A Note on Negative Shocks, Public Debt, and Interest Rates *

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Abstract

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*This research benefited from support from the Open Source Economics Laboratory at the University of Chicago. All Python code and documentation for the computational model is available at https://github.com/OpenSourceEcon/PubDebtNegShocks.

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1 Introduction

2 Economic Model

We study a simple 2-period-lived agent overlapping generations model in which the government promises to make a lump sum transfer $\bar{H} \geq 0$ from the young to the old each period. Ricardian equivalence holds in the sense that households have rational expectations and can forecast the effects of government budget imbalances. The constraints of the model generate states of the world in which the government can only make a transfer that is less that the promised amount $0 \leq H_t \leq \bar{H}$.

Our characterization of government budget insolvency relies on the assumption that when the state of the world is such that $\bar{H}$ generates negative consumption for the young, the agents in the economy resort to autarky rather than starvation (negative consumption). This shut-down result would not hold if the government merely reduced the size of the transfer program in the face of a shut down. Rational agents would expect this and incorporate that risk on the payment $\bar{H}$ in the second period of their lives.[1]

2.1 Household problem

A unit measure of identical consumer-worker households is born each period. A Household lives for exactly two periods indexed by $s = 1, 2$. They supply a unit of labor inelastically in both the young and old period of life $n_1 > 0$ and $n_2 \geq 0$ in all periods t.

In the first period of life, consumer-worker households choose how to divide their net labor income between age-1 consumption $c_{1,t}$ and capital investment (savings) with the firms $k_{2,t+1}$. The objective of a household is to maximize Epstein-Zin-Weil

[1] The argument here is that a proportional transfer program will never shut down a government. However, if the government is locked in to some degree of nonproportional transfer program, then there are states of the world in which the government must either shut down or default on that debt. If they default in a way that the consumption of the young does not go to zero, then the government has changed its nonproportional transfer program to look like a proportional transfer program.

utility subject to a period budget constraint and two nonnegativity constraints.[2]

$$\max_{c_{1,t},k_{2,t+1},c_{2,t+1}} (1-\beta)\ln(c_{1,t}) + \beta\frac{1}{1-\gamma}\ln\Big(E_t\big[(c_{2,t+1})^{1-\gamma}\big]\Big) \quad \forall t \tag{1}$$

$$\text{such that} \quad c_{1,t} + k_{2,t+1} = w_t n_1 + x_1 - H_t \tag{2}$$

$$\text{and} \quad c_{2,t+1} = \big(1+r_{t+1}\big)k_{2,t+1} + w_{t+1}n_2 + x_2 + H_{t+1} \tag{3}$$

$$\text{and} \quad c_{1,t}, c_{2,t+1}, k_{2,t+1} > 0 \tag{4}$$

The variables w_t and r_t are the wage and interest rate, respectively. And we follow Blanchard (2019) in including a potential exogenous transfer unrelated to the intergenerational transfer H_t in the first period of life x_1 and in the second period of life x_2. These solvency preserving income transfers are one of the key drivers of the results from Blanchard (2019).

Let new households have no initial capital $k_{1,t} = 0$. Note that the nonnegativity constraints on consumption $c_{1,t}$ and $c_{2,t+1}$ are strict inequalities in equilibrium due to the Inada conditions on the period utility functions. Furthermore, we also do not allow the government transfer program to zero out the consumption and savings of the young, as shown in Section 2.3 equation (13). Finally, the strict inequality on savings $k_{2,t+1} > 0$ is also an equilibrium condition that comes from the market clearing condition (15), that negative capital stock $K_t < 0$ is not defined in the production function (8), and that zero capital stock $K_t = 0$ would result in zero output $Y_t = 0$ from (8), zero wage $w_t = 0$ from (T.1.13), and an infinite interest rate $r_t = \infty$ from (T.1.12).

Consumption in the second period of life is characterized by the second period budget constraint.

$$c_{2,t+1} = (1+r_{t+1})k_{2,t+1} + w_{t+1}n_2 + x_2 + H_{t+1} \quad \forall t \tag{3}$$

Note that the nonnegativity constraint on old-age consumption $c_{2,t+1}$ will never bind because everything on the right-hand-side of (3) is weakly positive. Consumption

[2] See Epstein and Zin (2013) and Weil (1990).

in the first period of life $c_{1,t}$ and savings in the first period of life $k_{2,t+1}$ are jointly determined by the first period budget constraint (2) and by the Euler equation that characterizes the optimal young $s = 1$ consumption-savings decision that maximizes lifetime utility (1) subject to constraints (2), (3), and (4).

$$\frac{1-\beta}{c_{1,t}} = \beta\frac{E_t\left[(1+r_{t+1})\big(c_{2,t+1}\big)^{-\gamma}\right]}{E_t\left[\big(c_{2,t+1}\big)^{1-\gamma}\right]} \quad \forall t \tag{5}$$

By substituting the age $s = 1$ and $s = 2$ budget constraints (2) and (3) into the household Euler equation (5), we can see that the characterizing equation for savings $k_{2,t+1}$ is one equation with one unknown.

$$\frac{1-\beta}{w_t n_1 + x_1 - H_t - k_{2,t+1}} = \beta\frac{E_t\Big[(1+r_{t+1})\big([1+r_{t+1}]k_{2,t+1} + w_{t+1}n_2 + x_2 + H_{t+1}\big)^{-\gamma}\Big]}{E_t\Big[\big([1+r_{t+1}]k_{2,t+1} + w_{t+1}n_2 + x_2 + H_{t+1}\big)^{1-\gamma}\Big]} \quad \forall t \tag{6}$$

Equation (6) shows that the functional solution for household savings $k_{2,t+1}$ every period is a stationary function $\psi(\cdot)$ of the time path of transfers and prices over the lifetime of the household.

$$k_{2,t+1} = \psi\big(H_t, H_{t+1}, w_t, w_{t+1}, r_{t+1}\big) \tag{7}$$

2.2 Firm problem

A unit measure of identical perfectly competitive firms exist in this economy that hire aggregate labor L_t at wage w_t and rent aggregate capital K_t at rental rate r_t every period in order to produce consumption good Y_t according to a Cobb-Douglas production function,

$$Y_t = F(K_t, L_t, z_t) = A_t\Big[\alpha(K_t)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\alpha)(L_t)^{\frac{\varepsilon-1}{\varepsilon}}\Big]^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall t \tag{8}$$

where the capital share of income is given by $\alpha \in (0,1)$ and $\varepsilon > 0$ is the constant elasticity of substitution between capital and labor in the production process. Total factor productivity $A_t = e^{z_t} > 0$ is distributed log normally, and z_t follows a normally distributed $AR(1)$ process. Two important special parameterizations are the unit elasticity case $\varepsilon = 1$ in which the limit of (8) is the Cobb-Douglas production function and the perfectly elastic case $\varepsilon = \infty$ in which the production function is linear in K_t and L_t (perfect substitutes).

$$\begin{aligned} &z_t = \rho z_{t-1} + (1-\rho)\mu + \epsilon_t \\ &\quad \text{where} \quad \rho \in [0,1), \quad \mu \geq 0, \quad \text{and} \quad \epsilon_t \sim N(0,\sigma) \end{aligned} \tag{9}$$

The firm's problem each period is to choose how much capital K_t to rent and how much labor L_t to hire in order to maximize profits,

$$\max_{K_t,L_t} Pr_t = F(K_t,L_t,z_t) - w_t L_t - (r_t+\delta)K_t \quad \forall t \tag{10}$$

where δ is the per-period depreciation rate of capital. Profit maximization implies that the real wage and real rental rate are determined by the standard first order conditions for the firm.

$$r_t = \alpha (A_t)^{\frac{\varepsilon-1}{\varepsilon}} \left[\frac{Y_t}{K_t}\right]^{\frac{1}{\varepsilon}} - \delta \quad \forall t \tag{11}$$

$$w_t = (1-\alpha)(A_t)^{\frac{\varepsilon-1}{\varepsilon}} \left[\frac{Y_t}{L_t}\right]^{\frac{1}{\varepsilon}} \quad \forall t \tag{12}$$

Because the interest rate r_t in (T.1.12) is not defined when the capital stock is zero $K_t = 0$ and because the wage w_t in (T.1.13) is not defined when aggregate labor is zero $L_t = 0$, we know that both values must be strictly positive $K_t, L_t > 0$.

2.3 Government transfer program

We model a simple balanced budget public transfer program that takes an amount from the young each period H_t and gives that same amount to the old each period, as shown in the young and old budget constraints.

$$c_{1,t} + k_{2,t+1} = w_t n_1 + x_1 - H_t \quad \forall t \tag{2}$$

$$c_{2,t} = \big(1 + r_t\big) k_{2,t} + w_t n_2 + x_2 + H_t \quad \forall t \tag{3}$$

In contrast to the way the old-age ($s = 2$) budget constraint (3) is displayed in Section 2.1, we show the budget constraints here for a young household and old household both in period t (two separate individuals). The government budget is made up entirely of this transfer program, and the budget is always balanced because the government revenue taken from the young in period H_t is always equal to the transfers to the old H_t in all periods t.

In most periods, the government promises that the transfer will be $\bar{H} \geq 0$. However, in the case that $\bar{H} > 0$, there could exist states of the economy such that $\bar{H} \geq w_t n_1 + x_1$. In these cases, net labor income is less than or equal to zero, so the strict inequalities on $c_{1,t}$ and $k_{2,t+1}$ in (4) must be violated. To avoid negative consumption, we require that the most the government can take from the young in any period is all their income up to some arbitrarily small minimum consumption $c_{min} > 0$ and an arbitrarily small amount of savings $K_{min} > 0$.

$$\begin{aligned} H_t &\equiv \begin{cases} \bar{H} & \text{if} \quad w_t n_1 \geq \bar{H} - x_1 + c_{min} + K_{min} \\ w_t n_1 + x_1 - c_{min} - K_{min} & \text{if} \quad w_t n_1 < \bar{H} - x_1 + c_{min} + K_{min} \end{cases} \quad \forall t \\ &= \min\big(\bar{H}, w_t n_1 + x_1 - c_{min} - K_{min}\big) \quad \forall t \end{aligned} \tag{13}$$

This rule states that the government implements a balanced budget transfer program from the young to the old every period. And for $\bar{H} > 0$, once the wage dips low enough, the government can no longer take $\bar{H}$ from the young. In this case, the government takes all that it can from the young $H_t = w_t n_1 + x_1 - c_{min} - K_{min} < \bar{H}$ and transfers that amount to the old. The young are left with consumption and savings equal to the minimum $c_{1,t} = c_{min}$ and $k_{2,t+1} = K_{min}$, and the economy shuts down and devolves into autarky.

