Negative Shocks, Public Debt, and Interest Rates *

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Abstract

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1 Introduction

2 Economic Model

We study a simple 2-period-lived agent overlapping generations model in which the government promises to make a lump sum transfer $\bar{H} \geq 0$ from the young to the old each period. Ricardian equivalence holds in the sense that households have rational expectations and can forecast the effects of government budget imbalances. The constraints of the model generate states of the world in which the government can only make a transfer that is less that the promised amount $0 \leq H_t \leq \bar{H}$.

Our characterization of government budget insolvency relies on the assumption that when the state of the world is such that $\bar{H}$ generates negative consumption for the young, the agents in the economy resort to autarky rather than starvation (negative consumption). This shut-down result would not hold if the government merely reduced the size of the transfer program in the face of a shut down. Rational agents would expect this and incorporate that risk on the payment $\bar{H}$ in the second period of their lives.[1]

2.1 Household problem

A unit measure of identical consumer-worker households is born each period. A Household lives for exactly two periods indexed by $s = 1, 2$. They supply a unit of labor inelastically in both the young and old period of life $n_1 > 0$ and $n_2 \geq 0$ in all periods t.

In the first period of life, consumer-worker households choose how to divide their net labor income between age-1 consumption $c_{1,t}$ and capital investment (savings) with the firms $k_{2,t+1}$. The objective of a household is maximize utility subject to a

[1]The argument here is that a proportional transfer program will never shut down a government. However, if the government is locked in to some degree of nonproportional transfer program, then there are states of the world in which the government must either shut down or default on that debt. If they default in a way that the consumption of the young does not go to zero, then the government has changed its nonproportional transfer program to look like a proportional transfer program.

period budget constraint and two nonnegativity constraints,

$$\max_{c_{1,t},k_{2,t+1},c_{2,t+1}} \quad u(c_{1,t}) + \beta E_t\left[u(c_{2,t+1})\right] \quad \forall t \tag{1}$$

$$\text{where} \quad u(c_{s,t}) = \frac{(c_{s,t})^{1-\gamma}-1}{1-\gamma} \quad \text{for} \quad s=1,2 \tag{2}$$

$$\text{such that} \quad c_{1,t} + k_{2,t+1} = w_t n_1 - H_t \tag{3}$$

$$\text{and} \quad c_{2,t+1} = \big(1+r_{t+1}\big)k_{2,t+1} + w_{t+1}n_2 + H_{t+1} \tag{4}$$

$$\text{and} \quad c_{1,t}, c_{2,t+1}, k_{2,t+1} \geq 0 \tag{5}$$

Let new households have no initial capital $k_{1,t} = 0$. Note that the nonnegativity constraints on consumption $c_{1,t}$ and $c_{2,t+1}$ are strict inequalities. Nonpositive consumption is not defined in the utility function (2). Furthermore, we also do not allow the government transfer program to zero out the consumption and savings of the young, as shown in Section 2.3 equation (14). Finally, the strict inequality on savings $k_{2,t+1} > 0$ comes from the market clearing condition (16), that negative capital stock $K_t < 0$ is not defined in the production function (9), and that zero capital stock $K_t = 0$ would results in zero output $Y_t = 0$ from (9), zero wage $w_t = 0$ from (13), and therefore zero first period consumption $c_{1,t} = 0$ from (3).

Consumption in the second period of life is characterized by the second period budget constraint.

$$c_{2,t+1} = (1+r_{t+1})k_{2,t+1} + w_{t+1}n_2 + H_{t+1} \quad \forall t \tag{4}$$

Note that the nonnegativity constraint on old-age consumption $c_{2,t+1}$ will never bind because everything on the right-hand-side of (4) is weakly positive. Consumption in the first period of life $c_{1,t}$ and savings in the first period of life $k_{2,t+1}$ are jointly determined by the first period budget constraint (3) and by the Euler equation that characterizes the optimal young $s = 1$ consumption-savings decision that maximizes lifetime utility (1) subject to constraints (3), (4), and (5).

$$u'\big(c_{1,t}\big) = \beta E_t\Big[\big(1+r_{t+1}\big)u'\big(c_{2,t+1}\big)\Big] \tag{6}$$

By substituting the age $s = 1$ and $s = 2$ budget constraints (3) and (4) into the household Euler equation (6), we can see that the characterizing equation for savings $k_{2,t+1}$ is one equation with one unknown.

