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Disaster Risk and Business Cycles†

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Motivated by the evidence that risk premia are large and countercyclical, this paper studies a tractable real business cycle model with a small risk of economic disaster, such as the Great Depression. An increase in disaster risk leads to a decline of employment, output, investment, stock prices, and interest rates, and an increase in the expected return on risky assets. The model matches well data on quantities, asset prices, and particularly the relations between quantities and prices, suggesting that variation in aggregate risk plays a significant role in some business cycles. (*JEL* E13, E32, E44, G32)

The empirical finance literature has provided substantial evidence that risk premia vary over time, and that they are countercyclical.[1] Yet, standard business cycle models such as the real business cycle model, or the dynamic stochastic general equilibrium (DSGE) models used for monetary policy analysis, largely fail to replicate the level, the volatility, and the countercyclicality of risk premia. In these models, the variation in expected returns is entirely driven by variation in the risk-free interest rate. Is this a significant limitation of macroeconomic models? Do risk premia matter for macroeconomic dynamics?

To tackle this question requires a business cycle model where risk premia are time-varying. This paper builds on the work of Rietz (1988), Barro (2006), and Gabaix (2012), and introduces a tractable real business cycle model with a small risk of a large macroeconomic shock, an economic "disaster," such as the Great Depression. I model a disaster as a combination of permanent and transitory shocks to productivity, and a depreciation shock to capital, and show that this simple approach allows replicating accurately the response of consumption to a disaster, as estimated by Barro et al. (2011). An increase in the disaster probability affects the economy by lowering expectations, and by increasing risk. Because investors are risk averse, this higher risk leads to higher risk premia, and has significant implications both for business cycles and for asset prices: stock prices fall, employment and output contract, and investment especially declines. Demand for precautionary savings increases, leading the yield on less risky assets to fall, while expected excess returns

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[1] See Cochrane (2007) for a recent overview.

on risky securities increase. These dynamics occur in the absence of any change in total factor productivity.

The first contribution of this paper is to provide theoretical results characterizing the effect of disaster risk, in the special case of instantaneous and permanent disasters. When the probability of disaster is constant, macroeconomic quantities are exactly identical to those implied by a model without disasters, but a different discount factor β, that reflects the lower risk-adjusted return on capital. This "observational equivalence" (in a sample without disasters) is reminiscent of the numerical analysis of Tallarini (2000), who found that macroeconomic dynamics are essentially unaffected by the amount of risk or the degree of risk aversion. The Tallarini equivalence result holds exactly in my framework if the IES equals unity. On the other hand, when the probability of disaster is stochastic, a shock to this probability is observationally equivalent to a preference shift (i.e., a shock to the Euler equation). This provides an alternative interpretation of these shocks, which are an important part of DSGE models, and relates them explicitly to the risk premium, justifying the label "equity premium shocks" introduced by Smets and Wouters (2003). More importantly, this result shows that changes in aggregate risk can induce business cycles, if the IES differs from unity.

The second contribution of the paper is to demonstrate that, quantitatively, this parsimonious model matches key asset pricing facts while doing better than the RBC model in accounting for quantities. This is important since many asset pricing models which are successful in endowment economies do not generalize well to production economies.[2] Most interestingly, the model matches well the relations between macroeconomic aggregates (such as investment or output) and asset prices (such as expected returns, the price-to-book ratio, or the VIX index). As is well known, this connection between prices and quantities is problematic for most macroeconomic models. To my knowledge, this paper is the first to generate an empirically reasonable connection between risk premia and output or investment. Risk premia are important to understand, for instance, why investment is often low despite low riskless interest rates—the relevant user cost of capital, the expected return on capital, may well be high if the riskless interest rate is low precisely because of high disaster risk.

Overall, the results of this paper suggest an important role for time-varying risk in accounting for business cycles and asset prices. This result obtains in the context of a model which matches well data on prices, quantities, and the relations between quantities and prices, which in itself is a useful contribution.[3]

One possible interpretation of disaster risk is that it is a rational expectation. For instance, during the recent financial crisis, many commentators, including well-known macroeconomists, have highlighted the possibility that the US economy might fall into another Great Depression.[4] The paper demonstrates that even a small

[2] As explained in Jermann (1998); Lettau and Uhlig (2000); and Kaltenbrunner and Lochstoer (2010), it is often difficult to generate endogenously a large and time-varying market price of risk in a production economy.

[3] The model does not generate a large volatility of the return on unlevered capital, but it shows, however, that given a reasonable formulation for leverage, the model replicates a variety of empirical relationships.

[4] For instance: Greg Mankiw: "Looking back at [the great Depression], it's hard to avoid seeing parallels to the current situation. (…) Like Mr. Blanchard at the I.M.F., I am not predicting another Great Depression. But you should take that economic forecast, like all others, with more than a single grain of salt." (*New York Times*, October 25, 2008); Robert Barro: "… there is ample reason to worry about slipping into a depression. There is a roughly one-in-five chance that US GDP and consumption will fall by 10 percent or more, something not seen since the

probability of this scenario has in itself, a substantial effect on investment and stock returns. An alternative, "behavioral" interpretation of the model is that the time-varying disaster probability captures investors' beliefs, which may be excessively volatile. More generally, time-varying disaster risk captures the apparent oscillation between periods of optimism, where expected growth is high and uncertainty is low (such as the Great Moderation), versus periods of low expected growth and high uncertainty (such as 2008:IV).[5] This interpretation of the model is consistent with the view that some asset price variation ("bubbles," "animal spirits") is not obviously related to current or future productivity, but has a significant macroeconomic effect.

Introducing time-varying risk requires solving a model using nonlinear methods, i.e., going beyond the first-order approximation and considering higher order terms in the Taylor expansion. Researchers disagree on the importance of these higher order terms, and a fairly common view is that they are irrelevant for macroeconomic quantities. Lucas (2003), in his presidential address, summarizes:

> *Tallarini uses preferences of the Epstein-Zin type, with an intertemporal substitution elasticity of one, to construct a real business cycle model of the US economy. He finds an astonishing separation of quantity and asset price determination: The behavior of aggregate quantities depends hardly at all on attitudes toward risk, so the coefficient of risk aversion is left free to account for the equity premium perfectly.* [6]

My results show, however, that when the risk is large and varies over time, risk aversion affects macroeconomic dynamics in a significant way.

The paper is organized as follows: the rest of the introduction reviews the literature. Section I presents the model, and Section II provides a theoretical analysis in the case of instantaneous and permanent disasters. Section III parametrizes the model and studies its quantitative implications. Section IV discusses some extensions and robustness. Section V concludes. An online Appendix provides some additional robustness, and details the numerical computation and data.

Related Literature.—This paper is mostly related to four strands of literature. Starting with Jermann (1998) and Tallarini (2000), researchers have tried to connect business cycle models and asset return data.[7] Many of these papers consider only productivity shocks, and study the mean and standard deviations of return. My

early 1930s." (*Wall Street Journal*, March 4, 2009); and Paul Krugman: "This looks an awful lot like the beginning of a second Great Depression." (*New York Times*, January 4, 2009).

[5] In particular, researchers often interpret the 2008–2009 financial crisis as a "credit crunch" driven by a tightening of borrowing constraints or an increased financial intermediation wedge. This paper proposes an alternative mechanism: the high aggregate uncertainty had, in itself, a significant effect on investment and stock prices. Because both mechanisms work through the Euler equation, they have similar implications. The aggregate risk explanation has the advantage that it is consistent with the patterns observed in asset markets. More work is needed to distinguish these two plausible—and nonexclusive—interpretations.

[6] Tallarini (2000) actually picks the risk aversion coefficient to match the Sharpe ratio of equity. Since return volatility is very low in his model, the equity premium is much smaller in his model than in the data.

[7] A nonexhaustive list includes Boldrin, Christiano, and Fisher (2001); Lettau and Uhlig (2000); Kaltenbrunner and Lochstoer (2010); Fernández-Villaverde et al. (2010); Campanale et al. (2010); Croce (2010); Kuehn (2008); Uhlig (2007); Jaccard (2008); Dew-Becker (2011); Kung and Schmid (2011); Nezafat and Slavík (2011); and Rudebusch and Swanson (2012).

paper also considers disaster risk shocks, and goes beyond these moments to study the cyclical variation of risk premia, and the correlations between financial variables and macroeconomic quantities. Moreover, in contrast to my paper, many of these studies abstract from employment, a critical business cycle variable. Another important success is that the risk-free rate volatility is in line with the data. However, a limitation of my framework is that it relies on leverage to generate volatile cash flows, i.e., it does not address the volatility of the unlevered return on capital.

Second, the paper draws on recent research arguing that rare event risk can account for observed asset prices (Rietz 1988; Barro 2006; Gabaix 2012, 2011; Gourio 2008a, b; Wachter 2011; Weitzman 2007; and the criticisms of Julliard and Ghosh 2008 and Backus, Chernov, and Martin 2011).[8] This work is limited to endowment economies, and hence does not consider the feedback from time-varying risk to macroeconomic aggregates: in these models, disaster probability and output are assumed to evolve independently, so that risk premia are uncorrelated with the level of output, unlike the data. My model generates this correlation endogenously. Mendoza (2010) shows that collateral constraints can endogenously generate rare and deep recessions.

Third, an increase in disaster probability leads to an increase in risk, and hence naturally relates to the fast-growing literature on uncertainty shocks. Bloom (2009), and Bloom, Floetotto, and Jaimovich (2010) build models of heterogeneous firms facing fixed and linear costs to adjusting capital or labor, and show that a temporary increase in the (idiosyncratic or aggregate) variance of productivity shocks leads firms to freeze hiring, firing, and investment, leading to a recession and a reduction in endogenous aggregate TFP. My model also generates a recession in response to higher uncertainty, but there are several differences. The increase in disaster probability increases aggregate uncertainty about both productivity and depreciation. This higher aggregate uncertainty leads risk-averse consumers to invest less in risky capital; hence the mechanism is very different. Last, my model does not generate any change in TFP, which is attractive since some recessions, such as the recent financial crisis, occur without significant change in TFP. One important difference with most of the literature on uncertainty shocks is the focus on asset prices and risk premia. Recent estimations of DSGE models sometimes incorporate stochastic volatility (e.g., Justiniano and Primiceri 2008), but usually the volatility shock has little direct effect on business cycles; an exception is Fernández-Villaverde et al. (2011) who focus on a small open economy which faces large stochastic volatility in the interest rate.

Finally, the capital depreciation shock that I introduce has also been used by various authors, including Gertler and Karadi (2011) in a model with financial frictions, as well as Furlanetto and Seneca (2011) and Liu, Waggoner, and Zha (2011), and is also related to investment specific shocks (Justiniano, Primiceri, and Tambalotti 2010), as I discuss below.

[8] Gabaix (2012, 2011) independently obtained Propositions 1 and 2, and develops a macroeconomic framework where variation in the probability of disaster has no macroeconomic effect. In contrast, my paper uses the standard real business cycle model, and shows that a shock to the probability of disaster is equivalent to a preference shock (Proposition 3) and hence has a macroeconomic effect. As a result, my paper generates an empirically compelling correlation between asset prices or returns and macroeconomic quantities.

I. The Model

This section first presents the model setup, which extends the standard real business cycle framework by introducing time-varying disaster risk and recursive preferences. Next, I characterize the equilibrium and define asset prices.