2.4 Market clearing

Market clearing implies that the aggregate labor demand equals aggregate labor supply, aggregate capital demand equals aggregate capital supply, and output equals consumption plus investment in each period,

$$L_t = n_1 + n_2 \quad \forall t \tag{14}$$

$$K_t = k_{2,t} \quad \forall t \tag{15}$$

$$\begin{aligned} Y_t &= C_t + K_{t+1} - (1-\delta)K_t \quad \forall t \\ &\quad \text{where} \quad C_t \equiv c_{1,t} + c_{2,t} \end{aligned} \tag{16}$$

where the goods market clearing condition or resource constraint (16) is redundant by Walras' Law.

2.5 Equilibrium

In this section, we define a functional stationary equilibrium in which our definition of stationary is that the functional forms are not time dependent. That is, for a function $f(\boldsymbol{x})$ of vector of variables $\boldsymbol{x}$, the function does not change. Only the output values of the function changes in response to changing inputs $\boldsymbol{x}$.

Definition 1 (**Functional stationary equilibrium**)**.** A non-autarkic functional stationary equilibrium in the two-period-lived overlapping generations model with exogenous labor supply and aggregate shocks is defined by stationary price functions $r(k,z)$ and $w(k,z)$ and a stationary savings function $k' = \psi(k,z)$ for all current state wealth k and total factor productivity component z such that:

i. households optimize according to (2) and (3), and (5)

ii. firms optimize according to (T.1.12) and (T.1.13),

iii. markets clear according to (14) and (15).

We can solve for the stationary price functions analytically by substituting the market clearing conditions (14) and (15) into the firms' respective first order conditions (T.1.12) and (T.1.13).

$$r_t \equiv r\big(k_{2,t}, z_t\big) = \left(e^{z_t}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left[\alpha\frac{F(k_{2,t}, n_1+n_2, z_t)}{k_{2,t}}\right]^{\frac{1}{\varepsilon}} - \delta \quad \forall z_t \quad \text{and} \quad k_{2,t} > 0 \tag{17}$$

$$w_t \equiv w\big(k_{2,t}, z_t\big) = \left(e^{z_t}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left[(1-\alpha)\frac{F(k_{2,t}, n_1+n_2, z_t)}{n_1+n_2}\right]^{\frac{1}{\varepsilon}} \quad \forall z_t \quad \text{and} \quad k_{2,t} > 0 \tag{18}$$

We can also solve analytically for the equilibrium expression for the transfer each period H_t as a function of the wealth of the current-period old $k_{2,t}$ and the value of the normally distributed component z_t of the total factor productivity process (as well as the parameters of the promised transfer amount $\bar{H}$ and minimum values of young age consumption c_{min} and aggregate capital K_{min}) by substituting the equilibrium wage expression (18) into the expression for H_t (13).

$$H_t \equiv H\big(k_{2,t}, z_t\big) = \min\Big(\bar{H}, w(k_{2,t}, z_t)n_1 + x_1 - c_{min} - K_{min}\Big) \quad \forall z_t \quad \text{and} \quad k_{2,t} > 0 \tag{19}$$

Finally, if we substitute the equilibrium expressions for prices $r(k,z)$ and $w(k,z)$ and the transfer $H(k,z)$ from (17), (18), and (19) into the household Euler equation (6) and resulting policy function (7), it is clear that the equilibrium savings function $k' = \psi(k,z)$ is a function of the wealth of the current-period old $k_{2,t}$ and the value z_t

of the normally distributed component of total factor productivity,

$$
\begin{aligned}
&\frac{1-\beta}{w(k_{2,t},z_t)n_1 + x_1 - H(k_{2,t},z_t) - k_{2,t+1}} = \\
&\quad \beta\int_{z_{t+1}}\Biggl[\Bigl(1+r(k_{2,t+1},z_{t+1})\Bigr)\times \\
&\qquad\qquad \Bigl([1+r(k_{2,t+1},z_{t+1})]k_{2,t+1} + w(k_{2,t+1},z_{t+1})n_2 + H(k_{2,t+1},z_{t+1})\Bigr)^{-\gamma}\times \\
&\qquad\qquad\qquad f\bigl(z_{t+1}|\rho z_t + (1-\rho)\mu,\sigma\bigr)\Biggr]dz_{t+1} \div \\
&\quad \int_{z_{t+1}}\Biggl[\Bigl([1+r(k_{2,t+1},z_{t+1})]k_{2,t+1} + w(k_{2,t+1},z_{t+1})n_2 + H(k_{2,t+1},z_{t+1})\Bigr)^{1-\gamma}\times \\
&\qquad\qquad\qquad f\bigl(z_{t+1}|\rho z_t + (1-\rho)\mu,\sigma\bigr)\Biggr]dz_{t+1} \\
&\qquad\qquad\qquad\qquad \forall z_t, z_{t+1} \quad \text{and} \quad k_{2,t}, k_{2,t+1} > 0
\end{aligned}
\tag{20}
$$

$$
k_{2,t+1} = \psi\bigl(k_{2,t},z_t\bigr) > 0 \quad \forall z_t \quad \text{and} \quad k_{2,t} > 0 \tag{21}
$$

where $f(z_{t+1}|\rho z_t + (1-\rho)\mu,\sigma)$ is the probability density function of z_{t+1} distributed normally with mean $\rho z_t + (1-\rho)\mu$ and standard deviation σ.

2.6 One-period riskless bonds

In this section, we derive the return on a riskless bond. We make the simplifying assumption that the riskless bonds are zero absolute supply. However, this characterization can be generalized to cases in which the riskless bonds have exogenous positive supply. Because of our zero-supply assumption on the riskless bond, we can separate its derivation from the characterization of the household problem in Section 2.1. These zero-supply riskless bonds do not influence the rest of the economy. They simply represent another measure of the level of risk present in each period of the economy.

Assume that households have two potential instruments for saving. A household can invest income with the production sector $k_{2,t+1}$ and earn a stochastic risky return next period of r_t and they can buy $b_{2,t+1}$ units of a one-period riskless bond for price

p_t that returns exactly $b_{2,t+1}$ when old. It is clear that old-age households will have no demand for these bonds.

The maximization problem for a generic household can be characterized as choosing risky savings $k_{2,t+1}$ and riskless savings $b_{2,t+1}$ to maximize lifetime utility subject to budget constraints.

$$\max_{k_{2,t+1},b_{2,t+1}} (1-\beta)\ln(c_{1,t}) + \beta\frac{1}{1-\gamma}\ln\Big(E_t\big[(c_{2,t+1})^{1-\gamma}\big]\Big) \quad \forall t \tag{22}$$

$$\text{such that} \quad c_{1,t} + k_{2,t+1} + p_t b_{2,t+1} = w_t n_1 + x_1 - H_t \tag{23}$$

$$\text{and} \quad c_{2,t+1} = \big(1+r_{t+1}\big)k_{2,t+1} + b_{2,t+1} + w_{t+1}n_2 + x_2 + H_{t+1} \tag{24}$$

$$\text{and} \quad c_{1,t}, c_{2,t+1}, k_{2,t+1} \geq 0 \tag{25}$$

The optimality condition for risky savings $k_{2,t+1}$ is the same Euler equation as in Section 2.1.

$$\frac{1-\beta}{c_{1,t}} = \beta\frac{E_t\left[(1+r_{t+1})\big(c_{2,t+1}\big)^{-\gamma}\right]}{E_t\left[\big(c_{2,t+1}\big)^{1-\gamma}\right]} \quad \forall t \tag{5}$$

The Euler equation for riskless savings $b_{2,t+1}$ is the following,

$$\begin{aligned} \frac{1}{1+\bar{r}_t} \equiv p_t &= \left(\frac{\beta}{1-\beta}\right)\frac{(c_{1,t})E_t\left[\big(c_{2,t+1}\big)^{-\gamma}\right]}{E_t\left[\big(c_{2,t+1}\big)^{1-\gamma}\right]} \quad \forall t \\ \Rightarrow \quad \bar{r}_t &= \left(\frac{1-\beta}{\beta}\right)\frac{E_t\left[\big(c_{2,t+1}\big)^{1-\gamma}\right]}{(c_{1,t})E_t\left[\big(c_{2,t+1}\big)^{-\gamma}\right]} - 1 \quad \forall t \end{aligned} \tag{26}$$

where the price of the riskless bond p_t is defined as the reciprocal of the gross riskless return $1+\bar{r}_t$. The optimality conditions of the production sector are the same as in Section 2.2.

Euler equation (26) determines the demand for riskless bonds. We assume, generally, an exogenous supply of riskless bonds that is nonnegative $B_t \geq 0$ for all t. However, specifically in this model, we assume a zero supply of riskless bonds $B_t = 0$.

So the general version of our riskless bond market clearing condition is the following.

$$b_{2,t} = B_t \quad \forall t \tag{27}$$

With our zero supply assumption $B_t = 0$, the household demand for riskless bonds is set to zero through the market clearing condition,

$$b_{2,t} = 0 \quad \forall t \tag{28}$$

all the other endogenous variables are determined by the equilibrium described in Section 2.5, and the riskless return $\bar{r}_t$ is characterized by Euler equation (26).

If we were to relax our zero-supply assumption on riskless bonds $B_t > 0$, we would have to determine the riskless return $\bar{r}_t$ jointly with the rest of the endogenous variables.

And finally, because the agents in our model live for only two periods, it is intuitive that each model period must represent many years. If we assume that the average economic life is 60 years, then each model period represents 30 years. Let the parameter yrs be the number of years represented in a model period. Then we can report the riskless interest rate $\bar{r}_t$ characterized in (26) as an annual rate $\bar{r}_{t,an}$ using the following expression.

$$\bar{r}_{t,an} = \big(1 + \bar{r}_t\big)^{\frac{1}{yrs}} - 1 \quad \forall t \tag{29}$$

3 Simulations

We explore the properties of the model from Section 2 with respect to different values of the promised transfer $\bar{H}$, initial wealth $k_{2,0}$, and the extent and probability of low total factor productivity values A_t by calibrating the other parameters of the model and simulating a time series of the model 3,000 times for different combinations of $\bar{H}$, $k_{2,0}$, and the support and distribution of A_t. The first three rows of Table 1 show the different values of $\bar{H}$, $k_{2,0}$, and A_{min} that we test in our simulations. The remaining rows show our calibration of the other variables.[3]

Table 1: Calibration of 2-period-lived agent OG model with promised transfer $\bar{H}$

Parameter	Source to match	Value(s)
$\bar{H}$	Promised transfer amount	$[0.00, 0.05, 0.11, 0.17]$
$k_{2,0}$	Initial period wealth of old household	$[0.11, 0.14, 0.17]$
A_{min}	Minimum value in support of A_t	$[0.0, 0.75]$
z_0	Initial value of z_t TFP component	μ
n_1	Exogenous labor supply when young	1.0
n_2	Exogenous labor supply when old	0.0
β	Annual discount factor of 0.96	0.29
γ	Coefficient of relative risk aversion between 1.5 and 4.0	2.0
α	Capital share of income	0.35
δ	Annual capital depreciation of 0.05	0.79
ρ	AR(1) persistence of normally distributed shock to match annual persistence of 0.95	0.21
μ	AR(1) long-run average z_t level	0.0
σ	standard deviation of normally distributed z_t to match annual standard deviation of U.S. real GDP of 0.49	1.55
B_t	Exogenous supply of riskless bonds in every period	0
yrs	Number of years in a model period	30
T	Maximum number of periods to simulate in a given simulation	100
S	Number of simulated time series for a given parameterization	3,000

The Technical Appendix T-2 gives a detailed description of the calibration of all parameters.

In our simulations we study a first layer of the parameter space by simulating combinations of different values of government transfer $\bar{H} \in [0.00, 0.05, 0.11, 0.17]$ and different values of initial wealth $k_{2,0} \in [0.11, 0.14, 0.17]$. We first study the behavior of the economy with these combinations of $\bar{H}$ and $k_{2,0}$ when the total factor productivity value $A_t \equiv e^{z_t}$ is distributed lognormally as described in (T.1.10) with the range of A_t being $(0, \infty)$. This range of TFP shocks includes very small values close to zero, albeit with low probability, that can create fiscal insolvency when $\bar{H} > 0$.

[3] The code for these simulations is available at https://github.com/OpenSourceEcon/PubDebtNegShocks.

We then study those same combinations of $\bar{H}$ and $k_{2,0}$, but we assume a truncated support of the total factor productivity process $A_t \in [A_{min}, \infty)$, where $A_{min} > 0$. We implement this truncated TFP process by assuming that the shocks to the z_t process are truncated normal $TrN()$ with mean 0, standard deviation σ, and lower bound cutoff $\varepsilon_{t,min}$.[4]

$$\begin{aligned} &z_t = \rho z_{t-1} + (1-\rho)\mu + \varepsilon_t \\ &\quad \text{where} \quad \rho \in [0,1), \quad \mu \geq 0, \quad \text{and} \quad \varepsilon_t \sim TrN(0, \sigma, \varepsilon_{t,min}) \\ &\quad \text{and} \quad \varepsilon_{t,min} = \ln(A_{min}) - \rho z_{t-1} - (1-\rho)\mu \end{aligned} \tag{30}$$

We use these two alternative scenarios of $A_{min} = 0$ versus $A_{min} = 0.75$ to study how the properties of the economy change when there are more negative states of the world $A_{min} = 0$ versus fewer negative states of the world $A_{min} = 0.75$. The economy described by Blanchard (2019) is a case in which many negative states of the economy are assumed away. Many of the conclusions of Blanchard (2019) depend critically on these assumptions of relative safety. We show below that the properties of the economy and the costs of government promises change dramatically when the economy is faced with the possibility of more formidable negative shocks.

[4]The truncated normal distribution $TrN(0, \sigma, \varepsilon_{t,min})$ is the non-truncated normal distribution with mean 0 and standard deviation σ that is then truncated at $\varepsilon_{t,min}$ and rescaled to sum to 1. Technical Appendix T-3 has a description of this distribution.

Table 2: Initial values relative to median values: $H_t = \min\big(w_t n_1, \bar{H}\big)$**,** $A_{min} = 0.00$ **and** $z_0 = 0.0$

	$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
	w_{med} $\bar{H}/w_{med}$	k_{med} $k_{2,0}/k_{med}$	w_{med} $\bar{H}/w_{med}$	k_{med} $k_{2,0}/k_{med}$	w_{med} $\bar{H}/w_{med}$	k_{med} $k_{2,0}/k_{med}$
$\bar{H} = 0.00$	0.281 0.000	0.101 1.093	0.282 0.000	0.101 1.390	0.282 0.000	0.101 1.687
$\bar{H} = 0.05$	0.445 0.112	0.083 1.321	0.449 0.111	0.085 1.654	0.450 0.111	0.085 2.003
$\bar{H} = 0.11$	0.557 0.197	0.064 1.710	0.564 0.195	0.066 2.108	0.572 0.192	0.068 2.515
$\bar{H} = 0.17$	0.648 0.262	0.048 2.274	0.658 0.259	0.051 2.757	0.667 0.255	0.052 3.243

w_{med} is the median wage and k_{med} is the median capital stock across all 3,000 simulations before economic shut down.

Table 3: Initial values relative to median values: $H_t = \min\big(w_t n_1, \bar{H}\big)$**,** $A_{min} = 0.75$ **and** $z_0 = 0.0$

	$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
	w_{med} $\bar{H}/w_{med}$	k_{med} $k_{2,0}/k_{med}$	w_{med} $\bar{H}/w_{med}$	k_{med} $k_{2,0}/k_{med}$	w_{med} $\bar{H}/w_{med}$	k_{med} $k_{2,0}/k_{med}$
$\bar{H} = 0.00$	1.319 0.000	0.388 0.284	1.320 0.000	0.388 0.361	1.321 0.000	0.388 0.438
$\bar{H} = 0.05$	1.158 0.043	0.281 0.392	1.160 0.043	0.281 0.498	1.161 0.043	0.281 0.604
$\bar{H} = 0.11$	0.984 0.112	0.176 0.625	0.988 0.111	0.177 0.789	0.994 0.111	0.179 0.950
$\bar{H} = 0.17$	0.953 0.178	0.128 0.857	0.950 0.179	0.128 1.097	0.960 0.177	0.130 1.303

w_{med} is the median wage and k_{med} is the median capital stock across all 3,000 simulations before economic shut down.

Table 4: Initial values relative to median values: $H_t = \tau w_t n_1$, $A_{min} = 0.00$ and $z_0 = 0.0$

	$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
	w_{med} $H_{t,med}/w_{med}$	k_{med} $k_{2,0}/k_{med}$	w_{med} $H_{t,med}/w_{med}$	k_{med} $k_{2,0}/k_{med}$	w_{med} $H_{t,med}/w_{med}$	k_{med} $k_{2,0}/k_{med}$
$\tau = 0.00$	0.281 0.000	0.101 1.093	0.282 0.000	0.101 1.390	0.282 0.000	0.101 1.687
$\tau = 0.11$	0.248 0.110	0.069 1.592	0.248 0.110	0.069 2.024	0.248 0.110	0.069 2.457
$\tau = 0.20$	0.222 0.200	0.049 2.228	0.222 0.200	0.049 2.834	0.222 0.200	0.049 3.438
$\tau = 0.25$	0.208 0.250	0.040 2.722	0.208 0.250	0.040 3.461	0.208 0.250	0.040 4.200

w_{med} is the median wage and k_{med} is the median capital stock across all 3,000 simulations before economic shut down.

Table 5: Initial values relative to median values: $H_t = \tau w_t n_1$, $A_{min} = 0.75$ and $z_0 = 0.0$

	$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
	w_{med} $H_{t,med}/w_{med}$	k_{med} $k_{2,0}/k_{med}$	w_{med} $H_{t,med}/w_{med}$	k_{med} $k_{2,0}/k_{med}$	w_{med} $H_{t,med}/w_{med}$	k_{med} $k_{2,0}/k_{med}$
$\tau = 0.00$	1.319 0.000	0.388 0.284	1.320 0.000	0.388 0.361	1.321 0.000	0.388 0.438
$\tau = 0.11$	1.106 0.110	0.232 0.474	1.107 0.110	0.232 0.603	1.108 0.110	0.232 0.732
$\tau = 0.20$	0.943 0.200	0.145 0.758	0.943 0.200	0.145 0.964	0.944 0.200	0.145 1.170
$\tau = 0.25$	0.857 0.250	0.109 1.005	0.857 0.250	0.110 1.278	0.858 0.250	0.110 1.551

w_{med} is the median wage and k_{med} is the median capital stock across all 3,000 simulations before economic shut down.

Partial equilibrium effects on measures of central tendency. We test the effect on the measures of central tendency in Tables 2to 5. In the functions below the varialbe $pol \in \{H_t = \min(w_1 n_1, \bar{H}), \tau\}$.

$$w_{med}\big(pol, A_{min}, \bar{H} \text{ or } \tau, k_{2,0}\big)$$
$$k_{med}\big(pol, A_{min}, \bar{H} \text{ or } \tau, k_{2,0}\big)$$
$$\bar{H}/w_{med}\big(pol, A_{min}, \bar{H} \text{ or } \tau, k_{2,0}\big)$$
$$\text{or}$$
$$H_{t,med}/w_{med}\big(pol, A_{min}, \bar{H} \text{ or } \tau, k_{2,0}\big)$$
$$k_{2,0}/k_{med}\big(pol, A_{min}, \bar{H} \text{ or } \tau, k_{2,0}\big)$$

$$\frac{\partial w_{med}(A_{min}=0.00)}{\partial \bar{H}} > 0, \quad \frac{\partial w_{med}(A_{min}=0.75)}{\partial \bar{H}} < 0, \quad \frac{\partial w_{med}}{\partial \tau} < 0, \quad \frac{\partial w_{med}}{\partial k_{2,0}} \approx 0,$$
$$\frac{\partial w_{med}}{\partial pol} \leq 0, \quad \frac{\partial w_{med}}{\partial A_{min}} > 0$$

$$\frac{\partial k_{med}}{\partial \bar{H}}, \frac{\partial k_{med}}{\partial \tau} < 0, \quad \frac{\partial k_{med}}{\partial k_{2,0}} \approx 0, \quad \frac{\partial k_{med}}{\partial pol} \leq 0, \quad \frac{\partial k_{med}}{\partial A_{min}} > 0$$

$$\frac{\partial \bar{H}/w_{med}}{\partial \bar{H}}, \frac{\partial H_{t,med}/w_{med}}{\partial \tau} > 0, \quad \frac{\partial \bar{H}/w_{med}}{\partial k_{2,0}}, \frac{\partial H_{t,med}/w_{med}}{\partial k_{2,0}} \approx 0,$$
$$\frac{\partial H_{t,med}/w_{med}(A_{min}=0.00)}{\partial pol} \approx 0, \quad \frac{\partial H_{t,med}/w_{med}(A_{min}=0.75)}{\partial pol} \geq 0,$$
$$\frac{\partial H_{t,med}/w_{med}(pol=\bar{H})}{\partial A_{min}} \leq 0, \quad \frac{\partial H_{t,med}/w_{med}(pol=\tau)}{\partial A_{min}} = 0$$

$$\frac{\partial k_{2,0}/k_{med}}{\partial \bar{H}}, \frac{\partial k_{2,0}/k_{med}}{\partial \tau} > 0, \quad \frac{\partial k_{2,0}/k_{med}}{\partial k_{2,0}} > 0$$
$$\frac{\partial k_{2,0}/k_{med}}{\partial pol} > 0, \quad \frac{\partial k_{2,0}/k_{med}}{\partial A_{min}} < 0$$