$$u'\big(w_t n_1 - H_t - k_{2,t+1}\big) = \beta E_t\Big[\big(1+r_{t+1}\big)u'\big([1+r_{t+1}]k_{2,t+1} + w_{t+1}n_2 + H_{t+1}\big)\Big] \tag{7}$$

Equation (7) shows that the functional solution for household savings $k_{2,t+1}$ every period is a stationary function $\psi(\cdot)$ of the time path of transfers and prices over the lifetime of the household.

$$k_{2,t+1} = \psi\big(H_t, H_{t+1} w_t, w_{t+1}, r_{t+1}\big) \tag{8}$$

2.2 Firm problem

A unit measure of identical perfectly competitive firms exist in this economy that hire aggregate labor L_t at wage w_t and rent aggregate capital K_t at rental rate r_t every period in order to produce consumption good Y_t according to a Cobb-Douglas production function,

$$Y_t = F(K_t, L_t) = A_t K_t^\alpha L_t^{1-\alpha} \quad \forall t \tag{9}$$

where the capital share of income is given by $\alpha \in (0,1)$. Total factor productivity $A_t = e^{z_t} > 0$ is distributed log normally, and z_t follows a normally distributed $AR(1)$ process.

$$\begin{aligned} &z_t = \rho z_{t-1} + (1-\rho)\mu + \varepsilon_t \\ &\quad \text{where} \quad \rho \in [0,1), \quad \mu \geq 0, \quad \text{and} \quad \varepsilon_t \sim N(0,\sigma^2) \end{aligned} \tag{10}$$

The firm's problem each period is to choose how much capital K_t to rent and how much labor L_t to hire in order to maximize profits,

$$\max_{K_t,L_t} Pr_t = A_t K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - (r_t + \delta)K_t \quad \forall t \tag{11}$$

where δ is the per-period depreciation rate of capital. Profit maximization implies that the real wage and real rental rate are determined by the standard first order conditions for the firm.

$$r_t = \alpha e^{z_t}\left(\frac{L_t}{K_t}\right)^{1-\alpha} - \delta \quad \forall t \tag{12}$$

$$w_t = (1-\alpha)e^{z_t}\left(\frac{K_t}{L_t}\right)^{\alpha} \quad \forall t \tag{13}$$

Because the interest rate r_t in (12) is not defined when the capital stock is zero $K_t = 0$ and because the wage w_t in (13) is not defined when aggregate labor is zero $L_t = 0$, we know that both values must be strictly positive $K_t, L_t > 0$.

2.3 Government transfer program

We model a simple balanced budget public transfer program that takes an amount from the young each period H_t and gives that same amount to the old each period, as shown in the young and old budget constraints.

$$c_{1,t} + k_{2,t+1} = w_t n_1 - H_t \quad \forall t \tag{3}$$

$$c_{2,t} = \big(1 + r_t\big)k_{2,t} + w_t n_2 + H_t \quad \forall t \tag{4}$$

In contrast to the way the old-age ($s = 2$) budget constraint (4) is displayed in Section 2.1, we show the budget constraints here for a young household and old household both in period t (two separate individuals). The government budget is made up entirely of this transfer program, and the budget is always balanced because the government revenue taken from the young in period H_t is always equal to the transfers to the old H_t in all periods t.

In most periods, the government promises that the transfer will be $\bar{H} \geq 0$. However, in the case that $\bar{H} > 0$, there could exist states of the economy such that $\bar{H} \geq w_t n_1$. In these cases, net labor income is less than or equal to zero, so the strict inequalities on $c_{1,t}$ and $k_{2,t+1}$ in (5) must be violated. To avoid negative consumption, we require that the most the government can take from the young in any period is all their income up to some arbitrarily small minimum consumption $c_{min} > 0$ and an arbitrarily small amount of savings $K_{min} > 0$.

$$H_t \equiv \begin{cases} \bar{H} & \text{if} \quad w_t n_1 \geq \bar{H} + c_{min} + K_{min} \\ w_t n_1 - c_{min} - K_{min} & \text{if} \quad w_t n_1 < \bar{H} + c_{min} + K_{min} \end{cases} \quad \forall t$$
$$= \min\left(\bar{H}, w_t n_1 - c_{min} - K_{min}\right) \quad \forall t \tag{14}$$

This rule states that the government implements a balanced budget transfer program from the young to the old every period. And for $\bar{H} > 0$, once the wage dips low enough, the government can no longer take $\bar{H}$ from the young. In this case, the government takes all that it can from the young $H_t = w_t n_1 - c_{min} - K_{min} < \bar{H}$ and transfers that amount to the old. The young are left with consumption and savings equal to the minimum $c_{1,t} = c_{min}$ and $k_{2,t+1} = K_{min}$, and the economy shuts down and devolves into autarky.

2.4 Market clearing

Market clearing implies that the aggregate labor demand equals aggregate labor supply, aggregate capital demand equals aggregate capital supply, and output equals consumption plus investment in each period,

$$L_t = n_1 + n_2 \quad \forall t \tag{15}$$

$$K_t = k_{2,t} \quad \forall t \tag{16}$$

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t \quad \forall t \quad \text{where} \quad C_t \equiv c_{1,t} + c_{2,t} \tag{17}$$

where the goods market clearing condition or resource constraint (17) is redundant by Walras' Law.

2.5 Equilibrium

In this section, we define a functional stationary equilibrium in which our definition of stationary is that the functional forms are not time dependent. That is, for a function $f(\boldsymbol{x})$ of vector of variables $\boldsymbol{x}$, the function does not change. Only the output values of the function changes in response to changing inputs $\boldsymbol{x}$.

Definition 1 (**Functional stationary equilibrium**). A non-autarkic functional stationary equilibrium in the two-period-lived overlapping generations model with exogenous labor supply and aggregate shocks is defined by stationary price functions $r(k,z)$ and $w(k,z)$ and a stationary savings function $k' = \psi(k,z)$ for all current state wealth k and total factor productivity component z such that:

i. households optimize according to (3) and (4), and (6)

ii. firms optimize according to (12) and (13),

iii. markets clear according to (15) and (16).

We can solve for the stationary price functions analytically by substituting the market clearing conditions (15) and (16) into the firms' respective first order conditions (12) and (13).

$$r_t \equiv r\big(k_{2,t}, z_t\big) = \alpha e^{z_t}\left(\frac{n_1+n_2}{k_{2,t}}\right)^{1-\alpha} - \delta \quad \forall z_t \quad \text{and} \quad k_{2,t} > 0 \tag{18}$$

$$w_t \equiv w\big(k_{2,t}, z_t\big) = (1-\alpha)e^{z_t}\left(\frac{k_{2,t}}{n_1+n_2}\right)^{\alpha} \quad \forall z_t \quad \text{and} \quad k_{2,t} > 0 \tag{19}$$

We can also solve analytically for the equilibrium expression for the transfer each period H_t as a function of the wealth of the current-period old $k_{2,t}$ and the value of the normally distributed component z_t of the total factor productivity process (as well as the parameters of the promised transfer amount $\bar{H}$ and minimum values of young age consumption c_{min} and aggregate capital K_{min}) by substituting the equilibrium wage expression (19) into the expression for H_t (14).

$$H_t \equiv H\big(k_{2,t},z_t\big) = min\Big(\bar{H}, w(k_{2,t},z_t)n_1 - c_{min} - K_{min}\Big) \quad \forall z_t \quad \text{and} \quad k_{2,t}>0 \quad (20)$$

Finally, if we substitute the equilibrium expressions for prices $r(k,z)$ and $w(k,z)$ and the transfer $H(k,z)$ from (18), (19), and (20) into the household Euler equation (7) and resulting policy function (8), it is clear that the equilibrium savings function $k' = \psi(k,z)$ is a function of the wealth of the current-period old and the value z_t of the normally distributed component of total factor productivity,

$$\begin{aligned} & u'\Big(w(k_{2,t},z_t)n_1 - H(k_{2,t},z_t) - k_{2,t+1}\Big) = \\ & \quad \beta \int_{z_{t+1}} \Bigg[\big(1 + r(k_{2,t+1},z_{t+1})\big)\times \\ & \qquad\qquad u'\Big([1+r(k_{2,t+1},z_{t+1})]k_{2,t+1} + w(k_{2,t+1},z_{t+1})n_2 + H(k_{2,t+1},z_{t+1})\Big)\times \\ & \qquad\qquad\qquad f\big(z_{t+1}|\rho z_t + (1-\rho)\mu,\sigma\big)\Bigg] dz_{t+1} \\ & \qquad\qquad\qquad\qquad \forall z_t, z_{t+1} \quad \text{and} \quad k_{2,t}, k_{2,t+1} > 0 \end{aligned} \tag{21}$$

$$k_{2,t+1} = \psi\big(k_{2,t},z_t\big) \quad \forall z_t \quad \text{and} \quad k_{2,t}>0 \tag{22}$$

where $f(z_{t+1}|\rho z_t + (1-\rho)\mu,\sigma)$ is the probability density function of z_{t+1} distributed normally with mean $\rho z_t + (1-\rho)\mu$ and standard deviation σ.

2.6 One-period riskless bonds

In this section, we derive the return on a riskless bond. We make the simplifying assumption that the riskless bonds are zero absolute supply. However, this characterization can be generalized to cases in which the riskless bonds have exogenous or endogenous positive supply. Because of our zero-supply assumption on the riskless bond, we can separate its derivation from the characterization of the household problem in Section 2.1. These zero-supply riskless bonds do not influence the rest of the economy. They simply represent another measure of the level of risk present in each period of the economy.

Assume that households have two potential instruments for saving. A household can invest income with the production sector $k_{2,t+1}$ and earn a stochastic risky return next period of r_t and they can buy $b_{2,t+1}$ units of a one-period riskless bond for price p_t that returns exactly $b_{2,t+1}$ when old. It is clear that old-age households will have no demand for these bonds.

The maximization problem for a generic household can be characterized as choosing risky savings $k_{2,t+1}$ and riskless savings $b_{2,t+1}$ to maximize lifetime utility subject to budget constraints.

$$\max_{k_{2,t+1},b_{2,t+1}} \quad u(c_{1,t}) + \beta E_t\left[u(c_{2,t+1})\right] \quad \forall t \tag{23}$$

$$\text{such that} \quad c_{1,t} + k_{2,t+1} + p_t b_{2,t+1} = w_t n_1 - H_t \tag{24}$$

$$\text{and} \quad c_{2,t+1} = \big(1 + r_{t+1}\big)k_{2,t+1} + b_{2,t+1} + w_{t+1}n_2 + H_{t+1} \tag{25}$$

$$\text{and} \quad c_{1,t}, c_{2,t+1}, k_{2,t+1} \geq 0 \tag{26}$$

The optimality condition for risky savings $k_{2,t+1}$ is the same Euler equation as in Section 2.1.

$$u'\big(c_{1,t}\big) = \beta E_t\Big[\big(1 + r_{t+1}\big)u'\big(c_{2,t+1}\big)\Big] \tag{6}$$

The Euler equation for riskless savinge $b_{2,t+1}$ if the following,

$$\begin{aligned} \frac{1}{1+\bar{r}_t} \equiv p_t &= \beta\frac{E_t\Big[u'\big(c_{2,t+1}\big)\Big]}{u'\big(c_{1,t}\big)} \quad \forall t \\ \Rightarrow \quad \bar{r}_t &= \frac{u'\big(c_{1,t}\big)}{\beta E_t\Big[u'\big(c_{2,t+1}\big)\Big]} - 1 \quad \forall t \end{aligned} \tag{27}$$

where the price of the riskless bond p_t is defined as the reciprocal of the gross riskless return $1 + \bar{r}_t$. The optimality conditions of the production sector are the same as in Section 2.2.

Euler equation (27) determines the demand for riskless bonds. We assume, generally, an exogenous supply of riskless bonds that is nonnegative $B_t \geq 0$ for all t. However, specifically in this model, we assume a zero supply of riskless bonds $B_t = 0$. So the general version of our riskless bond market clearing condition is the following.

$$b_{2,t} = B_t \quad \forall t \tag{28}$$

With our zero supply assumption $B_t = 0$, the household demand for riskless bonds is set to zero through the market clearing condition,

$$b_{2,t} = 0 \quad \forall t \tag{29}$$

all the other endogenous variables are determined by the equilibrium described in Section 2.5, and the riskless return $\bar{r}_t$ is characterized by Euler equation (27).

If we were to relax our zero-supply assumption on riskless bonds $B_t > 0$, we would have to determine the riskless return $\bar{r}_t$ jointly with the rest of the endogenous variables.

And finally, because the agents in our model live for only two periods, it is intuitive that each model period must represent many years. If we assume that the average economic life is 60 years, then each model period represents 30 years. Let the parameter yrs be the number of years represented in a model period. Then we can report the riskless interest rate $\bar{r}_t$ characterized in (27) as an annual rate $\bar{r}_{t,an}$ using the following expression.

$$\bar{r}_{t,an} = \big(1 + \bar{r}_t\big)^{\frac{1}{yrs}} - 1 \quad \forall t \tag{30}$$

3 Simulations

We explore the properties of the model from Section 2 with respect to different values of the promised transfer $\bar{H}$, initial wealth $k_{2,0}$, and the extent and probability of low total factor productivity values A_t by calibrating the other parameters of the model and simulating a time series of the model 3,000 times for different combinations of $\bar{H}$, $k_{2,0}$, and the support and distribution of A_t. The first three rows of Table 1 show the different values of $\bar{H}$, $k_{2,0}$, and A_{min} that we test in our simulations. The remaining rows show our calibration of the other variables.[2]

Table 1: Calibration of 2-period-lived agent OG model with promised transfer $\bar{H}$

Parameter	Source to match	Value(s)
$\bar{H}$	Promised transfer amount	$[0.00, 0.05, 0.11, 0.17]$
$k_{2,0}$	Initial period wealth of old household	$[0.11, 0.14, 0.17]$
A_{min}	Minimum value in support of A_t	$[0.0, 0.75]$
z_0	Initial value of z_t TFP component	μ
n_1	Exogenous labor supply when young	1.0
n_2	Exogenous labor supply when old	0.0
β	Annual discount factor of 0.96	0.29
γ	Coefficient of relative risk aversion between 1.5 and 4.0	2.0
α	Capital share of income	0.35
δ	Annual capital depreciation of 0.05	0.79
ρ	AR(1) persistence of normally distributed shock to match annual persistence of 0.95	0.21
μ	AR(1) long-run average z_t level	0.0
σ	standard deviation of normally distributed z_t to match annual standard deviation of U.S. real GDP of 0.49	1.55
B_t	Exogenous supply of riskless bonds in every period	0
yrs	Number of years in a model period	30
T	Maximum number of periods to simulate in a given simulation	100
S	Number of simulated time series for a given parameterization	3,000

The Technical Appendix T-1 gives a detailed description of the calibration of all parameters.

[2]The code for these simulations is available at ?.

Table 2: Initial values relative to median values: $A_{min} = 0.00$ and $z_0 = 0.0$

	$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
	w_{med} $\bar{H}/w_{med}$	k_{med} $k_{2,0}/k_{med}$	w_{med} $\bar{H}/w_{med}$	k_{med} $k_{2,0}/k_{med}$	w_{med} $\bar{H}/w_{med}$	k_{med} $k_{2,0}/k_{med}$
$\bar{H} = 0.00$	0.281 0.000	0.101 1.093	0.282 0.000	0.101 1.390	0.282 0.000	0.101 1.687
$\bar{H} = 0.05$	0.445 0.112	0.083 1.321	0.449 0.111	0.085 1.654	0.450 0.111	0.085 2.003
$\bar{H} = 0.11$	0.557 0.197	0.064 1.710	0.564 0.195	0.066 2.108	0.572 0.192	0.068 2.515
$\bar{H} = 0.17$	0.648 0.262	0.048 2.274	0.658 0.259	0.051 2.757	0.667 0.255	0.052 3.243

w_{med} is the median wage and k_{med} is the median capital stock across all 3,000 simulations before economic shut down.

Table 3: Initial values relative to median values: $A_{min} = 0.75$ and $z_0 = 0.0$

	$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
	w_{med} $\bar{H}/w_{med}$	k_{med} $k_{2,0}/k_{med}$	w_{med} $\bar{H}/w_{med}$	k_{med} $k_{2,0}/k_{med}$	w_{med} $\bar{H}/w_{med}$	k_{med} $k_{2,0}/k_{med}$
$\bar{H} = 0.00$	1.319 0.000	0.388 0.284	1.320 0.000	0.388 0.361	1.321 0.000	0.388 0.438
$\bar{H} = 0.05$	1.158 0.043	0.281 0.392	1.160 0.043	0.281 0.498	1.161 0.043	0.281 0.604
$\bar{H} = 0.11$	0.984 0.112	0.176 0.625	0.988 0.111	0.177 0.789	0.994 0.111	0.179 0.950
$\bar{H} = 0.17$	0.953 0.178	0.128 0.857	0.950 0.179	0.128 1.097	0.960 0.177	0.130 1.303

w_{med} is the median wage and k_{med} is the median capital stock across all 3,000 simulations before economic shut down.

Table 4: Periods to shut down simulation statistics: $A_{min} = 0.00$ and $z_0 = 0.0$

		$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
		Periods	CDF	Periods	CDF	Periods	CDF
$\bar{H} = 0.00$	min	100	1.000	100	1.000	100	1.000
	med	100	1.000	100	1.000	100	1.000
	mean	100	1.000	100	1.000	100	1.000
	max	100	1.000	100	1.000	100	1.000
$\bar{H} = 0.05$	min	1	0.160	1	0.152	1	0.148
	med	4	0.517	4	0.512	4	0.507
	mean	6.1	0.693	6.2	0.689	6.2	0.686
	max	50	1.000	50	1.000	50	1.000
$\bar{H} = 0.11$	min	1	0.344	1	0.328	1	0.317
	med	2	0.534	2	0.522	2	0.512
	mean	3.5	0.713	3.6	0.705	3.6	0.697
	max	27	1.000	27	1.000	27	1.000
$\bar{H} = 0.17$	min	1	0.498	1	0.474	1	0.459
	med	2	0.683	2	0.670	2	0.658
	mean	2.5	0.758	2.6	0.745	2.6	0.732
	max	21	1.000	21	1.000	21	1.000

The "min", "med", "mean", and "max" rows in the "Periods" column represent the minimum, median, mean, and maximum number of periods, respectively, in which the simulated time series hit the economic shut down. The "CDF" column represents the percent of simulations that shut down in t periods or less, where t is the value in the "Periods" column. For the CDF value of the "mean" row, we used linear interpolation.

Table 5: Periods to shut down simulation statistics: $A_{min} = 0.75$ and $z_0 = 0.0$

		$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
		Periods	CDF	Periods	CDF	Periods	CDF
$\bar{H} = 0.00$	min	100	1.000	100	1.000	100	1.000
	med	100	1.000	100	1.000	100	1.000
	mean	100	1.000	100	1.000	100	1.000
	max	100	1.000	100	1.000	100	1.000
$\bar{H} = 0.05$	min	5	0.000	5	0.000	5	0.000
	med	100	1.000	100	1.000	100	1.000
	mean	99.8	0.008	99.8	0.007	99.8	0.007
	max	100	1.000	100	1.000	100	1.000
$\bar{H} = 0.11$	min	2	0.108	2	0.096	2	0.086
	med	23	0.500	25	0.506	25	0.502
	mean	34.1	0.620	34.9	0.608	35.2	0.613
	max	100	1.000	100	1.000	100	1.000
$\bar{H} = 0.17$	min	1	0.302	1	0.261	1	0.228
	med	3	0.506	4	0.521	4	0.501
	mean	8.5	0.692	9.0	0.674	9.4	0.680
	max	100	1.000	100	1.000	100	1.000

The "min", "med", "mean", and "max" rows in the "Periods" column represent the minimum, median, mean, and maximum number of periods, respectively, in which the simulated time series hit the economic shut down. The "CDF" column represents the percent of simulations that shut down in t periods or less, where t is the value in the "Periods" column. For the CDF value of the "mean" row, we used linear interpolation.

Table 6: Annualized Riskless return $\bar{r}_{t,an}$ simulation statistics: $A_{min} = 0.00$ and $z_0 = 0.0$

		$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
		$\bar{r}_{t,an}$	CDF	$\bar{r}_{t,an}$	CDF	$\bar{r}_{t,an}$	CDF
	$t = 0$	-2.06%	0.483	-2.12%	0.459	-2.18%	0.440
	min	-4.64%	0.000	-4.64%	0.000	-4.64%	0.000
$\bar{H} = 0.00$	med	-2.01%	0.500	-2.01%	0.500	-2.01%	0.500
	mean	-1.58%	0.645	-1.58%	0.645	-1.59%	0.645
	max	19.01%	1.000	19.01%	1.000	19.01%	1.000
	$t = 0$	-1.47%	0.589	-1.54%	0.563	-1.60%	0.541
	min	-4.39%	0.000	-4.39%	0.000	-4.39%	0.000
$\bar{H} = 0.05$	med	-1.69%	0.500	-1.70%	0.500	-1.71%	0.500
	mean	-1.14%	0.715	-1.14%	0.720	-1.14%	0.720
	max	36.56%	1.000	36.52%	1.000	36.49%	1.000
	$t = 0$	-1.71%	0.651	-1.80%	0.609	-1.87%	0.580
	min	-4.38%	0.000	-4.38%	0.000	-4.38%	0.000
$\bar{H} = 0.11$	med	-1.99%	0.500	-2.00%	0.500	-2.01%	0.500
	mean	-1.42%	0.750	-1.40%	0.759	-1.43%	0.756
	max	34.67%	1.000	36.29%	1.000	32.10%	1.000
	$t = 0$	-1.53%	0.743	-1.71%	0.704	-1.83%	0.674
	min	-4.37%	0.000	-4.37%	0.000	-4.37%	0.000
$\bar{H} = 0.17$	med	-2.13%	0.500	-2.16%	0.500	-2.18%	0.500
	mean	-1.51%	0.751	-1.53%	0.754	-1.59%	0.749
	max	35.69%	1.000	41.27%	1.000	35.76%	1.000

All riskless returns $\bar{r}_{t,an}$ are reported in percentage rates. The rate of return 0.0206 is reported in this table as 2.06%.

Table 7: Annualized Riskless return $\bar{r}_{t,an}$ simulation statistics: $A_{min} = 0.75$ and $z_0 = 0.0$

		$k_{2,0} = 0.11$		$k_{2,0} = 0.14$		$k_{2,0} = 0.17$	
		$\bar{r}_{t,an}$	CDF	$\bar{r}_{t,an}$	CDF	$\bar{r}_{t,an}$	CDF
	$t = 0$	3.91%	0.955	3.69%	0.940	3.51%	0.926
	min	-4.60%	0.000	-4.60%	0.000	-4.60%	0.000
$\bar{H} = 0.00$	med	0.47%	0.500	0.47%	0.500	0.47%	0.500
	mean	0.52%	0.509	0.52%	0.509	0.52%	0.509
	max	6.19%	1.000	6.19%	1.000	6.19%	1.000
	$t = 0$	5.40%	0.912	5.06%	0.895	4.80%	0.879
	min	-4.60%	0.000	-4.60%	0.000	-4.60%	0.000
$\bar{H} = 0.05$	med	1.15%	0.500	1.14%	0.500	1.14%	0.500
	mean	1.43%	0.537	1.43%	0.537	1.42%	0.537
	max	41.95%	1.000	41.95%	1.000	41.95%	1.000
	$t = 0$	7.62%	0.855	7.07%	0.836	6.65%	0.819
	min	-4.57%	0.000	-4.57%	0.000	-4.57%	0.000
$\bar{H} = 0.11$	med	2.26%	0.500	2.24%	0.500	2.23%	0.500
	mean	3.20%	0.588	3.16%	0.587	3.14%	0.586
	max	70.01%	1.000	76.72%	1.000	74.84%	1.000
	$t = 0$	9.56%	0.859	8.92%	0.838	8.43%	0.822
	min	-4.29%	0.000	-4.29%	0.000	-4.30%	0.000
$\bar{H} = 0.17$	med	3.13%	0.500	3.13%	0.500	3.09%	0.500
	mean	4.30%	0.584	4.26%	0.582	4.21%	0.584
	max	69.65%	1.000	67.07%	1.000	70.83%	1.000

All riskless returns $\bar{r}_{t,an}$ are reported in percentage rates. The rate of return 0.0206 is reported in this table as 2.06%.

4 Conclusion

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TECHNICAL APPENDIX

T-1 Description of calibration

This section details our calibration of the parameter values listed in Table 1. In our two-period-lived agent OG model, we assume that each period represents 30 years or, equivalently, a lifetime is 60 years. The model-period (30-year) discount factor β is set to match the annual discount factor common in the RBC literature of 0.96.

$$\beta = (0.96)^{30} \approx 0.2939 \tag{T.1.1}$$

We set the coefficient of relative risk aversion at a midrange value of $\gamma = 2$. This value lies in the midrange of values that have been used in the literature.[3] The capital share of income parameter is set to match the U.S. average $\alpha = 0.35$, and the model-period (30-year) depreciation rate δ is set to match an annual depreciation rate of 5 percent.

$$\delta = 1 - (1 - 0.05)^{30} \approx 0.7854 \tag{T.1.2}$$

The firms' production function in our model is the following,

$$Y_t = e^{z_t} K_t^{\alpha} L_t^{1-\alpha} \quad \forall t \tag{9}$$

where labor L_t is supplied inelastically and z_t is current-period normally distributed component of total factor productivity. We assume that z_t is an AR(1) process with normally distributed errors.

$$\begin{aligned} &z_t = \rho z_{t-1} + (1-\rho)\mu + \varepsilon_t \\ &\quad \text{where} \quad \rho \in [0,1), \quad \mu \geq 0, \quad \text{and} \quad \varepsilon_t \sim N(0,\sigma) \end{aligned} \tag{10}$$

This implies that the shock process e^{z_t} is lognormally distributed $LN(\rho z_t + (1-\rho)\mu, \sigma)$. The RBC literature calibrates the parameters on the shock process (10) to $\rho = 0.95$ and $\sigma = 0.4946$ for annual data.

For data in which one period is 30 years, we have to recalculate the analogous $\tilde{\rho}$ and $\tilde{\sigma}$.

$$\begin{aligned}
z_{t+1} &= \rho z_t + (1-\rho)\mu + \varepsilon_{t+1} \\
z_{t+2} &= \rho z_{t+1} + (1-\rho)\mu + \varepsilon_{t+2} \\
&= \rho^2 z_t + \rho(1-\rho)\mu + \rho\varepsilon_{t+1} + (1-\rho)\mu + \varepsilon_{t+2} \\
z_{t+3} &= \rho z_{t+2} + (1-\rho)\mu + \varepsilon_{t+3} \\
&= \rho^3 z_t + \rho^2(1-\rho)\mu + \rho^2\varepsilon_{t+1} + \rho(1-\rho)\mu + \rho\varepsilon_{t+2} + (1-\rho)\mu + \varepsilon_{t+3} \\
&\vdots \\
z_{t+j} &= \rho^j z_t + (1-\rho)\mu \sum_{s=1}^{j} \rho^{j-s} + \sum_{s=1}^{j} \rho^{j-s}\varepsilon_{t+s}
\end{aligned}$$

[3] Estimates of the coefficient of relative risk aversion γ mostly lie between 1 and 10. See Mankiw and Zeldes (1991), Blake (1996), Campbell (1996), Kocherlakota (1996), Brav et al. (2002), and Mehra and Prescott (1985).

With one period equal to thirty years $j = 30$, the shock process in our paper should be:

$$z_{t+30} = \rho^{30} z_t + (1 - \rho)\mu \sum_{s=1}^{30} \rho^{30-s} + \sum_{s=1}^{30} \rho^{30-s} \varepsilon_{t+s} \tag{T.1.3}$$

Then the persistence parameters in our one-period-equals-thirty-years model should be $\tilde{\rho} = \rho^{30} \approx 0.2146$. Define $\tilde{\varepsilon}_{t+30} \equiv \sum_{s=1}^{30} \rho^{30-s} \varepsilon_{t+s}$ as the summation term on the right-hand-side of (T.1.3). Then $\tilde{\varepsilon}_{t+30}$ is distributed:

$$\tilde{\varepsilon}_{t+30} \sim N\left(0, \left[\sum_{s=1}^{30} \rho^{30-s}\right] \sigma\right)$$

Using this formula, the annual persistence parameter $\rho = 0.95$, and the annual standard deviation parameter $\sigma = 0.4946$, the implied thirty-year standard deviation is $\tilde{\sigma} \approx 1.5471$. So our shock process should be,

$$z_t = \tilde{\rho} z_{t-1} + (1 - \rho)\tilde{\mu} + \tilde{\varepsilon}_t \quad \forall t \quad \text{where} \quad \tilde{\varepsilon} \sim N(0, \tilde{\sigma})$$

where $\tilde{\rho} = 0.2146$ and $\tilde{\sigma} = 1.5471$. We arbitrarily choose $\mu = \tilde{\mu} = 0$. However, we could have also chosen μ and the corresponding $\tilde{\mu}$ to his a median wage target.

Lastly, we set the size of the promised transfer $\bar{H}$ to be 32 percent of the median real wage. This level of transfers is meant to approximately match the average per capita real transfers in the United States to the average real wage in recent years. We get the median real wage by simulating a time series of the economy until it hits the shut down point, and we do this for 3,000 simulated time series. We take the median wage from those simulations. In order to reduce the effect of the initial values on the median, we take the simulation that lasted the longest number of periods before shutting down and remove the first 10 percent of the longest simulation's periods from each simulation for the calculation of the median.

T-2 Comments and Notes

- Interesting papers on debt and rare events: Rebelo et al. (2019), Reinhart et al. (2015)
- