \rho)\mu + \varepsilon_{t+3} \\ &\vdots \\ z_{t+j} &= \rho^j z_t + (1 - \rho)\mu \sum_{s=1}^j \rho^{j-s} + \sum_{s=1}^j \rho^{j-s} \varepsilon_{t+s} \end{aligned}$$

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<sup>8</sup>Estimates of the coefficient of relative risk aversion  $\gamma$  mostly lie between 1 and 10. See ?, ?, ?, ?, ?, and ?.

With one period equal to thirty years  $j = 30$ , the shock process in our paper should be:

$$z_{t+30} = \rho^{30} z_t + (1 - \rho) \mu \sum_{s=1}^{30} \rho^{30-s} + \sum_{s=1}^{30} \rho^{30-s} \varepsilon_{t+s} \quad (\text{T.2.3})$$

Then the persistence parameter in our one-period-equals-thirty-years model should be  $\tilde{\rho} = \rho^{30} \approx 0.2146$  and the unconditional mean should be  $\tilde{\mu} = \mu \sum_{s=1}^{30} \rho^{30-s} = 0$ . Define  $\tilde{\varepsilon}_{t+30} \equiv \sum_{s=1}^{30} \rho^{30-s} \varepsilon_{t+s}$  as the summation term on the right-hand-side of (T.2.3). Then  $\tilde{\varepsilon}_{t+30}$  is distributed:

$$\tilde{\varepsilon}_{t+30} \sim N\left(0, \left[\sum_{s=1}^{30} \rho^{2(30-s)}\right]^{\frac{1}{2}} \sigma\right)$$

Using this formula, the annual persistence parameter  $\rho = 0.95$ , and the annual standard deviation parameter  $\sigma = 0.4946$ , the implied thirty-year standard deviation is  $\tilde{\sigma} \approx 1.5471$ . So our shock process should be,

$$z_t = \tilde{\rho} z_{t-1} + (1 - \rho) \tilde{\mu} + \tilde{\varepsilon}_t \quad \forall t \quad \text{where} \quad \tilde{\varepsilon} \sim N(0, \tilde{\sigma})$$

where  $\tilde{\rho} = 0.2146$  and  $\tilde{\sigma} = 1.5471$ . We arbitrarily choose  $\mu = \tilde{\mu} = 0$ . However, we could have also chosen  $\mu$  and the corresponding  $\tilde{\mu}$  to his a median wage target.

Lastly, we set the size of the promised transfer  $\bar{H}$  to be 32 percent of the median real wage. This level of transfers is meant to approximately match the average per capita real transfers in the United States to the average real wage in recent years. We get the median real wage by simulating a time series of the economy until it hits the shut down point, and we do this for 3,000 simulated time series. We take the median wage from those simulations. In order to reduce the effect of the initial values on the median, we take the simulation that lasted the longest number of periods before shutting down and remove the first 10 percent of the longest simulation's periods from each simulation for the calculation of the median.



## **T-3    Truncated Normal Distribution**

Put a description of the properties of the truncated normal distribution here and how it interacts with the total factor productivity process.

## T-4 Other Results

In addition to the reduced endowment comparisons of Tables 4 and 5 to the Blanchard replication results in Table 3, I also show how a different type of increased risk affects the percent welfare increase results from Table 3. Tables 6 and 7 hold the endowment  $x_1$  constant at the original level and increase the standard deviation of the TFP shock by 5 percent and 10 percent respectively. In each case, we reduce the mean of the shock  $\mu$  by the corresponding amount that keeps the expected value of the shock  $E[e^{z_{t+1}}]$  constant. This mean-preserving spread increases risk in the economy while holding average values relatively constant.

**Table 6: Percent change in average lifetime utility from increased transfer  $\bar{H}$ : variable  $\mu$  as a function of  $E[R_{t+1}]$ ,  $\sigma = 1.05\sigma_{orig}$**

		Linear production $\varepsilon = \infty$			Cobb-Douglas $\varepsilon = 1$		
		average $\bar{R}$ (annual)			average $\bar{R}$ (annual)		
		-0.020	-0.005	0.010	-0.020	-0.005	0.010
average	0.00	-0.42%	-0.42%	n/a	-0.25%	-0.25%	n/a
$R_t$	0.02	-0.24%	-0.24%	-0.24%	-0.18%	-0.18%	-0.17%
(annual)	0.04	-0.14%	-0.14%	-0.14%	-0.14%	-0.13%	-0.13%

\* NOTE: The upper left element of each 3-by-3 set of percent changes in welfare is labeled “n/a” because that combination of average risky rate and average riskless rate implies a negative spread  $\text{avg. } R_t < \text{avg. } \bar{R}_t$ , which is not possible in equilibrium given equation (18). Averages calculated as average over 15 simulated time series of 25 periods each.

**Table 7: Percent change in average lifetime utility from increased transfer  $\bar{H}$ : variable  $\mu$  as a function of  $E[R_{t+1}]$ ,  $\sigma = 1.10\sigma_{orig}$**

		Linear production $\varepsilon = \infty$			Cobb-Douglas $\varepsilon = 1$		
		average $\bar{R}$ (annual)			average $\bar{R}$ (annual)		
		-0.020	-0.005	0.010	-0.020	-0.005	0.010
average	0.00	-0.42%	-0.42%	n/a	-0.25%	-0.25%	n/a
$R_t$	0.02	-0.24%	-0.24%	-0.24%	-0.19%	-0.18%	-0.18%
(annual)	0.04	-0.14%	-0.14%	-0.14%	-0.14%	-0.14%	-0.13%

\* NOTE: The upper left element of each 3-by-3 set of percent changes in welfare is labeled “n/a” because that combination of average risky rate and average riskless rate implies a negative spread  $\text{avg. } R_t < \text{avg. } \bar{R}_t$ , which is not possible in equilibrium given equation (18). Averages calculated as average over 15 simulated time series of 25 periods each.

As a check that the model’s theory is correctly coded and solved, the results in the right-side 3-by-3 panels of Tables 6 and 7 are identical to the right-side 3-by-3 panel of Table 3. The mean preserving spread of the TFP shock does not change the equilibrium results when production is linear. In the Cobb-Douglas cases, the mean

preserving increase in risk of either a 5-percent increase in the TFP standard deviation in Table 6 or a 10-percent increase in standard deviation in Table 7 have almost no effect on the welfare changes. The results in the left-side 3-by-3 panels of Tables 6 and 7 are different from Table 3, and the welfare changes decline monotonically as  $\sigma$  increases. But the changes are very small.