Table 6: Periods to shut down simulation statistics: $A_{min} = 0.00$ and $z_0 = 0.0$

		$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
		Periods	CDF	Periods	CDF	Periods	CDF
$\bar{H} = 0.00$	min	100	1.000	100	1.000	100	1.000
	med	100	1.000	100	1.000	100	1.000
	mean	100	1.000	100	1.000	100	1.000
	max	100	1.000	100	1.000	100	1.000
$\bar{H} = 0.05$	min	1	0.160	1	0.152	1	0.148
	med	4	0.517	4	0.512	4	0.507
	mean	6.1	0.693	6.2	0.689	6.2	0.686
	max	50	1.000	50	1.000	50	1.000
$\bar{H} = 0.11$	min	1	0.344	1	0.328	1	0.317
	med	2	0.534	2	0.522	2	0.512
	mean	3.5	0.713	3.6	0.705	3.6	0.697
	max	27	1.000	27	1.000	27	1.000
$\bar{H} = 0.17$	min	1	0.498	1	0.474	1	0.459
	med	2	0.683	2	0.670	2	0.658
	mean	2.5	0.758	2.6	0.745	2.6	0.732
	max	21	1.000	21	1.000	21	1.000

The "min", "med", "mean", and "max" rows in the "Periods" column represent the minimum, median, mean, and maximum number of periods, respectively, in which the simulated time series hit the economic shut down. The "CDF" column represents the percent of simulations that shut down in t periods or less, where t is the value in the "Periods" column. For the CDF value of the "mean" row, we used linear interpolation.

Table 7: Periods to shut down simulation statistics: $A_{min} = 0.75$ and $z_0 = 0.0$

		$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
		Periods	CDF	Periods	CDF	Periods	CDF
$\bar{H} = 0.00$	min	100	1.000	100	1.000	100	1.000
	med	100	1.000	100	1.000	100	1.000
	mean	100	1.000	100	1.000	100	1.000
	max	100	1.000	100	1.000	100	1.000
$\bar{H} = 0.05$	min	5	0.000	5	0.000	5	0.000
	med	100	1.000	100	1.000	100	1.000
	mean	99.8	0.008	99.8	0.007	99.8	0.007
	max	100	1.000	100	1.000	100	1.000
$\bar{H} = 0.11$	min	2	0.108	2	0.096	2	0.086
	med	23	0.500	25	0.506	25	0.502
	mean	34.1	0.620	34.9	0.608	35.2	0.613
	max	100	1.000	100	1.000	100	1.000
$\bar{H} = 0.17$	min	1	0.302	1	0.261	1	0.228
	med	3	0.506	4	0.521	4	0.501
	mean	8.5	0.692	9.0	0.674	9.4	0.680
	max	100	1.000	100	1.000	100	1.000

The "min", "med", "mean", and "max" rows in the "Periods" column represent the minimum, median, mean, and maximum number of periods, respectively, in which the simulated time series hit the economic shut down. The "CDF" column represents the percent of simulations that shut down in t periods or less, where t is the value in the "Periods" column. For the CDF value of the "mean" row, we used linear interpolation.

Partial equilibrium effects on statistics on periods to shut down across simulations. We test the effect on the statistics on periods to shut down in Tables 6 and 7.

$$
\begin{aligned}
&\frac{\partial \text{min}}{\partial \bar{H}} < 0, \quad \frac{\partial \text{min}}{\partial k_{2,0}} = 0, \frac{\partial \text{min}}{\partial A_{min}} > 0 \\
&\frac{\partial \text{med}}{\partial \bar{H}} > 0, \quad \frac{\partial \text{med}}{\partial k_{2,0}} > 0, \frac{\partial \text{med}}{\partial A_{min}} > 0 \\
&\frac{\partial \text{mean}}{\partial \bar{H}} > 0, \quad \frac{\partial \text{mean}}{\partial k_{2,0}} > 0, \frac{\partial \text{mean}}{\partial A_{min}} > 0 \\
&\frac{\partial \text{max}}{\partial \bar{H}} > 0, \quad \frac{\partial \text{max}}{\partial k_{2,0}} > 0, \frac{\partial \text{max}}{\partial A_{min}} > 0
\end{aligned}
$$

Table 8: Annualized Riskless return $\bar{r}_{t,an}$ simulation statistics: $H_t = \min\big(w_t n_1, \bar{H}\big)$, $A_{min} = 0.00$ and $z_0 = 0.0$

		$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
		$\bar{r}_{t,an}$	CDF	$\bar{r}_{t,an}$	CDF	$\bar{r}_{t,an}$	CDF
	$t = 0$	-2.06%	0.483	-2.12%	0.459	-2.18%	0.440
	min	-4.64%	0.000	-4.64%	0.000	-4.64%	0.000
$\bar{H} = 0.00$	med	-2.01%	0.500	-2.01%	0.500	-2.01%	0.500
	mean	-1.58%	0.645	-1.58%	0.645	-1.59%	0.645
	max	19.01%	1.000	19.01%	1.000	19.01%	1.000
	$t = 0$	-1.47%	0.589	-1.54%	0.563	-1.60%	0.541
	min	-4.39%	0.000	-4.39%	0.000	-4.39%	0.000
$\bar{H} = 0.05$	med	-1.69%	0.500	-1.70%	0.500	-1.71%	0.500
	mean	-1.14%	0.715	-1.14%	0.720	-1.14%	0.720
	max	36.56%	1.000	36.52%	1.000	36.49%	1.000
	$t = 0$	-1.71%	0.651	-1.80%	0.609	-1.87%	0.580
	min	-4.38%	0.000	-4.38%	0.000	-4.38%	0.000
$\bar{H} = 0.11$	med	-1.99%	0.500	-2.00%	0.500	-2.01%	0.500
	mean	-1.42%	0.750	-1.40%	0.759	-1.43%	0.756
	max	34.67%	1.000	36.29%	1.000	32.10%	1.000
	$t = 0$	-1.53%	0.743	-1.71%	0.704	-1.83%	0.674
	min	-4.37%	0.000	-4.37%	0.000	-4.37%	0.000
$\bar{H} = 0.17$	med	-2.13%	0.500	-2.16%	0.500	-2.18%	0.500
	mean	-1.51%	0.751	-1.53%	0.754	-1.59%	0.749
	max	35.69%	1.000	41.27%	1.000	35.76%	1.000

All riskless returns $\bar{r}_{t,an}$ are reported in percentage rates. The rate of return 0.0206 is reported in this table as 2.06%.

Table 9: Annualized Riskless return $\bar{r}_{t,an}$ simulation statistics: $H_t = \min\left(w_t n_1, \bar{H}\right)$, $A_{min} = 0.75$ and $z_0 = 0.0$

		$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
		$\bar{r}_{t,an}$	CDF	$\bar{r}_{t,an}$	CDF	$\bar{r}_{t,an}$	CDF
	$t = 0$	3.91%	0.955	3.69%	0.940	3.51%	0.926
	min	-4.60%	0.000	-4.60%	0.000	-4.60%	0.000
$\bar{H} = 0.00$	med	0.47%	0.500	0.47%	0.500	0.47%	0.500
	mean	0.52%	0.509	0.52%	0.509	0.52%	0.509
	max	6.19%	1.000	6.19%	1.000	6.19%	1.000
	$t = 0$	5.40%	0.912	5.06%	0.895	4.80%	0.879
	min	-4.60%	0.000	-4.60%	0.000	-4.60%	0.000
$\bar{H} = 0.05$	med	1.15%	0.500	1.14%	0.500	1.14%	0.500
	mean	1.43%	0.537	1.43%	0.537	1.42%	0.537
	max	41.95%	1.000	41.95%	1.000	41.95%	1.000
	$t = 0$	7.62%	0.855	7.07%	0.836	6.65%	0.819
	min	-4.57%	0.000	-4.57%	0.000	-4.57%	0.000
$\bar{H} = 0.11$	med	2.26%	0.500	2.24%	0.500	2.23%	0.500
	mean	3.20%	0.588	3.16%	0.587	3.14%	0.586
	max	70.01%	1.000	76.72%	1.000	74.84%	1.000
	$t = 0$	9.56%	0.859	8.92%	0.838	8.43%	0.822
	min	-4.29%	0.000	-4.29%	0.000	-4.30%	0.000
$\bar{H} = 0.17$	med	3.13%	0.500	3.13%	0.500	3.09%	0.500
	mean	4.30%	0.584	4.26%	0.582	4.21%	0.584
	max	69.65%	1.000	67.07%	1.000	70.83%	1.000

All riskless returns $\bar{r}_{t,an}$ are reported in percentage rates. The rate of return 0.0206 is reported in this table as 2.06%.

Table 10: Annualized Riskless return $\bar{r}_{t,an}$ simulation statistics: $H_t = \tau w_t n_1$, $A_{min} = 0.00$ and $z_0 = 0.0$

		$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
		$\bar{r}_{t,an}$	CDF	$\bar{r}_{t,an}$	CDF	$\bar{r}_{t,an}$	CDF
$\tau = 0.00$	$t = 0$	-2.06%	0.483	-2.12%	0.459	-2.18%	0.440
	min	-4.64%	0.000	-4.64%	0.000	-4.64%	0.000
	med	-2.01%	0.500	-2.01%	0.500	-2.01%	0.500
	mean	-1.58%	0.645	-1.58%	0.645	-1.59%	0.645
	max	20.00%	1.000	20.00%	1.000	20.00%	1.000
$\tau = 0.11$	$t = 0$	-2.08%	0.446	-2.15%	0.423	-2.20%	0.405
	min	-4.63%	0.000	-4.63%	0.000	-4.63%	0.000
	med	-1.93%	0.500	-1.93%	0.500	-1.93%	0.500
	mean	-1.34%	0.678	-1.34%	0.678	-1.35%	0.678
	max	22.45%	1.000	22.45%	1.000	22.45%	1.000
$\tau = 0.20$	$t = 0$	-2.07%	0.415	-2.13%	0.393	-2.19%	0.374
	min	-4.61%	0.000	-4.61%	0.000	-4.61%	0.000
	med	-1.81%	0.500	-1.81%	0.500	-1.81%	0.500
	mean	-1.06%	0.705	-1.06%	0.705	-1.06%	0.705
	max	24.51%	1.000	24.51%	1.000	24.51%	1.000
$\tau = 0.25$	$t = 0$	-2.04%	0.397	-2.11%	0.374	-2.17%	0.356
	min	-4.61%	0.000	-4.61%	0.000	-4.61%	0.000
	med	-1.72%	0.500	-1.72%	0.500	-1.72%	0.500
	mean	-0.86%	0.721	-0.86%	0.721	-0.86%	0.721
	max	25.71%	1.000	25.71%	1.000	25.71%	1.000

All riskless returns $\bar{r}_{t,an}$ are reported in percentage rates. The rate of return 0.0206 is reported in this table as 2.06%.

Table 11: Annualized Riskless return $\bar{r}_{t,an}$ simulation statistics: $H_t = \tau w_t n_1$, $A_{min} = 0.75$ and $z_0 = 0.0$

		$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
		$\bar{r}_{t,an}$	CDF	$\bar{r}_{t,an}$	CDF	$\bar{r}_{t,an}$	CDF
$\tau = 0.00$	$t = 0$	3.91%	0.955	3.69%	0.940	3.51%	0.926
	min	-4.60%	0.000	-4.60%	0.000	-4.60%	0.000
	med	0.47%	0.500	0.47%	0.500	0.47%	0.500
	mean	0.52%	0.509	0.52%	0.509	0.52%	0.509
	max	6.19%	1.000	6.19%	1.000	6.19%	1.000
$\tau = 0.11$	$t = 0$	4.62%	0.916	4.38%	0.896	4.19%	0.878
	min	-4.59%	0.000	-4.59%	0.000	-4.59%	0.000
	med	1.31%	0.500	1.31%	0.500	1.31%	0.500
	mean	1.33%	0.503	1.33%	0.503	1.33%	0.503
	max	7.80%	1.000	7.80%	1.000	7.80%	1.000
$\tau = 0.20$	$t = 0$	5.23%	0.872	4.98%	0.849	4.79%	0.829
	min	-4.57%	0.000	-4.57%	0.000	-4.57%	0.000
	med	2.15%	0.500	2.15%	0.500	2.15%	0.500
	mean	2.13%	0.497	2.12%	0.497	2.12%	0.497
	max	9.20%	1.000	9.20%	1.000	9.20%	1.000
$\tau = 0.25$	$t = 0$	5.59%	0.844	5.34%	0.819	5.13%	0.798
	min	-4.55%	0.000	-4.55%	0.000	-4.55%	0.000
	med	2.68%	0.500	2.68%	0.500	2.68%	0.500
	mean	2.63%	0.493	2.63%	0.493	2.63%	0.493
	max	10.03%	1.000	10.03%	1.000	10.03%	1.000

All riskless returns $\bar{r}_{t,an}$ are reported in percentage rates. The rate of return 0.0206 is reported in this table as 2.06%.

Table 12: Components of the equity premium in annual terms: $H_t = \min\left(w_t n_1, \bar{H}\right)$, $A_{min} = 0.00$ and $z_0 = 0.0$

		$k_{2,0} = 0.11$	$k_{2,0} = 0.14$	$k_{2,0} = 0.17$
$\bar{H} = 0.00$	Avg. $E[R_{t+1}]$	102.7%	102.7%	102.7%
	$\sigma(R_{t+1})$	5.090	5.090	5.089
	Avg. $\bar{R}_t$	98.4%	98.4%	98.4%
	Avg. eq. prem. $E[R_{t+1}] - \bar{R}_t]$	4.3%	4.3%	4.3%
	Avg. Sharpe ratio $\frac{E[R_{t+1}]-\bar{R}_t]}{\sigma(R_{t+1})}$	0.848	0.848	0.847
$\bar{H} = 0.05$	Avg. $E[R_{t+1}]$	104.1%	104.0%	103.9%
	$\sigma(R_{t+1})$	5.546	5.538	5.525
	Avg. $\bar{R}_t$	98.9%	98.9%	98.9%
	Avg. eq. prem. $E[R_{t+1}] - \bar{R}_t]$	5.2%	5.1%	5.1%
	Avg. Sharpe ratio $\frac{E[R_{t+1}]-\bar{R}_t]}{\sigma(R_{t+1})}$	0.938	0.925	0.918
$\bar{H} = 0.11$	Avg. $E[R_{t+1}]$	105.1%	104.9%	104.8%
	$\sigma(R_{t+1})$	5.593	5.516	5.473
	Avg. $\bar{R}_t$	98.6%	98.6%	98.6%
	Avg. eq. prem. $E[R_{t+1}] - \bar{R}_t]$	6.6%	6.3%	6.2%
	Avg. Sharpe ratio $\frac{E[R_{t+1}]-\bar{R}_t]}{\sigma(R_{t+1})}$	1.171	1.148	1.140
$\bar{H} = 0.17$	Avg. $E[R_{t+1}]$	106.4%	106.1%	105.8%
	$\sigma(R_{t+1})$	5.789	5.627	5.553
	Avg. $\bar{R}_t$	98.5%	98.5%	98.4%
	Avg. eq. prem. $E[R_{t+1}] - \bar{R}_t]$	7.9%	7.6%	7.4%
	Avg. Sharpe ratio $\frac{E[R_{t+1}]-\bar{R}_t]}{\sigma(R_{t+1})}$	1.367	1.352	1.337

The gross risky one-period return on capital is $R_{t+1} = 1 + r_{t+1}$ and the average expected gross risky return (Avg. $E[R_{t+1}]$) is the average value of R_{t+1} across simulations. The annualized gross risky one-period return is $(R_{t+1})^{1/30}$. The standard deviation of R_{t+1} is just the standard deviation of its realized value across simulations. The average riskless gross return (Avg. $\bar{R}_t$) is the average value across simulations, where $\bar{R}_t = 1 + \bar{r}_t$.

Table 13: Components of the equity premium in annual terms: $H_t = \min\big(w_t n_1, \bar{H}\big)$, $A_{min} = 0.75$ and $z_0 = 0.0$

		$k_{2,0} = 0.11$	$k_{2,0} = 0.14$	$k_{2,0} = 0.17$
$\bar{H} = 0.00$	Avg. $E[R_{t+1}]$	102.8%	102.8%	102.8%
	$\sigma(R_{t+1})$	3.944	3.941	3.938
	Avg. $\bar{R}_t$	100.5%	100.5%	100.5%
	Avg. eq. prem. $E[R_{t+1}] - \bar{R}_t]$	2.3%	2.3%	2.3%
	Avg. Sharpe ratio $\frac{E[R_{t+1}] - \bar{R}_t]}{\sigma(R_{t+1})}$	0.584	0.583	0.583
$\bar{H} = 0.05$	Avg. $E[R_{t+1}]$	103.7%	103.7%	103.6%
	$\sigma(R_{t+1})$	4.418	4.412	4.408
	Avg. $\bar{R}_t$	101.4%	101.4%	101.4%
	Avg. eq. prem. $E[R_{t+1}] - \bar{R}_t]$	2.2%	2.2%	2.2%
	Avg. Sharpe ratio $\frac{E[R_{t+1}] - \bar{R}_t]}{\sigma(R_{t+1})}$	0.504	0.504	0.504
$\bar{H} = 0.11$	Avg. $E[R_{t+1}]$	105.1%	105.0%	105.0%
	$\sigma(R_{t+1})$	5.462	5.436	5.408
	Avg. $\bar{R}_t$	103.2%	103.2%	103.1%
	Avg. eq. prem. $E[R_{t+1}] - \bar{R}_t]$	1.9%	1.9%	1.9%
	Avg. Sharpe ratio $\frac{E[R_{t+1}] - \bar{R}_t]}{\sigma(R_{t+1})}$	0.345	0.347	0.347
$\bar{H} = 0.17$	Avg. $E[R_{t+1}]$	106.3%	106.2%	106.1%
	$\sigma(R_{t+1})$	6.257	6.131	6.030
	Avg. $\bar{R}_t$	104.3%	104.3%	104.2%
	Avg. eq. prem. $E[R_{t+1}] - \bar{R}_t]$	2.0%	1.9%	1.9%
	Avg. Sharpe ratio $\frac{E[R_{t+1}] - \bar{R}_t]}{\sigma(R_{t+1})}$	0.324	0.318	0.310

The gross risky one-period return on capital is $R_{t+1} = 1 + r_{t+1}$ and the average expected gross risky return (Avg. $E[R_{t+1}]$) is the average value of R_{t+1} across simulations. The annualized gross risky one-period return is $(R_{t+1})^{1/30}$. The standard deviation of R_{t+1} is just the standard deviation of its realized value across simulations. The average riskless gross return (Avg. $\bar{R}_t$) is the average value across simulations, where $\bar{R}_t = 1 + \bar{r}_t$.

Table 14: Components of the equity premium in annual terms: $H_t = \tau w_t n_1$, $A_{min} = 0.00$ and $z_0 = 0.0$

		$k_{2,0} = 0.11$	$k_{2,0} = 0.14$	$k_{2,0} = 0.17$
	Avg. $E[R_{t+1}]$	102.7%	102.7%	102.7%
	$\sigma(R_{t+1})$	5.090	5.090	5.089
$\tau = 0.00$	Avg. $\bar{R}_t$	98.4%	98.4%	98.4%
	Avg. eq. prem. $E[R_{t+1}] - \bar{R}_t]$	4.3%	4.3%	4.3%
	Avg. Sharpe ratio $\frac{E[R_{t+1}]-\bar{R}_t]}{\sigma(R_{t+1})}$	0.848	0.848	0.847
	Avg. $E[R_{t+1}]$	103.4%	103.4%	103.4%
	$\sigma(R_{t+1})$	5.273	5.272	5.272
$\tau = 0.11$	Avg. $\bar{R}_t$	98.7%	98.7%	98.7%
	Avg. eq. prem. $E[R_{t+1}] - \bar{R}_t]$	4.7%	4.7%	4.7%
	Avg. Sharpe ratio $\frac{E[R_{t+1}]-\bar{R}_t]}{\sigma(R_{t+1})}$	0.895	0.895	0.894
	Avg. $E[R_{t+1}]$	104.0%	103.9%	103.9%
	$\sigma(R_{t+1})$	5.417	5.417	5.416
$\tau = 0.20$	Avg. $\bar{R}_t$	98.9%	98.9%	98.9%
	Avg. eq. prem. $E[R_{t+1}] - \bar{R}_t]$	5.0%	5.0%	5.0%
	Avg. Sharpe ratio $\frac{E[R_{t+1}]-\bar{R}_t]}{\sigma(R_{t+1})}$	0.925	0.925	0.925
	Avg. $E[R_{t+1}]$	104.3%	104.3%	104.3%
	$\sigma(R_{t+1})$	5.493	5.493	5.493
$\tau = 0.25$	Avg. $\bar{R}_t$	99.1%	99.1%	99.1%
	Avg. eq. prem. $E[R_{t+1}] - \bar{R}_t]$	5.2%	5.2%	5.1%
	Avg. Sharpe ratio $\frac{E[R_{t+1}]-\bar{R}_t]}{\sigma(R_{t+1})}$	0.938	0.938	0.938

The gross risky one-period return on capital is $R_{t+1} = 1 + r_{t+1}$ and the average expected gross risky return (Avg. $E[R_{t+1}]$) is the average value of R_{t+1} across simulations. The annualized gross risky one-period return is $(R_{t+1})^{1/30}$. The standard deviation of R_{t+1} is just the standard deviation of its realized value across simulations. The average riskless gross return (Avg. $\bar{R}_t$) is the average value across simulations, where $\bar{R}_t = 1 + \bar{r}_t$.

Table 15: Components of the equity premium in annual terms: $H_t = \tau w_t n_1$, $A_{min} = 0.75$ and $z_0 = 0.0$

		$k_{2,0} = 0.11$	$k_{2,0} = 0.14$	$k_{2,0} = 0.17$
$\tau = 0.00$	Avg. $E[R_{t+1}]$	102.8%	102.8%	102.8%
	$\sigma(R_{t+1})$	3.944	3.941	3.938
	Avg. $\bar{R}_t$	100.5%	100.5%	100.5%
	Avg. eq. prem. $E[R_{t+1}] - \bar{R}_t]$	2.3%	2.3%	2.3%
	Avg. Sharpe ratio $\frac{E[R_{t+1}]-\bar{R}_t]}{\sigma(R_{t+1})}$	0.584	0.583	0.583
$\tau = 0.11$	Avg. $E[R_{t+1}]$	103.8%	103.8%	103.8%
	$\sigma(R_{t+1})$	4.193	4.190	4.187
	Avg. $\bar{R}_t$	101.3%	101.3%	101.3%
	Avg. eq. prem. $E[R_{t+1}] - \bar{R}_t]$	2.5%	2.5%	2.5%
	Avg. Sharpe ratio $\frac{E[R_{t+1}]-\bar{R}_t]}{\sigma(R_{t+1})}$	0.592	0.592	0.592
$\tau = 0.20$	Avg. $E[R_{t+1}]$	104.7%	104.7%	104.7%
	$\sigma(R_{t+1})$	4.404	4.401	4.399
	Avg. $\bar{R}_t$	102.1%	102.1%	102.1%
	Avg. eq. prem. $E[R_{t+1}] - \bar{R}_t]$	2.6%	2.6%	2.6%
	Avg. Sharpe ratio $\frac{E[R_{t+1}]-\bar{R}_t]}{\sigma(R_{t+1})}$	0.594	0.593	0.593
$\tau = 0.25$	Avg. $E[R_{t+1}]$	105.3%	105.3%	105.3%
	$\sigma(R_{t+1})$	4.523	4.520	4.518
	Avg. $\bar{R}_t$	102.6%	102.6%	102.6%
	Avg. eq. prem. $E[R_{t+1}] - \bar{R}_t]$	2.7%	2.7%	2.7%
	Avg. Sharpe ratio $\frac{E[R_{t+1}]-\bar{R}_t]}{\sigma(R_{t+1})}$	0.593	0.593	0.593

The gross risky one-period return on capital is $R_{t+1} = 1+r_{t+1}$ and the average expected gross risky return (Avg. $E[R_{t+1}]$) is the average value of R_{t+1} across simulations. The annualized gross risky one-period return is $(R_{t+1})^{1/30}$. The standard deviation of R_{t+1} is just the standard deviation of its realized value across simulations. The average riskless gross return (Avg. $\bar{R}_t$) is the average value across simulations, where $\bar{R}_t = 1 + \bar{r}_t$.

4 Conclusion

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TECHNICAL APPENDIX

T-1 Blanchard (2019) model and calibration

This section shows how the Blanchard (2019) model is a nested case of the model described in the body of this paper, and it derives and computes some of the results from Blanchard (2019).

T-1.1 Households

Blanchard (2019) assumes that a unit measure of identical households is born each period and live for two periods. A household supplies a unit of labor inelastically when young $n_{1,t} = 1$ for all t and does not work when old $n_{2,t} = 0$ for all t. Young households have lump sum amount $\bar{H}$ taken from them and given to the current period old each period. Young households also receive an endowment x_1 each period. This endowment comes exogenously from some other economy and does not figure into this economy's government budget constraint. Households choose how much to consume each period $c_{1,t}$ and $c_{2,t+1}$ and how much to save in terms of risky savings $k_{2,t+1}$ and riskless bonds $b_{2,t+1}$. The household maximization problem is the following,

$$\max_{k_{2,t+1},b_{2,t+1}} (1-\beta)\ln(c_{1,t}) + \beta\frac{1}{1-\gamma}\ln\Big(E_t\big[(c_{2,t+1})^{1-\gamma}\big]\Big) \quad \forall t \tag{1}$$

$$\text{such that} \quad c_{1,t} + k_{2,t+1} + p_t b_{2,t+1} = w_t + x_1 - \bar{H} \tag{T.1.1}$$

$$\text{and} \quad c_{2,t+1} = R_{t+1}k_{2,t+1} + b_{2,t+1} + \bar{H} \tag{T.1.2}$$

$$\text{and} \quad c_{1,t}, c_{2,t+1}, k_{2,t+1} \geq 0 \tag{T.1.3}$$

where R_t is the gross return on risky savings, w_t is the wage on the unit of inelastically supplied labor by the young, and p_t is the price per unit of the riskless bond. The resulting Euler equation for optimal risky savings $k_{2,t+1}$ is the following.

$$\frac{1-\beta}{c_{1,t}} = \beta\frac{E_t\Big[R_{t+1}\big(c_{2,t+1}\big)^{-\gamma}\Big]}{E_t\Big[\big(c_{2,t+1}\big)^{1-\gamma}\Big]} \quad \forall t \tag{T.1.4}$$

And the resulting Euler equation for optimal riskless savings $b_{2,t+1}$ is the following.

$$\begin{aligned} \frac{1}{\bar{R}_t} \equiv p_t &= \left(\frac{\beta}{1-\beta}\right)\frac{(c_{1,t})E_t\Big[\big(c_{2,t+1}\big)^{-\gamma}\Big]}{E_t\Big[\big(c_{2,t+1}\big)^{1-\gamma}\Big]} \quad \forall t \\ \Rightarrow \quad \bar{R}_t &= \left(\frac{1-\beta}{\beta}\right)\frac{E_t\Big[\big(c_{2,t+1}\big)^{1-\gamma}\Big]}{(c_{1,t})E_t\Big[\big(c_{2,t+1}\big)^{-\gamma}\Big]} \quad \forall t \end{aligned} \tag{T.1.5}$$

Substituting the period budget constraints (T.1.1) and (T.1.2) into the two Euler equations (T.1.4) and (T.1.5), we can show that optimal risky savings $k_{2,t+1}$ and riskless savings $b_{2,t+1}$ are functions $\psi(\cdot)$ and $\phi(\cdot)$, respectively, of the time paths of transfers and prices over the lifetime of the household,

$$k_{2,t+1} = \psi\big(\bar{H}, w_t, R_{t+1}\big) \quad \forall t \tag{T.1.6}$$

$$b_{2,t+1} = \phi\big(\bar{H}, w_t, R_{t+1}\big) \quad \forall t \tag{T.1.7}$$

where R_{t+1} is in the expectations operator.

Implicit in Blanchard (2019) is the assumption of, generally, an exogenous supply of riskless bonds that is nonnegative $B_t \geq 0$ for all t. However, this model specifically assumes a zero supply of riskless bonds $B_t = 0$. So the general version of our riskless bond market clearing condition is the following.

$$b_{2,t} = B_t \quad \forall t \tag{T.1.8}$$

With the zero supply assumption $B_t = 0$, the household demand for riskless bonds is zero in equilibrium through the market clearing condition,

$$b_{2,t} = 0 \quad \forall t \tag{T.1.9}$$

all the other endogenous variables are determined by the equilibrium described without the riskless bonds, and the riskless return $\bar{R}_t$ is characterized by Euler equation (T.1.5).

T-1.2 Firms

A unit measure of identical perfectly competitive firms exist in this economy that hire aggregate labor L_t at wage w_t and rent aggregate capital K_t at rental rate r_t every period in order to produce consumption good Y_t according to a Cobb-Douglas production function,

$$Y_t = F(K_t, L_t, z_t) = A_t\Big[\alpha(K_t)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\alpha)(L_t)^{\frac{\varepsilon-1}{\varepsilon}}\Big]^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall t \tag{8}$$

where the capital share of income is given by $\alpha \in (0,1)$ and $\varepsilon > 0$ is the constant elasticity of substitution between capital and labor in the production process. Total factor productivity $A_t \equiv e^{z_t} > 0$ is distributed log normally, and z_t follows a normally distributed $AR(1)$ process. Two important special parameterizations are the unit elasticity case $\varepsilon = 1$ in which the limit of (8) is the Cobb-Douglas production function and the perfectly elastic case $\varepsilon = \infty$ in which the production function is linear in K_t and L_t (perfect substitutes).

$$\begin{aligned} z_t &= \rho z_{t-1} + (1-\rho)\mu + \epsilon_t \\ &\text{where} \quad \rho \in [0,1), \quad \mu \geq 0, \quad \text{and} \quad \epsilon_t \sim N(0,\sigma) \end{aligned} \tag{T.1.10}$$

The firm's problem each period is to choose how much capital K_t to rent and how much labor L_t to hire in order to maximize profits,

$$\max_{K_t, L_t} Pr_t = F(K_t, L_t, z_t) - w_t L_t - R_t K_t \quad \forall t \tag{T.1.11}$$

where this equation implies full depreciation of capital each period $\delta = 1$. Profit maximization implies that the real wage and real rental rate are determined by the standard first order conditions for the firm.

$$R_t = \alpha (A_t)^{\frac{\varepsilon-1}{\varepsilon}} \left[\frac{Y_t}{K_t}\right]^{\frac{1}{\varepsilon}} \quad \forall t \tag{T.1.12}$$

$$w_t = (1-\alpha)(A_t)^{\frac{\varepsilon-1}{\varepsilon}} \left[\frac{Y_t}{L_t}\right]^{\frac{1}{\varepsilon}} \quad \forall t \tag{T.1.13}$$

Because the risky interest rate R_t in (T.1.12) is not defined when the capital stock is zero $K_t = 0$ and because the wage w_t in (T.1.13) is not defined when aggregate labor is zero $L_t = 0$, we know that both values must be strictly positive $K_t, L_t > 0$.

Blanchard (2019) looks at two cases of the production function. Perfect substitutes ($\varepsilon = \infty$) is the simplest case in which the production function simplifies to the following linear function of K_t and L_t and the first order conditions become independent of K_t and L_t and simply functions of α and A_t.

$$Y_t = A_t\big[\alpha K_t + (1-\alpha)L_t\big] \quad \forall t \tag{T.1.14}$$

$$R_t = \alpha A_t \quad \forall t \tag{T.1.15}$$

$$w_t = (1-\alpha)(A_t) \quad \forall t \tag{T.1.16}$$

The second case is that of unit elasticity ($\varepsilon = 1$), which results in a Cobb-Douglas production function of K_t and L_t with the corresponding first order conditions.

$$Y_t = A_t\big(K_t\big)^{\alpha}\big(L_t\big)^{1-\alpha} \quad \forall t \tag{T.1.17}$$

$$R_t = \alpha A_t \left(\frac{L_t}{K_t}\right)^{1-\alpha} \quad \forall t \tag{T.1.18}$$

$$w_t = (1-\alpha)(A_t)\left(\frac{K_t}{L_t}\right)^{\alpha} \quad \forall t \tag{T.1.19}$$

T-1.3 Government budget constraint and market clearing

The government budget constraint in Blanchard (2019) is a simple balanced one in which revenues taken lump sum from the young in every period $\bar{H}$ equal transfer expenditures given to the old each period $\bar{H}$. This does not cause any default or inability for the young to pay because the Blanchard analysis assumes the endowment x_1 is big enough so that no adverse shock will make $\bar{H} \geq w_t + x_1$.

The model includes four market clearing conditions, only three of which are necessary for the solution–the labor market (T.1.20), risky capital market (15), riskless bond market (T.1.21), and the goods market (T.1.22). We will leave the goods market clearing condition (T.1.22) out of the solution method due to its redundancy by

Walras' Law.

$$L_t = 1 \quad \forall t \tag{T.1.20}$$
$$K_t = k_{2,t} \quad \forall t \tag{15}$$
$$B_t = 0 \quad \forall t \tag{T.1.21}$$
$$\begin{split} &Y_t = C_t + K_{t+1} \quad \forall t \\ &\quad \text{where} \quad C_t \equiv c_{1,t} + c_{2,t} \quad \text{and} \quad K_{t+1} = I_t \end{split} \tag{T.1.22}$$

T-1.4 Equilibrium

The equilibrium in Blanchard (2019) is similar to Definition 1 from Section 2.5.

Definition 2 (Blanchard (2019) functional stationary equilibrium). A non-autarkic functional stationary equilibrium in the two-period-lived overlapping generations model with exogenous labor supply and aggregate shocks in Blanchard (2019) is defined by stationary price functions $R(k,z)$, $w(k,z)$, and $\bar{R}(k,z)$ and a stationary risky savings function $k' = \psi(k,z)$ for all current state wealth k and total factor productivity component z such that:

i. households optimize according to (T.1.1) and (T.1.2), (T.1.4), and (T.1.5)

ii. firms optimize according to (T.1.12) and (T.1.13),

iii. markets clear according to (14) and (15).

T-1.4.1 Zero transfers and perfect substitutes

When transfers are zero $\bar{H} = 0$ and capital and labor are perfect substitutes in production $\varepsilon = \infty$, the equilibrium has an analytical solution.

$$R_t = \alpha e^{z_t} \quad \forall z_t \tag{T.1.23}$$
$$w_t = (1-\alpha)e^{z_t} \quad \forall z_t \tag{T.1.24}$$
$$\bar{R}_t = \alpha e^{\rho z_t + (1-\rho)\mu + \frac{\sigma^2(1-2\gamma)}{2}} \quad \forall z_t \tag{T.1.25}$$
$$c_{1,t} = (1-\beta)\Big([1-\alpha]e^{z_t} + x_1\Big) \quad \forall z_t \tag{T.1.26}$$
$$k_{2,t+1} = \beta\Big([1-\alpha]e^{z_t} + x_1\Big) \quad \forall z_t \tag{T.1.27}$$
$$c_{2,t} = \alpha e^{z_t} k_{2,t} \quad \forall k_{2,t}, z_t \tag{T.1.28}$$

An important relationship that comes out of the equilibrium solution described above is the percent spread between the expected risky gross return next period and the current riskless return.

$$E_t\big[R_{t+1}\big] = \alpha e^{\rho z_t + (1-\rho)\mu + \frac{\sigma^2}{2}} \quad \forall t \tag{T.1.29}$$
$$\ln\Big(E_t\big[R_{t+1}\big]\Big) - \ln\big(\bar{R}_t\big) = \gamma\sigma^2 \quad \forall t \tag{T.1.30}$$

T-1.4.2 Zero transfers and unit elasticity

When transfers are zero $\bar{H} = 0$ and capital and labor have unit elasticity in the production function $\varepsilon = 1$, the equilibrium also has an analytical solution.

$$R_t = \alpha e^{z_t}\big(k_{2,t}\big)^{\alpha-1} \quad \forall k_{2,t}, z_t \tag{T.1.31}$$

$$w_t = (1-\alpha)e^{z_t}\big(k_{2,t}\big)^{\alpha} \quad \forall k_{2,t}, z_t \tag{T.1.32}$$

$$\bar{R}_t = \frac{\alpha e^{\rho z_t + (1-\rho)\mu + \frac{\sigma^2(1-2\gamma)}{2}}}{\Big(\beta\big[(1-\alpha)e^{z_t}(k_{2,t})^{\alpha} + x_1\big]\Big)^{1-\alpha}} \quad \forall k_{2,t}, z_t \tag{T.1.33}$$

$$c_{1,t} = (1-\beta)\Big([1-\alpha]e^{z_t}\big(k_{2,t}\big)^{\alpha} + x_1\Big) \quad \forall k_{2,t}, z_t \tag{T.1.34}$$

$$k_{2,t+1} = \beta\Big([1-\alpha]e^{z_t}\big(k_{2,t}\big)^{\alpha} + x_1\Big) \quad \forall k_{2,t}, z_t \tag{T.1.35}$$

$$c_{2,t} = \alpha e^{z_t}\big(k_{2,t}\big)^{\alpha} \quad \forall k_{2,t}, z_t \tag{T.1.36}$$

The analogous relationship to (T.1.30) that comes out of the equilibrium solution described above is the percent spread between the expected risky gross return next period and the current riskless return.

$$E_t\big[R_{t+1}\big] = \alpha e^{\rho z_t + (1-\rho)\mu + \frac{\sigma^2}{2}}\big(k_{2,t+1}\big)^{\alpha-1} \quad \forall t \tag{T.1.37}$$

$$\ln\Big(E_t\big[R_{t+1}\big]\Big) - \ln\big(\bar{R}_t\big) = \gamma\sigma^2 \quad \forall t \tag{T.1.30}$$

T-1.5 Calibration

Blanchard assumes that households inelastically supply a unit of labor when young $n_{1,t} = 1$ and supply no labor when old $n_{2,t} = 0$ for all t. He calibrates the capital share of income parameter $\alpha = 1/3$. He calibrates the annual standard deviation of the normally distributed component of z_t the total factor productivity process to be $\sigma_{an} = 0.2$, which implies a model 25-year standard deviation of $\sigma \approx 0.615$. Blanchard assumes full depreciation of capital each period $\delta = 1$.

Given a calibrated value for σ, Blanchard (2019, p. 1213) identifies the value of μ independently of β using the linear production expression for the expected value of the marginal product of capital (T.1.29),

$$E_t\big[R_{t+1}\big] = \alpha e^{\rho z_t + (1-\rho)\mu + \frac{\sigma^2}{2}} \quad \forall t \tag{T.1.29}$$

and calibrates γ from the difference in the expected marginal product from the riskless rate (T.1.30), which expression holds in both the linear and Cobb-Douglas production cases.

$$\ln\Big(E_t\big[R_{t+1}\big]\Big) - \ln\big(\bar{R}_t\big) = \gamma\sigma^2 \quad \forall t \tag{T.1.30}$$

He then identifies β independent of μ using the Cobb-Douglas expression for the expected value of the marginal product of capital, the derivation of which is given below in the lead up to (T.1.45). This use of two separate models to identify two respective parameters to be used in the same model is justified given the independence of the identifying equations on the other parameter.

To derive the independent expression for β from the Cobb-Douglas specification of the model, we must solve for the long run average of R_{t+1}, $k_{2,t}$ and w_t, of which x_1 is a function. The long-run expected value version of the expected marginal product of capital from (T.1.37) is the following.

$$E\big[R_{t+1}\big] = \alpha e^{\mu+\frac{\sigma^2}{2}}\beta^{\alpha-1}\big[(1-\alpha)e^{\mu}(\bar{k}_2)^{\alpha} + x_1\big]^{\alpha-1} \tag{T.1.38}$$

We solve for the average capital stock as the expected value of savings tomorrow $E_t\big[k_{2,t+2}\big]$.

$$E_t\big[k_{2,t+2}\big] = \beta\Big[(1-\alpha)e^{\rho z_t+(1-\rho)\mu+\frac{\sigma^2}{2}}\big(k_{2,t+1}\big)^{\alpha} + x_1\Big] \quad \forall t \tag{T.1.39}$$

Then let $\bar{k}_2$ be the average $k_{2,t}$ across a simulation by setting $k_{2,t} = \bar{k}_2$ for all t in (T.1.39) and set z_t to its average value $z_t = \mu$ for all t.

$$\bar{k}_2 = \beta\Big[(1-\alpha)e^{\mu+\frac{\sigma^2}{2}}\big(\bar{k}_2\big)^{\alpha} + x_1\Big] \tag{T.1.40}$$

We solve for the average wage as the expected value of the wage tomorrow $E_t[w_{t+1}]$.

$$E_t\big[w_{t+1}\big] = (1-\alpha)e^{\rho z_t+(1-\rho)\mu+\frac{\sigma^2}{2}}\big(k_{2,t+1}\big)^{\alpha} \quad \forall t \tag{T.1.41}$$

Then set $k_{2,t} = \bar{k}_2$ and $z_t = \mu$ for all t, and the average wage $\bar{w}$ is the following.

$$\bar{w} = (1-\alpha)e^{\mu+\frac{\sigma^2}{2}}\big(\bar{k}_2\big)^{\alpha} \tag{T.1.42}$$

If we calibrate x_1 to be 100 percent of the average wage, then we can rewrite (T.1.42).

$$x_1 = (1-\alpha)e^{\mu+\frac{\sigma^2}{2}}\big(\bar{k}_2\big)^{\alpha} \tag{T.1.43}$$

Substituting (T.1.43) into (T.1.40) gives the following equation.

$$\bar{k}_2 = 2\beta x_1 \tag{T.1.44}$$

Then dividing (T.1.43) by (T.1.38) and substituting in (T.1.44) gives an expression for β independent of μ, x_1, and $\bar{k}_2$.

$$\beta = \left(\frac{\alpha}{1-\alpha}\right)\frac{1}{2E[R_{t+1}]} \tag{T.1.45}$$

Substituting (T.1.44) into (T.1.43), we can solve for x_1 as a function of μ and β.

$$x_1 = \Big[(1-\alpha)e^{\mu+\frac{\sigma^2}{2}}(2\beta)^{\alpha}\Big]^{\frac{1}{1-\alpha}} \tag{T.1.46}$$

Finally, we solve for the long-run value of wealth $\bar{k}_2$ but substituting the expression for x_1 from (T.1.46) into (T.1.44).

$$\bar{k}_2 = 2\beta\Big[(1-\alpha)e^{\mu+\frac{\sigma^2}{2}}(2\beta)^{\alpha}\Big]^{\frac{1}{1-\alpha}} \tag{T.1.47}$$

T-1.6 Problems with Blanchard (2019) approach

i. The calibration approach for β and μ is contradictory. Blanchard calibrates μ based on one model, the model with perfect substitutes production $\varepsilon = \infty$. He assumes the unit elasticity Cobb-Douglas case $\varepsilon = 1$ to calibrate β based on the independence of the $E_t[R_{t+1}]$ equation of the respective parameter in each case. But β and μ are calibrated with two different models. [NOTE: The identification of γ is nice using the $\ln(E_t[R_{t+1}] - \ln(\bar{R}_t) = \gamma\sigma^2$ equation.]

T-2 Description of calibration

This section details our calibration of the parameter values listed in Table 1. In our two-period-lived agent OG model, we assume that each period represents 30 years or, equivalently, a lifetime is 60 years. The model-period (30-year) discount factor β is set to match the annual discount factor common in the RBC literature of 0.96.

$$\beta = (0.96)^{30} \approx 0.2939 \tag{T.2.1}$$

We set the coefficient of relative risk aversion at a midrange value of $\gamma = 2$. This value lies in the midrange of values that have been used in the literature.[5] The capital share of income parameter is set to match the U.S. average $\alpha = 0.35$, and the model-period (30-year) depreciation rate δ is set to match an annual depreciation rate of 5 percent.

$$\delta = 1 - (1 - 0.05)^{30} \approx 0.7854 \tag{T.2.2}$$

The firms' production function in our model is the following,

$$Y_t = e^{z_t} K_t^{\alpha} L_t^{1-\alpha} \quad \forall t \tag{8}$$

where labor L_t is supplied inelastically and z_t is current-period normally distributed component of total factor productivity. We assume that z_t is an AR(1) process with normally distributed errors.

$$\begin{aligned} z_t &= \rho z_{t-1} + (1-\rho)\mu + \varepsilon_t \\ &\quad \text{where} \quad \rho \in [0,1), \quad \mu \geq 0, \quad \text{and} \quad \varepsilon_t \sim N(0,\sigma) \end{aligned} \tag{T.1.10}$$

This implies that the shock process e^{z_t} is lognormally distributed $LN(\rho z_t + (1-\rho)\mu, \sigma)$. The RBC literature calibrates the parameters on the shock process (T.1.10) to $\rho = 0.95$ and $\sigma = 0.4946$ for annual data.

For data in which one period is 30 years, we have to recalculate the analogous $\tilde{\rho}$ and $\tilde{\sigma}$.

$$\begin{aligned} z_{t+1} &= \rho z_t + (1-\rho)\mu + \varepsilon_{t+1} \\ z_{t+2} &= \rho z_{t+1} + (1-\rho)\mu + \varepsilon_{t+2} \\ &= \rho^2 z_t + \rho(1-\rho)\mu + \rho\varepsilon_{t+1} + (1-\rho)\mu + \varepsilon_{t+2} \\ z_{t+3} &= \rho z_{t+2} + (1-\rho)\mu + \varepsilon_{t+3} \\ &= \rho^3 z_t + \rho^2(1-\rho)\mu + \rho^2\varepsilon_{t+1} + \rho(1-\rho)\mu + \rho\varepsilon_{t+2} + (1-\rho)\mu + \varepsilon_{t+3} \\ &\vdots \\ z_{t+j} &= \rho^j z_t + (1-\rho)\mu \sum_{s=1}^{j} \rho^{j-s} + \sum_{s=1}^{j} \rho^{j-s}\varepsilon_{t+s} \end{aligned}$$

[5]Estimates of the coefficient of relative risk aversion γ mostly lie between 1 and 10. See **?**, **?**, **?**, **?**, **?**, and **?**.

With one period equal to thirty years $j = 30$, the shock process in our paper should be:

$$z_{t+30} = \rho^{30} z_t + (1-\rho)\mu \sum_{s=1}^{30} \rho^{30-s} + \sum_{s=1}^{30} \rho^{30-s} \varepsilon_{t+s} \tag{T.2.3}$$

Then the persistence parameter in our one-period-equals-thirty-years model should be $\tilde{\rho} = \rho^{30} \approx 0.2146$ and the unconditional mean should be $\tilde{\mu} = \mu \sum_{s=1}^{30} \rho^{30-s} = 0$. Define $\tilde{\varepsilon}_{t+30} \equiv \sum_{s=1}^{30} \rho^{30-s} \varepsilon_{t+s}$ as the summation term on the right-hand-side of (T.2.3). Then $\tilde{\varepsilon}_{t+30}$ is distributed:

$$\tilde{\varepsilon}_{t+30} \sim N\left(0, \left[\sum_{s=1}^{30} \rho^{2(30-s)}\right]^{\frac{1}{2}} \sigma\right)$$

Using this formula, the annual persistence parameter $\rho = 0.95$, and the annual standard deviation parameter $\sigma = 0.4946$, the implied thirty-year standard deviation is $\tilde{\sigma} \approx 1.5471$. So our shock process should be,

$$z_t = \tilde{\rho} z_{t-1} + (1-\rho)\tilde{\mu} + \tilde{\varepsilon}_t \quad \forall t \quad \text{where} \quad \tilde{\varepsilon} \sim N(0, \tilde{\sigma})$$

where $\tilde{\rho} = 0.2146$ and $\tilde{\sigma} = 1.5471$. We arbitrarily choose $\mu = \tilde{\mu} = 0$. However, we could have also chosen μ and the corresponding $\tilde{\mu}$ to his a median wage target.

Lastly, we set the size of the promised transfer $\bar{H}$ to be 32 percent of the median real wage. This level of transfers is meant to approximately match the average per capita real transfers in the United States to the average real wage in recent years. We get the median real wage by simulating a time series of the economy until it hits the shut down point, and we do this for 3,000 simulated time series. We take the median wage from those simulations. In order to reduce the effect of the initial values on the median, we take the simulation that lasted the longest number of periods before shutting down and remove the first 10 percent of the longest simulation's periods from each simulation for the calculation of the median.

T-3 Truncated Normal Distribution

Put a description of the properties of the truncated normal distribution here and how it interacts with the total factor productivity process.

T-4 Comments and Notes

- Interesting papers on debt and rare events: **?**, **?**
- Equity premium puzzle explanations
  - General: **?**, **?**
  - Prospect theory by Kahneman and Tversky
  - the role of personal debt
  - the importance of credit risk and liquidity: **?**
  - the impact of government regulation
  - consideration of taxes
  - rare events/disasters: see references in **?**, including **?**, **?**
- Our current calibration does not match the NBER paper because I am not sure I like the way we calculated the median (see last paragraph of Technical Appendix T-2.).
- Rick thinks we should use language of "fiscal limit", and not use the more extreme terms such as "shutdown" or "game over".
- Does Ricardian equivalence hold in this model? Agents have rational expectations, and they smooth consumption. But the government runs a balanced budget in each period. My intuition is that this is a Ricardian model because agents expect that the government will eventually have to default on its promised transfer $\bar{H}$.
- In describing the transfer program, justify lump sum transfers as approximating a degree of fiscal inertia or fiscal stickiness. Policy stickiness could either speed up the expected time until the economy hits its fiscal limit, or it could delay policy responses after hitting the limit which make outcomes worse. Papers that incorporate policy stickiness into stochastic OLG models are **????** and **?**. **?** discuss the foundations of fiscal stickiness.
- List of papers that focus "on important intergenerational and distributional consequences of fiscal stress and fiscal limits": **?**, **???**, **??**, **?**, **?**, **?**, **?**, **?**, and **?**.