A. *Model Setup*

The representative consumer has recursive preferences (Epstein and Zin 1989):

$$V_t = \left(U_t^{1-\psi} + \beta E_t\left(V_{t+1}^{1-\gamma}\right)^{\frac{1-\psi}{1-\gamma}}\right)^{\frac{1}{1-\psi}}, \tag{1}$$

where the utility index U_t depends on consumption C_t as well as hours worked N_t, and takes the following standard Cobb-Douglas form, consistent with balanced growth:

$$U_t = u(C_t, N_t) = C_t(1 - N_t)^{\upsilon}.$$

For this specification, γ is the risk aversion coefficient and the parameter ψ is inversely related to elasticity of substitution (IES). Specifically, the IES is $1/\hat{\psi}$, where $\hat{\psi} = 1 - (1 + \upsilon)(1 - \psi)$, and it is larger than unity if and only if $\psi < 1$.[9]

Firms produce output using a Cobb-Douglas production function:

$$Y_t = K_t^{\alpha}(z_t N_t)^{1-\alpha}, \tag{2}$$

where z_t is productivity, which is decomposed into a permanent component $z_{p,t}$ and a transitory component $z_{r,t}$:

$$\log z_t = \log z_{p,t} + \log z_{r,t}, \tag{3}$$

and each of these components follows an exogenous stochastic process described below. Capital is accumulated according to

$$K_{t+1} = (1 - \delta)K_t + \phi\left(\frac{I_t}{K_t}\right)K_t, \tag{4}$$

where ϕ is an increasing and concave function, which curvature captures physical adjustment costs. The resource constraint is

$$C_t + I_t = Y_t. \tag{5}$$

To describe the dynamics of shocks, it is useful to distinguish between disasters and "normal times" (i.e., no disasters).

[9] The IES is not equal to $1/\psi$ because U_t is not homogeneous of degree one. An alternative, equivalent definition of preferences starts from $U_t = C_t^{\lambda}(1 - N_t)^{1-\lambda}$, in which case the IES is $1/\psi$, and risk aversion is $\lambda\gamma$ (Swanson 2012).

Normal Times.—In normal times, the permanent component of productivity evolves according to a random walk with drift,

$$\log z_{p,t} = \log z_{p,t-1} + \mu + \varepsilon_t,$$

where ε_t is the usual, normally distributed "small shock" of real business cycle theory: ε_t i.i.d. $N(0, \sigma_\varepsilon^2)$. The transitory component of productivity reverts to zero:

$$\log z_{r,t} = \rho_z \log z_{r,t-1},$$

with $\rho < 1$. As a result, an economy that has not had a disaster in the recent past will have $\log z_{r,t} \simeq 0$.[10]

Disasters.—The economy switches from "normal times" to "disaster states" with probability p_t each period. Once it has entered the disaster state, the economy remains there the next period with probability q. The disaster is modelled as a combination of a productivity shock and a "depreciation shock" to the capital stock (or "capital quality shock"). I discuss the interpretation in detail below. First, in each period while the economy is in a disaster state, a "depreciation shock" ξ_{t+1} hits the capital stock, where ξ_{t+1} is i.i.d. $N(\mu_\xi - \frac{1}{2}\sigma_\xi^2, \sigma_\xi^2)$. Hence, the law of motion for capital accumulation during disasters is

$$K_{t+1} = \left((1 - \delta) K_t + \phi\left(\frac{I_t}{K_t}\right) K_t\right) e^{\xi_{t+1}}.$$

Second, productivity is affected by both a shock to the transitory component and a shock to the permanent component. While previous research has largely focused on the case where disasters are instantaneous and permanent (i.e., there is no transitory component), the data clearly favor a specification where disasters occur over several years and are followed by partial recoveries (Gourio 2008a, Barro et al. 2011). To capture these features, I use the flexible specification introduced by Barro et al. (2011). Section III discusses how to map their empirical estimates into parameters for my model. Each period while the economy is in a disaster state, the permanent component of productivity is affected by a factor θ_t i.i.d. $N(\mu_\theta - \frac{1}{2}\sigma_\theta^2, \sigma_\theta^2)$:

$$\log z_{p,t} = \log z_{p,t-1} + \mu + \varepsilon_t + \theta_t.$$

The transitory component is affected by a different factor φ_t, which is i.i.d. $N(\mu_\varphi - \frac{1}{2}\sigma_\varphi^2, \sigma_\varphi^2)$:

$$\log z_{r,t} = \rho_z \log z_{r,t-1} + \varphi_t - \theta_t.$$

[10] For simplicity I assume that the small, normally distributed shock ε_t affects productivity permanently; alternatively one may assume that its effect is transitory, leading to $\log z_{r,t} = \rho \log z_{r,t-1} + \varepsilon_t$ instead. This has little effect on the results.

This implies that as long as the economy is in a disaster state, productivity is hit by a shock that has a short-run effect ϕ_t, but then recovers at a speed governed by ρ_z, to its long-run impact θ_t. This U-shape productivity pattern generates a large recession followed by a recovery. Because the shocks are normally distributed, in some cases the long-run effect of a disaster is actually positive, perhaps reflecting the effect of political or economic reforms in reaction to the disaster.

Finally, in contrast to Barro et al. (2011), I allow the probability of entering a disaster state p_t to be time-varying: specifically, p_t follows a Markov chain which approximates the following AR(1) process,

$$\log(p_t) = \rho_p \log(p_{t-1}) + (1 - \rho_p)\log\bar{p} + \varepsilon_t^p, \tag{6}$$

with ε_t^p i.i.d. $N(0, \sigma_p^2)$.

Model Summary.—Denote by x_t an indicator equal to one if there is a disaster ongoing at time t, and zero if not. We can then summarize the modeling assumptions succinctly:

$$\log z_{p,t} = \log z_{p,t-1} + \mu + \varepsilon_t + x_t\theta_t, \tag{7}$$

$$\log z_{r,t} = \rho_z \log z_{r,t-1} + (\varphi_t - \theta_t)x_t, \tag{8}$$

$$K_{t+1} = \left((1 - \delta)K_t + \phi\left(\frac{I_t}{K_t}\right)K_t\right)e^{x_{t+1}\xi_{t+1}}, \tag{9}$$

$$\Pr(x_{t+1} = 1 \mid x_t = 1) = \max(q, p_t), \quad \text{and} \quad \Pr(x_{t+1} = 1 \mid x_t = 0) = p_t. \tag{10}$$

Discussion of the Modeling Assumptions.—The incorporation of disaster risk is motivated by recent empirical work (Barro 2006, Barro and Ursúa 2008) that documents, using a cross-country panel, a long history of very large macroeconomic shocks, that are usually caused by wars or economic depressions. In a standard neoclassical model there are two natural approaches to model these macroeconomic disasters—as a reduction in total factor productivity, or as capital destruction. My formulation allows for both. TFP appears to play an important role during economic depressions (Kehoe and Prescott 2007). While economists do not understand well the sources of fluctuations in total factor productivity, large and persistent declines in TFP may be linked to poor government policies (e.g., confiscatory taxes or tariffs), or to disruptions in financial intermediation, if these lead to inefficient capital allocation.

Capital destruction is clearly realistic for wars or natural disasters, but obviously not for economic depressions. The assumption requires in this case a broader interpretation as a shock to the "quality" of capital. Perhaps it is not the physical capital but the intangible capital (customer and employee value) that is destroyed during prolonged economic depressions. Moreover, economic crises often lead to microeconomic volatility and large reallocation, implying that some specialized capital goods may become worthless. Finally, expropriation of capital may be equivalent to capital destruction, if the capital is taken away and not used as effectively.

The theoretical results of Section II show that the model mechanism requires two ingredients: (i) disasters are events with high marginal utility of consumption; (ii) the realized return on capital is low when a disaster hits. These assumptions are certainly realistic. Introducing a large TFP shock is the simplest way to obtain (i) in a neoclassical model, and introducing a depreciation shock is the simplest way to obtain (ii), but the results are likely robust to alternative modeling assumptions.[11]

B. *Equilibrium Characterization*

The equilibrium is characterized by three equations: first, the resource constraint (5); second, the standard labor market condition

$$-\frac{u_N(C_t,N_t)}{u_C(C_t,N_t)} = \frac{\upsilon C_t}{1-N_t} = W_t = (1-\alpha)\frac{Y_t}{N_t}; \tag{11}$$

and last the Euler condition

$$E_t\left(M_{t+1}R^K_{t+1}\right) = 1, \tag{12}$$

where the stochastic discount factor M_{t+1} is the marginal rate of substitution of the representative household, i.e., with recursive preferences:

$$M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\psi}\left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-\psi)\upsilon}\frac{V_{t+1}^{\psi-\gamma}}{E_t\left(V_{t+1}^{1-\gamma}\right)^{\frac{\psi-\gamma}{1-\gamma}}}, \tag{13}$$

and the return on capital is

$$R^K_{t+1} = e^{x_{t+1}\xi_{t+1}}\phi'\left(\frac{I_t}{K_t}\right)\left(\frac{1-\delta+\phi\left(\frac{I_{t+1}}{K_{t+1}}\right)}{\phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)} + \alpha\frac{Y_{t+1}}{K_{t+1}} - \frac{I_{t+1}}{K_{t+1}}\right). \tag{14}$$

This expression for the return on capital follows from standard Q-theory, adjusted for the depreciation shock ξ_{t+1}.[12]

Recursive Formulation.—I now set up a recursive formulation of the problem, which is used in Section II to prove analytical results, and in Section III to derive quantitative implications. The model has five state variables: capital K, the permanent and transitory components of technology z_p and z_r, the probability of disaster p, and the current disaster state $x \in \{0,1\}$. There are three shocks: the realization of disaster $x' \in \{0,1\}$, the draw of the new probability of disaster p', and the "small

[11] An alternative to depreciation shocks is to introduce steep adjustment costs and rely only on a large TFP shock. Since investment declines significantly when TFP falls, the price of capital (Tobin's Q) also falls, generating endogenously a low return on capital during disasters. It is somewhat difficult to calibrate the curvature of adjustment costs to generate this effect, and at the same time maintain realistic business cycle dynamics, so in the interest of simplicity I do not pursue this approach.

[12] For a proof, see for instance Kaltenbrunner and Lochstoer (2010).

TFP shock" ε'. In the case of a disaster realization, three further shocks determine the disaster size: φ', θ', and ξ'.

Denote $V(K, z_p, z_r, p, x)$ the value function of the household. Homogeneity of the utility function, production function, and adjustment costs, and the fact that the permanent component of technology follows a random walk, imply that we can write

$$V(K, z_p, z_r, p, x) = z_p g(k, z_r, p, x),$$

where $k = K/z_p$, and the function g satisfies

$$g(k, z_r, p, x)^{1-\psi} = \max_{c,i,N} u(c, N)^{1-\psi} + \beta E\left(e^{(1-\gamma)(\mu+\varepsilon'+x'\theta')} g(k', z_r', p', x')^{1-\gamma}\right)^{\frac{1-\psi}{1-\gamma}}, \tag{15}$$

subject to

$$c + i = k^{\alpha}(z_r N)^{1-\alpha}, \tag{16}$$

and

$$k' = \frac{e^{x'\xi'}\left((1 - \delta)k + \phi\left(\frac{i}{k}\right)k\right)}{e^{\mu+\varepsilon'+x'\theta'}}. \tag{17}$$

In this equation, $c = C/z_p$ and $i = I/z_p$ are consumption and investment detrended by the permanent component of technology, and the expectation is taken over x', p', ε', φ', θ', and ξ'. This homogeneity argument simplifies the problem substantially: it delivers some analytical results, and makes the numerical analysis simpler: first, k is stationary; second, the dimension of the state space is reduced.

C. *Asset Prices*

Given expression (13) for the stochastic discount factor M_{t+1}, it is straightforward to compute asset prices in this economy. The mapping between the data requires some discussion, first for interest rates, then for equity values. The price of a one-period real risk-free bond is $E_t(M_{t+1})$, however this asset may not have an observable counterpart: even government bonds often default during disasters.[13] Following Barro (2006), I model this default by assuming that in a disaster, government bonds have a recovery rate $r < 1$. The price of a T-bill (a one period government bond) is $Q_{1,t} = E_t\big(M_{t,t+1}(1 + x_{t+1}(r - 1))\big)$. The term structure of government bonds is derived using the standard recursion: $Q_{n,t} = E_t\big(M_{t+1}(1 + x_{t+1}(r - 1))Q_{n-1,t+1}\big)$, with $Q_{0,t} = 1$. This assumes that in a disaster all maturities are affected proportionately.

Turning to equity values, start with the value P_t of the capital stock. This value satisfies

$$P_t = E_t\big(M_{t+1}(D_{t+1} + P_{t+1})\big), \tag{18}$$

[13] Empirically, default often takes the form of high inflation which reduces the real value of nominal government debt.

where $D_t = Y_t - w_t N_t - I_t$ is the net payout of the representative firm. The asset return is then $R_{t+1} = (P_{t+1} + D_{t+1})/P_t$, which equals the return on capital defined in equation (14), as shown by Hayashi (1982). This model suffers from a well-known deficiency: the volatility of profits in the model equals the volatility of output; as a result the cash flows implied by the model are not as risky (volatile and procyclical) as the data, making it difficult to replicate the volatility of equity returns in the model. To make cash flows more risky, I follow the literature and incorporate leverage in the model. Each period, the firm issues bonds in a proportion ζ to its capital stock; these bonds have a maturity of 10 years (similar to the data, and to the lifetime length of capital), and this debt is assumed to default in a disaster, with a recovery rate $r_c < r$.[14] In Section IV, I show that the results are not highly sensitive to the details of this leverage policy. Leverage here should be interpreted not only as financial leverage, but also operating leverage (e.g., fixed costs and labor contracts).[15] Because the Modigliani and Miller theorem holds, the only effect of leverage is to modify the payout process and subsequently the properties of equity returns. Hence, all the other results of the paper are unaffected by this assumption.

II. Theoretical Results

While the model incorporates a rich specification of disasters, which is important to reproduce quantitatively the dynamics of consumption, output and other macro variables during disasters, the key mechanism of the paper can be studied analytically in a simple special case, when disasters are instantaneous and have only permanent effects. Under this assumption, Proposition 1 characterizes the dynamics of quantities and returns following a disaster. Next, Propositions 2, 3, 4, and 5 discuss the effect of disaster risk on quantities and expected returns.

Formally, I will use the following assumptions:

ASSUMPTION 1: *The effect of a disaster on productivity is permanent:* $\varphi_t = \theta_t$, *and hence* $z_{r,t} = 1$, *for all* $t \geq 0$.

ASSUMPTION 2: *Disasters are not intrinsically persistent:* $q = 0$, i.e., $\Pr(x_{t+1} = 1) = p_t$, *for all* $t \geq 0$.

Under Assumptions 1 and 2, x and z_r are not state variables, and equation (15) simplifies to

$$(19)\qquad g(k,p)^{1-\psi} = \max_{c,i,N} u(c,N)^{1-\psi} + \beta E\left(e^{(1-\gamma)(\mu+\varepsilon'+x'\theta')} g(k',p')^{1-\gamma}\right)^{\frac{1-\psi}{1-\gamma}},$$

[14] Given this exogenously specified recovery rate r_c, it is possible that, in a very large disaster, the firms' liabilities exceed the value of its assets. For simplicity, the calculations of the paper do not take into account these bankruptcies. See Gourio (2011) for an analysis that explicitly incorporates limited liability and a capital structure choice with endogenous default.

[15] A previous version of the paper followed a specification of leverage that is common in the asset pricing literature (e.g., Abel 1999, Bansal and Yaron 2004, Wachter 2011) and defined the equity as a claim to "dividends" $D_t = Y_t^{\lambda}$, where λ is the leverage parameter. This formulation implies that the growth rate of dividends is $\Delta \log D_t = \lambda \Delta \log Y_t$, which is more volatile than output assuming $\lambda > 1$. The results were similar to the ones reported in the paper, and are also similar if the "equity" is defined as a claim to C_t^{λ}.

subject to (16) and (17).

A further simplification arises when the depreciation shock and the permanent productivity shock are exactly equal.

ASSUMPTION 3: *The capital quality shock equals the permanent productivity shock:* $\theta_t = \xi_t$, *for all* $t \geq 0$.

PROPOSITION 1: *Under Assumptions* 1, 2, *and* 3, *a disaster leads productivity, consumption, investment, capital, and output to be instantaneously and permanently multiplied by a factor* $e^{\xi} = e^{\theta}$. *Hours are unaffected. The return on capital is multiplied by* $e^{\xi} = e^{\theta}$ *on impact, while the return on government bonds is multiplied by* $r < 1$.

PROOF:

Solving equation (19) yields the policy functions $c(k,p), i(k,p), N(k,p)$ and $y(k,p) = k^{\alpha} N(k,p)^{1-\alpha}$ which express the solution as a function of the probability of disaster p (the exogenous state variable) and the detrended capital k (the endogenous state variable). The detrended capital evolves according to the shocks $\varepsilon', x', p', \theta', \varphi', \xi'$ through equation (17); however since $\xi' = \theta' = \varphi'$, this equation simplifies to

$$k' = \frac{(1 - \delta)k + \phi\left(\frac{i(k,p)}{k}\right)k}{e^{\mu+\varepsilon'}},$$

i.e., k' is independent of the realization of disaster x'. As a result, the realization of a disaster does not affect c, i, N, y, since k is unchanged, and hence it leads consumption $C = cz$, investment $I = iz$, output $Y = yz$, and technology z to be multiplied by the same factor $e^{\xi'} = e^{\theta'}$ on impact. Furthermore, the disaster has no additional effect after this first period since endogenous dynamics are captured by k, which is unaffected. The statement regarding returns follows from equation (14): the investment-capital ratio and output-capital ratios are unaffected by the disaster, hence the only effect of the disaster is to multiply R^K_{t+1} by the factor $e^{\xi_{t+1}}$, and similarly for the government bond returns.

Intuitively, the economy simply shifts from one steady-state to a lower steady-state. The lower TFP implies a lower desired capital-labor ratio, and the preferences imply that the steady-state level of hours worked is unaffected by a change in TFP. Given $\theta = \xi$, the amount of capital destruction is exactly what is required for the economy to reach its new steady-state instantaneously, and there are no transitional dynamics. In contrast, when $\theta \neq \xi$, a disaster has both impact effects and transitional dynamics. For instance, a large capital destruction without a change in productivity leads to high investment and a recovery as the economy converges back to its initial steady-state. Inversely, a productivity decline without capital destruction leads to a persistently low level of investment as the economy adjusts gradually to reach its new steady-state. The benchmark model analyzed numerically in Section III allows for these realistic scenarios, but the key mechanism does not rely on these dynamics.

Note that under Assumptions 1, 2, and 3, there is a single shock characterizing the disaster, since $\theta = \xi = \varphi$. To guarantee that this shock is "bad," we make the following assumption.

ASSUMPTION 4: *Either* $\mu_\xi < 0$, *or* $\sigma_\xi > 0$, *or both.*

Assumption 4 requires either that a disaster shifts the mean of future ξ, θ, φ down, or that it increases uncertainty over these variables, or both. Our key results hold regardless, as we discuss below.

PROPOSITION 2: *Under Assumptions* 1, 2, 3, *and if the probability of disaster p is constant*, *the policy functions* $c(k)$, $i(k)$, $N(k)$, *and* $y(k)$ *are the same as in a model without disasters* $(p = 0)$, *but with a different discount factor* β^*, *where*

$$\beta^* = \beta E\left(e^{(1-\gamma)\xi' x'}\right)^{\frac{1-\psi}{1-\gamma}} = \beta\left(1 - p + p e^{(1-\gamma)(\mu_\xi - \frac{1}{2}\gamma\sigma_\xi^2)}\right)^{\frac{1-\psi}{1-\gamma}}.$$

Under Assumption 4, *the equivalent discount factor* β^* *is decreasing in p if and only if the IES is greater than unity*, *i.e.*, $\psi < 1$.

PROOF:

Following Proposition 1, note that k' is independent of the realization of disaster x'. Given that p is constant, we can simplify the Bellman equation (19):

$$\begin{aligned} g(k)^{1-\psi} &= \max_{c,i,N} u(c,N)^{1-\psi} + \beta E\left(e^{(1-\gamma)\xi' x'}\right)^{\frac{1-\psi}{1-\gamma}} E\left(e^{(1-\gamma)(\mu+\varepsilon')} g(k')^{1-\gamma}\right)^{\frac{1-\psi}{1-\gamma}}, \\ &= \max_{c,i,N} u(c,N)^{1-\psi} + \beta^* e^{\mu(1-\psi)} E\left(e^{(1-\gamma)\varepsilon'} g(k')^{1-\gamma}\right)^{\frac{1-\psi}{1-\gamma}}. \end{aligned}$$

This is the same Bellman equation as in a standard neoclassical model with discount rate β^*. The law of motion for detrended capital and the resource constraint are also the same. As a result, the policy functions $c(k)$,$N(k)$,$i(k)$, and $y(k)$ are the same as a standard neoclassical model. The fact that β^* is decreasing in p is easy to verify directly.

This result has several implications. First, in a sample where no disasters are realized, the quantities implied by the model (consumption, investment, hours, output, and capital) are *exactly* the same as those implied by the standard RBC model, provided that the discount factor is adjusted. In particular, the response of quantities to the standard TFP shock ε_t is unaffected. As can be anticipated, the model nevertheless generates a significant equity premium (see Proposition 5). This provides an example of model with large risk premia and reasonable business cycle dynamics, addressing the question studied by Jermann (1998) and Boldrin, Christiano, and Fisher (2001).[16] Second, this analytical result clarifies the numerical findings of Tallarini (2000), who found, in a model where the IES is unity, that increasing

[16] However, this model does not generate a large volatility of stock returns.

risk aversion has little effect on business cycle quantity dynamics, a finding often interpreted as "macroeconomists [can] safely go on ignoring finance" (Cochrane 2007, p. 297). In my model, if the IES is unity the equivalence of dynamics is an *exact* result: in this case, $\beta^* = \beta$, and disaster risk does not affect business cycle dynamics, despite affecting risk premia. Third, if the IES differs from unity, the discounting change will lead to a change in the steady-state—a point that will be clarified in Proposition 4.

The key intuition is that disaster risk leads to a lower risk-adjusted physical return on capital $E(R_K^{1-\gamma})^{\frac{1}{1-\gamma}}$. A lower risk-adjusted return makes people save less if and only if the IES is larger than unity. The decrease in the risk-adjusted return is driven by a combination of a first and second moment effect. On the one hand, a higher probability of disaster simply decreases the expected return on capital, because of the lower conditional mean of future productivity, and the higher expected depreciation. On the other hand, a higher probability of disaster increases the uncertainty, because (i) whether the disaster will hit is uncertain; (ii) conditional on the disaster occurring, its size ξ is uncertain. Both the first and second moment effects contribute to lowering the risk-adjusted return on investment. Some special cases are especially clear. First, if $p = 1$ and $\sigma_\xi = 0$, the disaster and its size are certain, so there is no uncertainty at all, and $\beta^* = \beta e^{(1-\psi)\mu_\xi}$. The presence of disasters lowers the effective discount factor, i.e., $\beta^* < \beta$, provided that $\mu_\xi < 0$ and $\psi < 1$ (and independent of risk aversion). This is the first-moment effect of disasters. Second, suppose that $p = 1$, and $\mu_\xi = 0$, so that the disaster is certain, but its size is uncertain, and has mean zero—a pure "uncertainty shock." In this case, $\beta^* = \beta e^{-\frac{1}{2}\gamma(1-\psi)\sigma_\xi^2}$, which is lower than β if $\sigma_\xi > 0$ and $\psi < 1$: the uncertainty leads to lower investment if people are risk averse. This is the second moment effect of disaster size. Finally, to illustrate the second moment effect of the disaster realization risk, suppose that $p < 1$ but $\sigma_\xi = 0$, so that the disaster size is known, but its realization is uncertain; in this case $\beta^* = \beta(1 - p + pe^{(1-\gamma)\mu_\xi})^{\frac{1-\psi}{1-\gamma}}$, which (assuming positive risk aversion) is lower than $\beta(1 - p + pe^{\mu_\xi})^{1-\psi}$, the equivalent β^* if the household was certain to receive a shock with same expected size $1 - p + pe^{\mu_\xi}$. Hence, in this model there is a direct equivalence between an uncertainty shock and a pure first moment shock.[17]

Proposition 2 can be extended to the case where p is itself stochastic.

PROPOSITION 3: *Under Assumptions* 1, 2, *and* 3, *and if p follows a stationary Markov process*, *the policy functions* $c(k,p)$, $i(k,p)$, $N(k,p)$, *and* $y(k,p)$ *are the same as in a model without disasters* $(p = 0)$, *but where the discount factor* β *follows an exogenous stationary process*, $\beta(p) = \beta\left(1 - p + pe^{(1-\gamma)\left(\mu_\xi - \frac{1}{2}\gamma\sigma_\xi^2\right)}\right)^{\frac{1-\psi}{1-\gamma}}$. *Moreover*, *under Assumption* 4, β *is inversely related to p if and only if* $\psi < 1$.

[17] More generally, the agent is indifferent between the certainty of a shock μ_ζ and the disaster, if and only if

$$e^{(1-\gamma)\mu_\zeta} = 1 - p + pe^{(1-\gamma)(\mu_\xi - \frac{1}{2}\gamma\sigma_\xi^2)}.$$

It may seem counterintuitive that an uncertainty shock has the same implications as a first-moment shock. However, this reflects that both first and second moments affect the risk-adjusted return on investment (i.e., they work through the same channel in this case). Moreover, the higher risk is not observed in a sample that does not include disaster realizations, making it impossible to distinguish the two.

PROOF:

Assumptions 1–3 imply that k' is independent of x', and we can again simplify the expectation inside the Bellman equation (15):

$$g(k,p)^{1-\psi} = \max_{c,i,N} u(c,N)^{1-\psi} + \beta E\left(e^{(1-\gamma)x'\xi'}\right)^{\frac{1-\psi}{1-\gamma}} E\left(e^{(1-\gamma)(\mu+\varepsilon')} g(k',p')^{1-\gamma}\right)^{\frac{1-\psi}{1-\gamma}},$$

$$= \max_{c,i,N} u(c,N)^{1-\psi} + \beta(p)\, e^{\mu(1-\psi)} E\left(e^{(1-\gamma)\varepsilon'} g(k',p')^{1-\gamma}\right)^{\frac{1-\psi}{1-\gamma}},$$

i.e., the same equation (and constraints) as a real-business cycle model with time-varying β, but no disasters.

This result shows that time-varying risk of disaster has the same implications for quantities as a preference shock. It is well known that these shocks have a significant effect on macroeconomic quantities. Because $\beta(p)$ depends on risk aversion, this version of the model breaks the "separation theorem" of Tallarini (2000): when risk varies over time, risk aversion has an effect on quantities. Asset prices will also respond to this higher risk, generating correlations of risk premia and quantities.

A large number of DSGE models, such as Smets and Wouters (2003), also feature a shock affecting the Euler equation, often labeled an "equity premium shock." Recent empirical estimates, such as Galí, Smets, and Wouters (2010), suggest that this type of shock plays an important role during some episodes, for instance at the beginning of the recent financial crisis (2008:IV), but Chari, Kehoe, and McGrattan (2009) criticize these shocks for their lack of microfoundations. My model provides a simple microfoundation, that allows to tie these shocks to asset prices precisely, and justifies the name "equity premium shock."[18] While slightly different, the investment shocks emphasized by Justiniano, Primiceri, and Tambalotti (2010) are closely related, because they primarily affect the consumption-savings decision. Several models of financial frictions also lead to a reduced form Euler equation shock—a wedge between the interest rate faced by firms and households. For this reason, recent macroeconomic analyses of the 2008 financial crisis often feature such a shock (e.g., Christiano, Eichenbaum, and Rebelo 2011). The disaster risk shock has similar implications: people desire to save more in riskless assets, leading the riskless interest rate to fall (and potentially creating a zero-lower bound problem in a monetary economy). An advantage of my formulation is that investment falls sharply despite the low interest rate, because the risk premium rises.

The last two theoretical results clarify the effect of a change in p on quantities and returns. The case of a constant probability of disaster p, and no TFP shocks ($\sigma_\varepsilon = 0$) is especially simple, as the balanced growth path has a constant k_t, i_t, N_t, y_t, c_t.[19]

[18] Of course, my model is significantly simpler than the DSGE models discussed here, but I conjecture that the equivalence result holds, at least approximately, in many models.

[19] The level variables K_t, I_t, C_t, and Y_t grow at a constant trend μ, with random, infrequent disasters that affect these levels permanently by the factor e^{ξ}, but the ratios $k_t = K_t/z_t$, $i_t = I_t/z_t$, etc., are constant along this balanced growth path, which is reached asymptotically from any initial condition.

PROPOSITION 4: *Under Assumptions* 1–4, *if the probability of disaster p is constant*, *if* $\sigma_\varepsilon = 0$, *and* $\psi < 1$, *the balanced growth path values* k^*, i^*, N^*, y^*, c^* *are all decreasing in p.*

PROOF:

Equation (12) along the balanced growth path reads

$$\beta^* e^{-\mu\psi}\left(1 - \delta + \alpha\left(\frac{k^*}{N^*}\right)^{\alpha-1}\right) = 1,$$

and hence a higher p, leading to a lower β^*, implies a lower capital-labor ratio. Since $c^* = k^{*\alpha}N^{*1-\alpha} - (\delta + \mu)k^*$, equation (11) implies that

$$\frac{N^*}{1 - N^*} = \frac{1}{\upsilon}\frac{(1 - \alpha)}{1 - (\delta + \mu)\left(\frac{k^*}{N^*}\right)^{1-\alpha}},$$

and hence N^* falls. It follows that k^* falls, and so do i^*, y^*, and c^*.

All model quantities shift to a lower steady-state if disaster risk is increased, because higher investment risk reduces savings if the IES is larger than unity (a result similar to Angeletos 2007; in contrast, higher productivity risk typically increases savings). The sign of this effect depends on the IES, but its strength depends on the risk aversion γ. The last proposition analyzes returns.

PROPOSITION 5: *Under Assumptions* 1–4, *if the probability of disaster p is constant*, *and* $\sigma_\varepsilon = 0$, *the risk-free rate is decreasing in p and the expected excess return on capital is increasing in p*, *at least for p small enough.*

PROOF:

For simplicity, this proof assumes that the economy is on its balanced growth path (the results can be extended at a notational cost), with $k_t = k^*$. The stochastic discount factor is a function of the disaster realization and size, which are now the only shocks:

$$M(x',\xi') = \frac{\beta e^{-\mu\psi-\gamma x'\xi'}}{E\left(e^{(1-\gamma)\xi' x'}\right)^{\frac{\psi-\gamma}{1-\gamma}}},$$

and the return on capital is, along this balanced growth path:

$$R^K(x',\xi') = e^{x'\xi'}\left(1 - \delta + \alpha\left(\frac{k^*}{N^*}\right)^{\alpha-1}\right).$$

The first order condition $E(M(x',\xi')R^K(x',\xi')) = 1$ leads, after some manipulation, to

$$E\left(R^K(x',\xi')\right) = \frac{E\left(e^{\xi' x'}\right)}{\beta e^{-\mu\psi}E\left(e^{(1-\gamma)\xi' x'}\right)^{\frac{1-\psi}{1-\gamma}}},$$

and the risk-free rate is constant, equal to

$$R^f = \frac{E\left(e^{(1-\gamma)x'\xi'}\right)^{\frac{\psi-\gamma}{1-\gamma}}}{\beta e^{-\mu\psi} E\left(e^{-\gamma\xi' x'}\right)}.$$

Simple but tedious algebra shows that this is decreasing in p, if p is small enough. Last, the expected excess return is

$$\frac{E\left(R^K(x',\xi')\right)}{R^f} = \frac{E\left(e^{\xi' x'}\right)E\left(e^{-\gamma\xi' x'}\right)}{E\left(e^{(1-\gamma)\xi' x'}\right)},$$

which is increasing in p, if p is small enough.

These formulas are the same as those that arise in the endowment economy model of Barro (2006), because the consumption dynamics assumed in that paper are generated endogenously by my production economy. Hence applying a calibration similar to that of Barro (2006) will lead to the same (substantial) equity premium. An increase in the disaster probability lowers expected growth, and increases the risk at least if p is small.[20] This is why, for small p, the interest rate falls and the excess return on capital rises.[21]

Overall, the analytical results show that, provided that $\psi < 1$, an increase in disaster risk lowers investment, employment, output, and interest rates, while increasing excess returns. This suggests that the model can generate endogenously countercyclical risk premia. To evaluate this mechanism quantitatively, we now return to the general model.

III. Quantitative Results

This section first discusses the calibration, and shows that the model generates disaster dynamics that mimic the data. The section then explores the implications of variation in disaster risk for business cycle quantities, asset prices, and for the relations between asset prices and quantities (the central result of this paper). In general, the model cannot be solved analytically, leading me to resort to a numerical approximation.[22]

A. *Calibration*

Parameters are listed in Table 1. The time period is one quarter. A first set of parameters follows the business cycle literature $(\alpha, \delta, \upsilon, \beta)$. The parameters for the

[20] For p larger than 0.5, and σ_ξ small, the risk may decrease, since the disaster becomes more certain (recall that the variance of a binomial distribution is $p(1-p)$).

[21] The effect on the expected return on capital is generally ambiguous, because lower expected productivity lowers expected cash flows and reduces the expected return, while the higher risk premium tends to increase it. However, in the case of a pure uncertainty shock ($\mu_\xi = 0$), the first effect does not operate, so the expected return unambiguously rises with p. The benchmark model of Section III incorporates leverage, which makes equity more exposed to disaster risk than the capital stock is, amplifying the effect on expected returns.

[22] A nonlinear method is crucial to analyze the effect of time-varying risk. I use projection methods that approximate policy and value functions using Chebychev polynomials; see the online Appendix for details.

Table 1—Parameter Values for the Benchmark Model

Capital share	α	0.34	Mean permanent shock	μ_θ	−0.007
Depreciation rate	δ	0.02	Std. dev. permanent shock	σ_θ	0.092
Leisure preference	υ	2.33	Mean transitory shock	μ_φ	−0.055
Discount factor	β	0.999	Std. dev. transitory shock	σ_φ	0.041
Adjustment cost	η	1.7	Persistence of productivity	ρ_z	0.71
Trend growth of TFP	μ	0.005	Persistence of disaster state	q	0.914
Std. dev. of TFP shock	σ_ε	0.01	Persistence of $\log(p)$	ρ_p	0.90
Elasticity of substitution	$1/\hat{\psi}$	2	Std. dev. of $\log(p)$	$\sigma_p\big/\sqrt{1-\rho_p^2}$	2.80
Risk aversion	γ	3.8	Average prob. of disaster		0.72

Note: The distribution of transitory shocks is truncated to negative values, as in Barro et al. (2011).

TFP process, μ and σ_ε are obtained by a direct measurement of total factor productivity. Adjustment costs take a standard quadratic form, $\phi(x) = x - \eta(x - \delta - \mu)^2/2$, with the parameter $\eta = 1.7$ chosen to match the volatility of investment.[23] The risk aversion parameter γ is picked to replicate the mean equity premium, leading to a value of 3.8.

The intertemporal elasticity of substitution of consumption (IES) is set equal to 2. As Section II shows, an IES larger than unity is required for the model to generate that increases in disaster risk lead to declines in investment, and hence to make risk premia countercyclical, as in the data. Hence, a low IES would have counterfactual implications. There is of course a controversial debate surrounding the value of the IES.[24] I discuss the sensitivity of the model results to the IES (and to other parameters) in Section IV.

The first critical element of the calibration regards the frequency and size of disasters, which are characterized by the parameters $(q, \mu_\xi, \sigma_\xi, \mu_\varphi, \sigma_\varphi, \mu_\theta, \sigma_\theta, \rho_z, \overline{p})$. Barro et al. (2011) estimate a time series process for consumption using annual data for a large panel of countries, that explicitly allows for disaster states. My calibration strategy is to pick parameters so that my model, when time-aggregated to annual data, reproduces their estimated response of consumption during disasters, depicted in Figure 1. Some of their estimates can readily be mapped into my model; for instance, the average probability of entering the disaster is 0.72 percent per quarter. Similarly, their estimate of the permanent shock for consumption directly maps into the permanent shock to productivity $(\mu_\theta, \sigma_\theta)$, because in the neoclassical growth model, the long-run response of consumption equals the long-run response of productivity. The mapping is more complicated for the other parameters, since consumption in my model is endogenously less volatile and more persistent than productivity because of consumption smoothing. For parsimony, I first assume that the

[23] Because the function ϕ is such that adjustment costs are zero in steady-state, the average adjustment costs are small, around 0.02 percent of output. But adjustment costs nevertheless have a substantial effect on investment volatility.

[24] Most direct estimates using aggregate data find low numbers (e.g., Hall 1988, but see Section IVF), but this view has been challenged by several authors (see among others Bansal and Yaron 2004, Guvenen 2006, Gruber 2006, Mulligan 2004, Vissing-Jorgensen 2002, Fernández-Villaverde et al. 2010). As emphasized by Bansal and Yaron (2004), a low IES has the counterintuitive effects that higher expected growth lowers asset prices, and higher uncertainty increases asset prices.

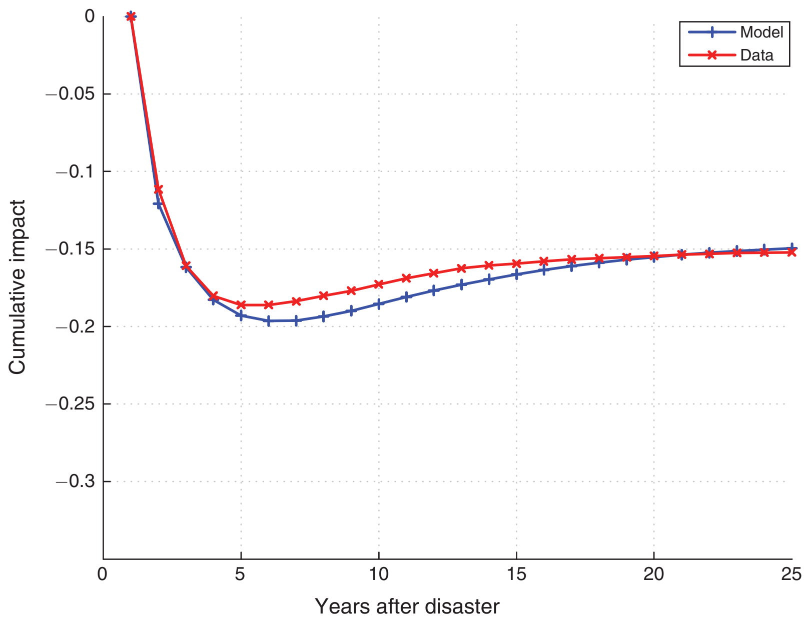

Figure 1. Impulse Response Function of Consumption to a Disaster Realization: Model versus Data (*Estimate of Barro et al. 2011*)

capital destruction shocks are equal to the permanent productivity shocks, $\xi_t = \theta_t$.[25] Next, the parameters $(q, \rho_z, \mu_\varphi, \sigma_\varphi)$ are picked so as to replicate the estimated response of consumption according to Barro et al. (2011); this is required to make productivity more volatile, and less persistent, than the consumption process estimated by these authors. (As in Barro et al. 2011, the distribution of transitory shocks φ_t is truncated to keep only negative values.) Last, I show in Section IVB, generating the large initial drop of consumption estimated by Barro et al. (2011) and the smaller reductions that follow requires that the initial productivity and capital shocks be twice larger than the shocks in subsequent periods. Summarizing, while the rich dynamic specification of disasters introduces several parameters, the calibration is fairly tightly constrained by the difficult target—the full path of the response of consumption, which is well replicated as shown in Figure 1. (Moreover, the variance around this average response is also approximately reproduced.) The next section further shows that the model's implications for other variables than consumption are also reasonable.[26]

The second critical element of the calibration is the persistence and volatility of movements in this probability of disaster, which appear in equation (6). While Berkman, Jacobsen, and Lee (2011) provide evidence that disaster risk is time-varying, it is difficult to offer an a priori estimate of these parameters. My strategy is to choose the unconditional standard deviation to approximately match the volatility

[25] A more general specification where ξ_t depends on θ_t and φ_t is of course possible, but has little additional implications. The results hold as long as there is a significant correlation between ξ_t and permanent productivity shocks.

[26] A reasonable concern is that the United States may face a lower risk of disaster than the rest of the world (the estimates of Barro et al. 2011 pool all the world data). The model implications survive with a smaller disaster size or probability, provided that the risk aversion is correspondingly adjusted to replicate the mean equity premium.

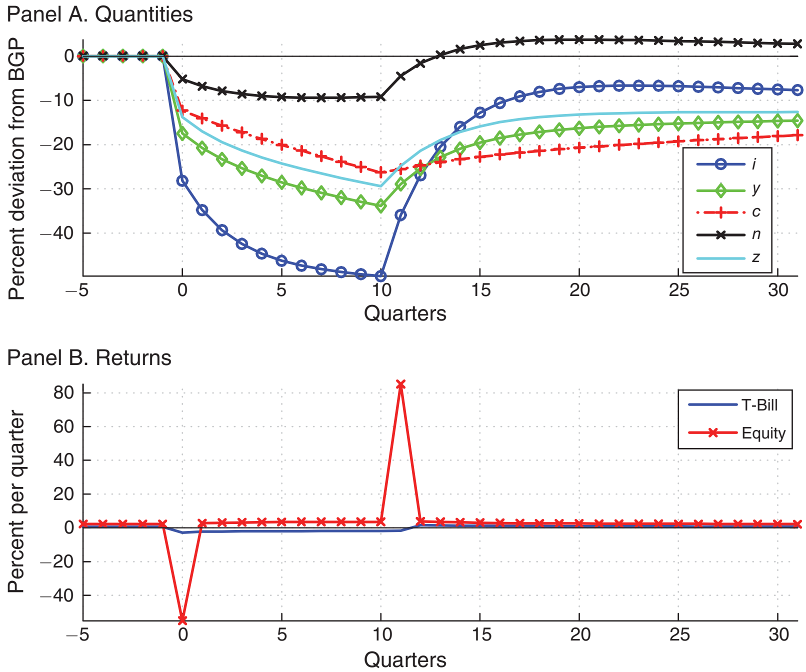

Figure 2. Response of Macroeconomic Quantities and Asset Returns to a Typical Disaster in the Model

of equity returns ($\sigma_p = 2.8$). I set $\rho_p = 0.90$, but the results are not highly sensitive to that value (see Section IVC).

Finally, consistent with Barro (2006), government recovery rates r are correlated with the size of disaster, and the default loss is 20 percent of the size of the capital shock, while corporate bonds default by 38 percent. The parameter ζ is set to 0.018: each quarter, firms issue a debt with market value 1.8 percent of the firms' assets, which implies an average leverage of around 50 percent. This level of leverage allows the model to generate the large stock market crashes observed during disasters. In the data, financial leverage is lower, around 30 percent. But, as discussed in Section IC, my interpretation of leverage is broader than simply financial leverage, and appears consistent with the large exposure of dividends and profits to disasters (Longstaff and Piazzesi 2004). Section IVC provides some sensitivity analysis.

B. *A Disaster Realization*

Figure 2 illustrates the response of the economy to a typical disaster, that lasts for 10 quarters, with realizations of θ_t and φ_t equal to the mean of their distributions. The large decline in total factor productivity and in the capital stock leads to a reduction in labor demand that further reduces output, despite the wealth effect that pushes labor supply up. Given that disasters are partly transitory, investment falls more than consumption by permanent income logic. The decline of investment is amplified by the higher riskiness of capital, which is hit by the shock ξ_t as long as the disaster continues. As productivity keeps falling, so do output and consumption, and the deviations from the initial balanced growth increase continuously. At some point the economy exits

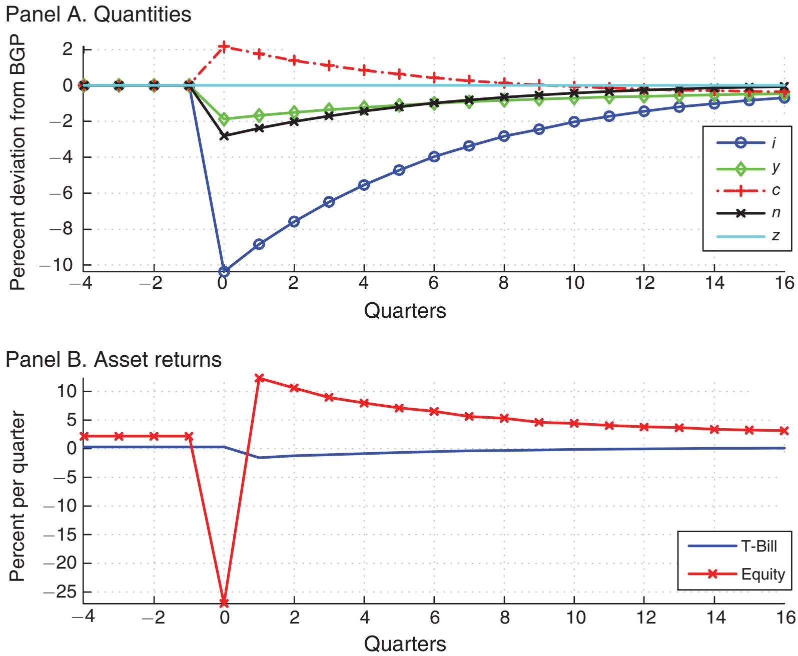

Figure 3. Impulse Response Function to a Temporary Increase in Disaster Probability from 0.72 Percent to 4 Percent

the disaster state, following which the transitory component of productivity converges back to zero, and only the permanent effect on productivity remains. The economy then recovers fairly swiftly to its new-steady-state, with a boom of investment, an increase in employment, and a slower adjustment of consumption. The speed of these transitions depends on capital adjustment costs, elasticities of substitution, and risk aversion.

Turning to asset returns, interest rates fall following the disaster as expected growth is low in the short-term and risk is high. Stock prices fall by a very large amount on impact, discounting lower future cash flows at a higher rate given the ongoing risk, and recover when the economy exits the disaster state.[27]

As Figure 1 shows, the model replicates well the impulse response of consumption to a disaster estimated by Barro et al. (2011).[28] Unfortunately, there is little systematic evidence on the behavior of investment, output, or stock prices during disasters, that would allow to evaluate the fit of the model, but these other implications appear fairly reasonable. For instance they are broadly consistent with the Great Depression. Overall, the model appears to capture well the dynamics of disasters, especially consumption, which is the principal determinant of the stochastic discount factor.

[27] Given that investors perfectly understand that the disaster is likely very persistent, stock prices keep falling after impact only if the disaster realizations θ_t or φ_t are very negative, which is not the case for this experiment.

[28] The impulse response to a disaster is (by definition) the average response conditional on a disaster starting, and hence averages over all the possible shock realizations (various sizes and duration of disaster). This explains that Figure 1 is smoother than Figure 2. The same difference is visible in Barro et al. (2011) between their Figures 2 and 3.

C. *A Temporary Increase in Disaster Probability*

We can now turn to the key experiment of this paper—an increase in disaster risk. Figure 3 presents the response of quantities and returns when the probability of disaster increases from its long-run average of 0.72 percent per quarter to 4 percent, and then mean-reverts to its long-run average according to equation (6). Investment falls by about 11 percent, as firms reduce their holdings of physical capital, which is now more risky. Hence, even a relatively small risk of disaster has a substantial effect on investment. Employment and output contract by about 3 percent and 2 percent respectively: the decline of output is driven entirely by employment, since there is no change in current total factor productivity or the capital stock. This is one of the main results of the paper: an increase in disaster risk leads to a recession. Intuitively, the economy has less need for output as investment falls, reducing the incentive for production. Technically, the decline of employment is driven by intertemporal labor supply: the return on savings is low, making work today less attractive.[29] Similarly, consumption increases initially because the elasticity of substitution is assumed to be greater than unity. The model hence predicts imperfect comovement between consumption on one side, and investment, employment, and output on the other; I return to this issue in Section IVA.

The bottom of Figure 3 reveals that, following the shock, the short-term interest rate decreases—a "flight to quality." Investors try to shift their portfolio towards safer assets, but in equilibrium, the net supply of these assets is zero, so the price has to adjust. Second, the higher risk increases the spread between the two returns, i.e., the risk premium goes up. Hence, in the model, an increase in risk premia coincides with a recession. Last, on impact, equity prices drop substantially, mostly through a discount rate effect. Finally, it is noteworthy that, even though consumption rises on impact, states of nature with high probability of disaster have high marginal utility (as can be verified by calculating the stochastic discount factor). With recursive preferences, marginal utility depends not only on current consumption and leisure, but also on continuation utility, which is reduced by the lower mean and higher uncertainty, since there is preference for early resolution of uncertainty (the risk aversion is larger than the inverse of the IES). As a result, securities that pay off badly in states when p is high bear a risk premium, i.e., the p-shock is a priced risk factor, just like volatility is a priced factor in Bansal and Yaron (2004).

Two parameters play a critical role in these results. First, the elasticity of intertemporal substitution determines the sign of the investment and output response, as shown analytically in Section II. Second, in the absence of depreciation shocks, capital is a fairly safe asset, and an increase in disaster risk leads to an investment and output boom, as people accumulate assets to smooth the potential disaster. Depreciation shocks are hence necessary to reproduce that output and investment are low when risk premia are high. Section IVC provides more comparative statics.

[29] This is in spite of a negative wealth effect which tends to push employment up; given the large IES the substitution effect overwhelms the wealth effect.

Table 2—Business Cycle Statistics for Consumption, Investment, Output, and Employment

		$\sigma(Y)$	$\sigma(C)$	$\sigma(I)$	$\sigma(N)$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{N,Y}$
1	Data	0.99	0.54	2.75	0.93	0.48	0.61	0.74
		(0.06)	(0.03)	(0.17)	(0.06)	(0.06)	(0.05)	(0.04)
2	No disasters	0.82	0.48	1.49	0.25	0.99	1.00	0.99
		(0.04)	(0.03)	(0.08)	(0.01)	(0.00)	(0.00)	(0.01)
3	Constant p	0.82	0.48	1.56	0.25	0.99	1.00	0.99
		(0.04)	(0.03)	(0.08)	(0.01)	(0.00)	(0.00)	(0.01)
4	Benchmark	0.93	0.69	2.96	0.69	0.32	0.87	0.75
		(0.07)	(0.10)	(0.64)	(0.17)	(0.24)	(0.03)	(0.04)

Notes: All series are in growth rate. Standard errors in parentheses. Data 1947:I–2010:IV.

Table 3—Financial Statistics

		$E(R_f)$	$E(R_b)$	$E(R_e - R_b)$	$\sigma(R_f)$	$\sigma(R_b)$	$\sigma(R_e - R_b)$
1	Data	—	0.19	1.84	—	0.82	8.20
			(0.05)	(0.52)		(0.06)	(0.44)
2	No disasters	0.56	0.56	0.01	0.05	0.05	1.03
		(0.03)	(0.03)	(0.07)	(0.01)	(0.01)	(0.06)
3	Constant p	−0.17	0.04	2.30	0.05	0.05	0.61
		(0.03)	(0.03)	(0.02)	(0.01)	(0.01)	(0.03)
4	Benchmark	0.16	0.28	2.24	1.00	0.74	7.67
		(0.30)	(0.21)	(0.42)	(0.56)	(0.41)	(1.85)

Notes: Mean and standard deviation of the risk-free, T-bill, and equity excess return. Standard errors in parentheses. Data 1947:I–2010:IV.

D. *First and Second Moments of Quantities and Asset Returns*

Table 2 reports business cycle moments obtained from model simulations, together with standard US post WWII statistics.[30] Row 2 presents the basic RBC model results (i.e., when there is no disaster risk): consumption is less volatile than output, while investment is more volatile than output, and the volatility of hours is fairly low, a well-known limitation of that model. Introducing a constant probability of disaster, in row 3, does not change the moments significantly: even though the assumptions of Proposition 3 are not verified (because disasters are not permanent and instantaneous), and even though the IES is not unity (so that $\beta^* \neq \beta$), its result holds approximately. Finally, making the probability of disaster time-varying leads to significant new dynamics, which are visible in row 4. The addition of a second shock leads to more volatility in all time series, but especially investment and employment. The correlation of consumption with output is reduced. Overall the model is significantly closer to the data than the basic RBC model, driven only by TFP shocks; especially noteworthy is the substantial increase in employment volatility.

[30] Given the absence of disasters in these data, the model moments are reported for a sample where no disaster actually takes place, i.e., agents fear a disaster but it does not occur in sample; see Section IVC for the population moments.

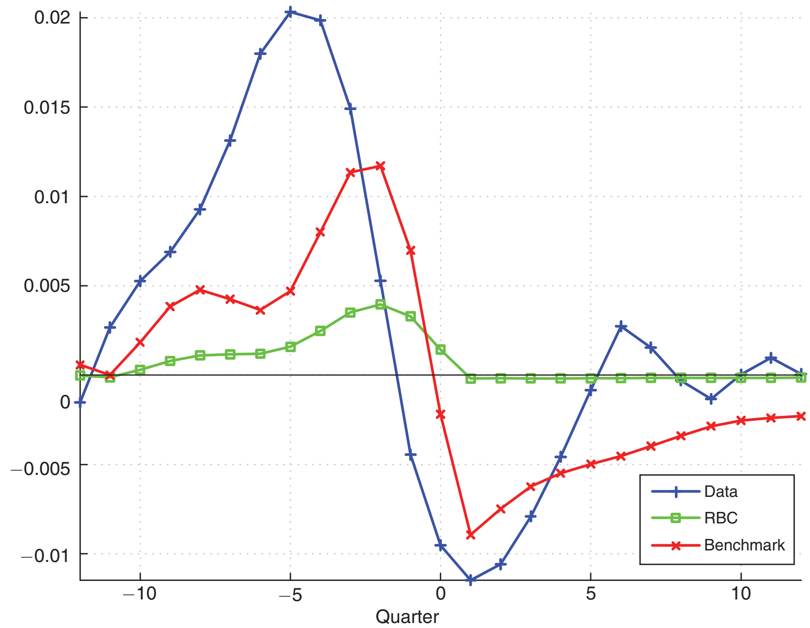

Figure 4. Cross-Covariogram of (One-Sided Filtered) GDP and Excess Stock Returns in the Data and in the RBC and Benchmark Models

Turning to returns, Table 3 shows that the RBC model (row 2) generates risk premia that are very small, and the volatility of returns is also much smaller than the data. Row 3 shows that adding constant disaster risk leads to a large risk premium (2.30 percent per quarter), but does not create return volatility. Finally, the benchmark model (row 4) generates an equity premium in line with the data, of 2.24 percent per quarter, and the volatility is also almost in line with the data (7.67 percent per quarter). Importantly, the model matches the low volatility of short-term interest rates (0.74 percent versus 0.82 percent in the data), in contrast to studies of Jermann (1998) and Boldrin, Christiano, and Fisher (2001) which implied highly volatile interest rates.

E. *Relations between Asset Prices and Macroeconomic Quantities*

This section evaluates the ability of the model to reproduce some empirical relations between asset prices and macroeconomic quantities.

Countercyclicality of Risk Premia.—An important feature of the data is that risk premia are countercyclical. This has been illustrated strikingly by the recent recession, during which the yield on risky assets such as corporate bonds went up while the yield on safe assets such as government bonds went down. This pattern is actually common to most US recessions, yet DSGE models are unable to capture it. To illustrate it simply, Figure 4 reports the covariance between detrended output $\tilde{y}_t$ and stock excess returns at different leads and lags, i.e., $\text{Cov}(\tilde{y}_t, R^e_{t+k} - R^f_{t+k})$, for $k = -12$ to $k = 12$ quarters. In the data (full line), this covariance is positive for $k < 0$, reflecting the well-known fact that stock returns lead GDP, but this covariance becomes negative for $k > 0$, implying that output negatively leads excess returns, i.e., *risk premia are lower when output is high*.[31]

The fact that returns lead GDP (i.e., the left side of the figure, $k < 0$), while interesting, might be rationalized by features such as advance information and time-to-build. Indeed, even the basic RBC model (without disaster risk) generates some of this pattern, since high returns reflect positive TFP shocks, and positive TFP shocks lead to above-trend output for some time.

More interesting, and more discriminating, is the right-side of this picture ($k > 0$), i.e., low output is associated with high future excess returns. The basic RBC model does not generate any variation in risk premia, so the model-implied covariance is essentially zero. This covariance would also be very close to zero for all standard DSGE models with low risk aversion. In contrast, my model generates about the right comovement of output and risk premia. This is a validation of the model key mechanism: changes in risk drive both expected returns and output. Similarly, investment is low when expected returns are high, consistent with the user cost of capital condition. The risk adjustment is critical, because risk-free interest rates are low when expected returns are high: a naïve econometrician would be surprised that investment does not respond to the low interest rates.

Volatility, *VIX and GDP*.—Recent work emphasizes a negative relation between volatility and economic activity. In an influential study, Bloom (2009) shows using a reduced-form VAR that shocks to the VIX index have a significant negative effect on output. The VIX index is a measure of the implied volatility of the SP500, constructed by the Chicago Board of Options Exchange from index option prices with different strikes. In the model, VIX can be calculated as the conditional standard deviation of the stock market return, under the risk-neutral measure,

$$VIX_t = \sqrt{\frac{E_t(M_{t+1}R_{t+1}^2)}{E_t(M_{t+1})} - \left(\frac{E_t(M_{t+1}R_{t+1})}{E_t(M_{t+1})}\right)^2},$$

whereas the expected "physical" volatility is simply

$$VOL_t = \sigma_t(R_{t+1}) = \sqrt{E_t(R_{t+1}^2) - E_t(R_{t+1})^2}.$$

Figure 5 reproduces Bloom's result by depicting the impulse response of output to a one-standard deviation shock to VIX in the data. This impulse response function is computed using a Cholesky decomposition, under the orthogonalization assumption

[31] I use this particular statistic because it can be calculated in both model and data. Of course there are many other variables (not present in the model) that forecast stock returns better, such as the unemployment rate. Moreover, I concentrate on the covariance rather than the correlation because the size of the association is critical—correlations can appear satisfactory even if there is only a tiny variation, provided it has the right sign. Last, GDP is detrended using the one-sided version of the Baxter-King (1999) filter to avoid look-ahead bias. A two-sided filter defines output as low relative to future output, and hence the covariance also capture variations in realized returns, rather than solely expected returns.

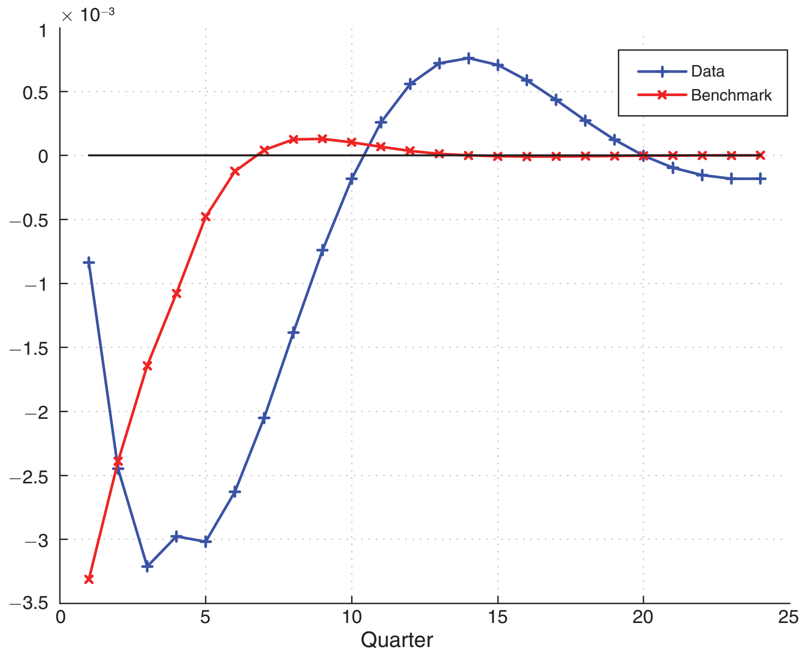

Figure 5. Impulse Response of GDP to a Shock to VIX in a Bivariate VAR, in the Data, and in the Benchmark Model

that the GDP shock has no impact effect on VIX.[32] The figure shows that running the VAR on model-generated data yields a response that is fairly similar to the real data. In the model, VIX is largely driven by the fear of a disaster, i.e., VIX is nearly one-to-one with the state variable p. Because increases in p lead to a decline in output, the model captures quantitatively well the observed response; however the model does not capture the gradual decline of GDP in the two quarters following the shock, because adjustment in the model is instantaneous.[33]

Investment and Asset Prices.—One enduring puzzle in macroeconomics and finance is the relation between investment and the stock market. While the Q-theory correctly predicts a positive correlation, the level of adjustment costs required to match the investment and the stock market is widely considered excessive. In contrast, the model captures well the magnitude of the relation between the stock market and investment, even with small adjustment costs. To establish this result, Figure 6 presents the cross-covariogram $\gamma_j = \text{Cov}(i_t, \log(P_{t+j}/K_{t+j}))$, where i_t is HP-filtered log investment, and P_{t+j}/K_{t+j} is HP-filtered log price-to-book ratio, for $j = -12$ to $j = 12$ quarters. In the data, this covariance is positive, and the stock market leads investment. The basic RBC model implies a very small covariance between the stock market investment, because TFP shocks have little effect on the stock market

[32] The orthogonalization assumption has little impact on these results. Following Bloom, both GDP and VIX are logged and HP-filtered, but this is not critical either. The VAR has four lags. Last, it makes little difference whether we use the implied volatility VIX or the physical volatility VOL.

[33] The RBC model cannot reproduce this pattern, because VIX is essentially constant.

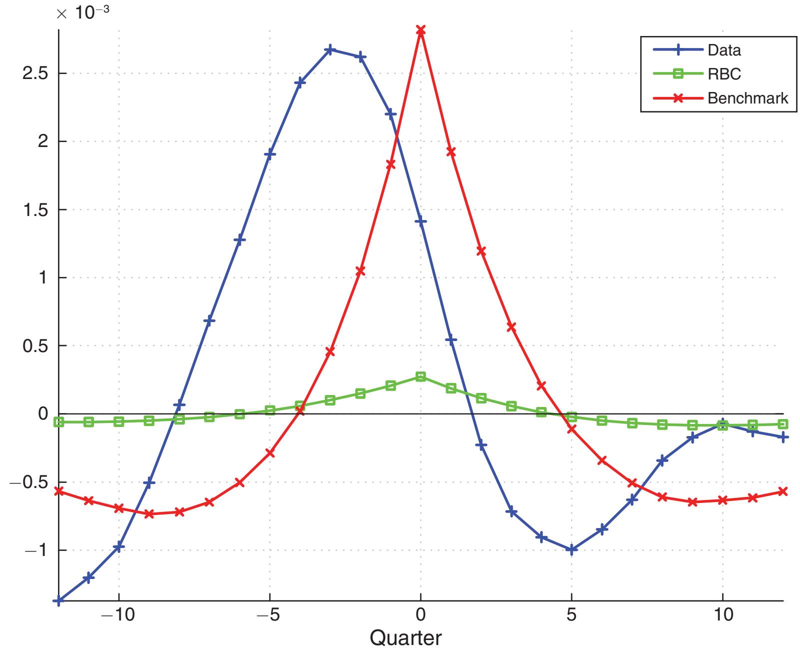

FIGURE 6. CROSS-COVARIOGRAM OF INVESTMENT AND PRICE TO BOOK RATIO IN THE DATA, AND IN THE RBC AND BENCHMARK MODELS

value. The benchmark model, however, matches well the magnitude of the association between stock prices and investment. Here too, the model is unable to replicate the exact timing of the association between the stock market and investment, suggestive of additional frictions such as time-to-build.

Additional Asset Pricing Implications.—This section discusses briefly several additional implications of the model. First, the model generates reasonable levels of the P-D ratio (15.3) and volatility of the log price-dividend ratio (0.27), and it is consistent with the evidence that equity returns are predictable. The regression

$$R^e_{t\to t+k} - R^f_{t\to t+k} = \alpha + \beta \frac{P_t}{D_t} + \varepsilon_{t+k},$$

yields a slope coefficient of 15.8 and a $R^2 = 25$ percent at the four-year horizon $(k = 4)$ in the data, and a slope coefficient of 9.9 and a $R^2 = 43$ percent in the model. In both data and model, the results are similar if the left-hand-side variable is returns rather than excess returns.

Second, a consumption claim in the model has a significantly smaller risk premium than the equity claim, with a mean excess return of 0.52 percent per quarter and a standard deviation of 0.64 percent per quarter. This is in line with the results of Lustig, Van Nieuwerburgh, and Verdelhan (2010), who show that the price of a claim to consumption is very high, because consumption is much safer than dividends.

Third, the lack of perfect comovement of consumption with other variables, while it is a limitation from a business cycle standpoint, can help explain various asset pricing puzzles. It is consistent with the weak correlation of consumption and asset returns. Moreover, estimating the IES in model-generated data, by running a standard univariate regression of consumption growth on the T-bill rate yields a small estimate of 0.36, consistent with the Hall (1988) estimates, and much lower than the true IES of 2. The regression is severely biased because it does not control for changes in precautionary savings induced by changes in probability of disaster, that are correlated with the interest rate.

Fourth, the model implies a downward-sloping term structure of interest rates. This negative term premium is not driven by disaster risk, since short-term bonds and long-term bonds are assumed to default by the same amount. As usual, TFP shocks generate very small risk premia. The term premium is thus driven by the third shock, i.e., the shock to the probability of disaster. An increase in the probability of disaster reduces interest rates, as the demand for precautionary savings rises. As a result, long-term bond prices rise (provided they do not default too much in disaster). Hence, long term bonds hedge against the shock to the probability of disaster, which implies that they have lower average return than the short-term bonds, leading to a yield curve that is on average downward sloping. The mean excess return on a 10-year government bond is -1.3 percent, and the standard deviation of this bond return is 5.4 percent per quarter. However, because my model does not incorporate inflation, it is difficult to compare these numbers precisely to the data. It seems likely, however, that the volatility of long-term bond returns is too high, and the mean return too low, compared to the data.

Last, Gourio (2011) shows that the model also offers a compelling framework to account for corporate bond spreads.

IV. Robustness and Extensions

This section first presents an extension of the model that addresses the consumption comovement. It then discusses different approaches to modeling disasters, and finally performs sensitivity analysis.

A. *Comovement and Consumption Response*

As illustrated in Figure 3, the model implies imperfect comovement of key macroeconomic aggregates: when the disaster probability rises, output, investment, and employment fall while consumption rises on impact. This result is easy to understand in light of the equivalence between disaster risk shocks and preference shocks (Proposition 3). As demonstrated by Barro and King (1984), given that labor demand does not shift following a preference shock, it is impossible to generate positive comovement (on impact) between consumption and hours worked. The many macroeconomic models that incorporate shocks to the Euler equation of course face the same issue. As a result, the same ingredients that help these studies generate comovement likely apply to this paper as well.

To illustrate this point, this section presents a minimal extension of the model that generates comovement, while maintaining the key other implications of the model.

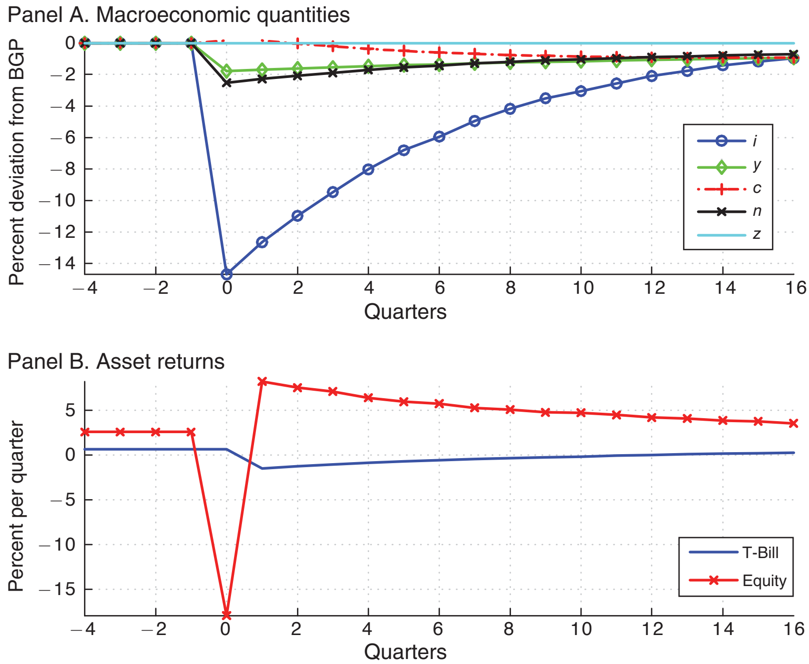

FIGURE 7. IMPULSE RESPONSE FUNCTION TO A TEMPORARY INCREASE IN DISASTER PROBABILITY FROM 0.72 PERCENT TO 2 PERCENT, IN THE MODEL OF SECTION IVA

The extension is adapted from a recent paper studying financial friction shocks (Hall 2011), and makes two changes to the benchmark model. First, preferences are assumed to exhibit complementarity between consumption and hours worked, with the following formulation

$$V_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \chi C_t^{1-\sigma} N_t^{1+\phi} - \alpha \frac{N_t^{1+\phi}}{1+\phi} + \beta E_t \left(V_{t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}.$$

The complementarity (middle) term mechanically forces employment and consumption to move more synchronously. There are various possible microfoundations for this specification, ranging from household production (Aguiar and Hurst 2005) to hand-to-mouth workers. Second, markups are assumed to be countercyclical; for simplicity, I follow Hall (2011) and do not model the sources of this cyclical variation, and simply postulate a reduced form, $\chi_t = \chi(y_t)$ where y_t is detrended GDP, and $\chi' < 0$. As a result, the labor demand now reads

$$(1-\alpha)\left(\frac{K_t}{z_t N_t}\right)^{\alpha} \frac{1}{\chi(y_t)} = W_t.$$

As output falls, markups increase and labor demand shifts down, leading to a further output decline, which helps obtain comovement.

Consistent with the intuition, Figure 7 shows that the initial increase in consumption is almost entirely eliminated (the consumption response is slightly positive for the first two quarters, but extremely small). Meanwhile, the key model implications for investment, output, and asset returns, are largely maintained. Countercyclical markups amplify significantly the effect on quantities.[34] These results suggest that the implications of the model for consumption are not robust to realistic extensions.[35] This justifies the focus of the paper on investment and output, on one side, and on asset prices on the other side, which are the more robust implications of the model.[36]

B. *Modeling Disasters*

The model assumes that a disaster is driven by permanent and transitory shocks to productivity, as well as depreciation shocks. This section discusses how alternative approaches to modeling disasters affect the results. In the absence of permanent productivity shocks, the model economy eventually recovers to its initial steady-state. This directly contradicts Figure 1, which shows that the long-run average effect of the disaster on consumption is significantly negative. In a wide class of model, this long-run consumption response pins down exactly the permanent productivity shocks.

On the other hand, one may wonder if depreciation shocks, or transitory productivity shocks, are critical to the model. The online Appendix produces the equivalent of Figure 1 for various parametrizations of the model, together with the key business cycles and financial statistics (Tables A1 and A2). First, if there is no transitory component of productivity, consumption actually rises on impact (Figure A1), because output does not fall a lot, and investment is now risky. As a result, the model does not replicate the consumption response well. (Still, all the key model results hold in this case, since permanent shocks are the ones that matter most for marginal utility.)

Second, if there is no capital destruction, there are two offsetting effects: on the one hand, investment is now much less risky, and it allows to smooth consumption if the disaster continues, so it may fall less than in the benchmark. On the other hand, the capital has not taken an initial hit, and it is now too high since productivity has fallen, which leads investment to fall more. For our calibration, the first effect dominates, and consumption falls too much initially (Figure A2), so that the shape does not fit at all the data. Moreover, the depreciation shock is critical to our business cycles and financial statistics. If capital is safe, an increase in disaster risk leads agent to accumulate safe capital, generating an investment and output boom, and hence a counterfactual positive correlation between investment and risk premia.

Third, the benchmark calibration assumes that the initial productivity shocks are twice larger than subsequent shocks; Figure A3 in the online Appendix shows that this is necessary to generate the magnitude of the drop of output during the first two years.

[34] The preferences parameters differ from Hall, as I need an IES of consumption larger than unity for my results. This simulation assumes $\sigma = 2/3, \chi = -0.1, \phi = 0.2, \alpha = 5$, and an elasticity of markup to output equal to 0.5. The figure depicts the effect of an increase of the probability of disaster to 2 percent (rather than 6 percent), in the case of permanent shocks alone (for numerical reasons), but the size of the investment response is similar, which is the sign of substantial amplification.

[35] Hall (2011) argues that this model captures the key features of sticky price models; Basu and Bundick (2011) explicitly incorporate sticky prices in a model with time-varying aggregate uncertainty and generate on-impact comovement.

[36] Another possible mechanism to generate comovement is to allow the disaster probability to be correlated with other macroeconomic shocks, for instance productivity shocks. This can be motivated by learning: in reality people revise their beliefs about the likelihood of a disaster, based on the current state of the economy. For instance, a string of negative TFP realizations might lead people to think a disaster is now more likely.

Table 4—Sensitivity Analysis: Business Cycle Statistics

		$\sigma(Y)$	$\sigma(C)$	$\sigma(I)$	$\sigma(N)$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{N,Y}$
1	Data	0.99	0.54	2.75	0.93	0.48	0.61	0.74
2	Benchmark	0.93	0.69	2.96	0.69	0.31	0.87	0.75
3	Sample with disasters	1.57	1.39	5.98	1.14	0.48	0.79	0.62
4	$1/\hat{\psi} = 1.5$	0.87	0.61	2.31	0.52	0.56	0.88	0.74
5	$\gamma = 1/\hat{\psi} = 0.5$	0.83	0.51	1.64	0.30	0.91	0.96	0.89
6	$\hat{\psi} = \gamma = 2$	0.74	0.58	1.16	0.15	0.98	0.98	0.85
7	$\rho_p = 0.8$	0.92	0.68	2.82	0.66	0.35	0.87	0.75
8	$\sigma_p = 1.7$	0.88	0.62	2.46	0.56	0.49	0.88	0.74

Table 5—Sensitivity Analysis: Financial Statistics

		$E(R_f)$	$E(R_b)$	$E(R_e - R_b)$	$\sigma(R_f)$	$\sigma(R_b)$	$\sigma(R_e - R_b)$
1	Data	—	0.19	1.84	—	0.89	8.20
2	Benchmark	0.16	0.28	2.24	1.00	0.74	7.67
3	Sample with disasters	0.23	0.33	2.28	0.94	0.80	10.93
4	$1/\hat{\psi} = 1.5$	0.19	0.31	2.21	0.99	0.73	7.46
5	$\gamma = 1/\hat{\psi} = 0.5$	0.50	0.53	0.25	0.29	0.22	2.16
6	$\hat{\psi} = \gamma = 2$	0.66	0.69	0.50	0.36	0.28	2.80
7	$\rho_p = 0.8$	0.13	0.26	2.32	1.26	0.94	8.20
8	$\sigma_p = 1.7$	0.03	0.19	2.25	0.81	0.60	5.68

C. *Sensitivity Analysis*

Tables 4 and 5 report model moments for alternative parameter values. First, calculating the model moments in a full sample (rather than a sample that does not include disaster realizations) does not change the equity premium significantly. On the one hand, the Peso problem (sample selection) is reduced as the worst realizations are now included; but on the other hand, disasters are periods of high uncertainty and hence high expected returns, which boosts the (arithmetic) mean equity premium. The volatility of macroeconomic quantities and returns increases significantly as they now incorporate the largest absolute changes.

The intertemporal elasticity of substitution is a critical parameter. As Section II shows, if the IES equals one, disaster risk has no effect on quantities, at least in the case of instantaneous and permanent disasters. Consistent with this finding, an IES of 1.5 reduces the volatility of investment, from 2.96 percent to 2.31 percent, and similarly for employment and output (row 4). This change in IES, however, does not affect markedly the asset pricing properties of the model.

One important difference between this model and standard DSGE models is that it incorporates recursive preferences. Reducing risk aversion to 0.5 (rather than 3.8 in the benchmark) leads to expected utility preferences. The *qualitative* results of the model, such as the impulse response function (Figure 3) are unaffected by this change, but the *magnitude* of both risk premia, and the response of quantities to disaster risk, is now substantially smaller (row 5), with an investment volatility of 1.64 percent: the size of business cycles is affected by risk aversion.

Rather than decreasing the risk aversion coefficient, one can reach expected utility by decreasing the IES, as in row 6. With an IES lower than unity, the model performs well according to the metrics of this table, but it implies—counterfactually—that risk premia are procyclical (i.e., the right-side Figure 4 is flipped). Hence, recursive preferences, while not critical to the qualitative results, are important quantitatively as they allow the combination of a high IES, and a high risk aversion.

The process for the disaster probability obviously affects the results: decreasing the volatility of p to $\sigma_p = 1.7$ leads to smoother quantities and returns; for instance the volatility of investment falls to 2.46 percent. The change has little effect on the mean equity premium. A lower persistence of p has relatively little effect on the results, holding the unconditional volatility of p_t fixed.

Finally, the leverage process is important to generate large risk premia, consistent with the data, but the results are robust to reasonable changes. First, if one assumes that firms manage their debt to keep their leverage constant, rather than not adjusting debt until it matures, the mean and volatility of returns are similar, respectively 1.99 percent and 7.09 percent. Second, a higher leverage ratio leads of course to higher mean and volatility of returns, and this effect is nonlinear: for $\zeta = 0.016$, the mean excess return is 1.7 percent and the volatility is 5.8 percent, while for $\zeta = 0.02$, the mean excess return is 3.0 percent and the volatility is 11.2 percent. Third, a higher default on debt leads to lower mean and volatility of returns: if the recovery rate is 0.42 instead of 0.38, the mean falls to 2.1 percent and the volatility to 7.3 percent. Finally, using a lower maturity of debt of 5 years reduces somewhat the mean and volatility of returns, to 2.1 percent and 6.9 percent. Given that the Modigliani and Miller theorem holds in this model, all these changes have no effect on business cycle statistics.

V. Conclusion

Introducing time-varying disaster risk into a standard real business cycle model improves its fit of asset return data, preserves its implications for quantities in response to TFP shocks, and creates some interesting new macroeconomic dynamics. The model replicates not only the second moments of quantities and asset returns, but also a variety of empirical relationships between macroeconomic quantities and asset prices, which have so far largely eluded researchers. In particular, risk premia are endogenously countercyclical, which significantly affects investment dynamics. The results of this paper suggest that fluctuations in macroeconomic risk contribute to business cycles.

The tractability of this framework makes it feasible to embed disaster risk into a richer model, with several additional realistic features, such as sticky prices or financial frictions. One could then study how private choices and public policies affect the severity of potential economic disasters, which is important for welfare.

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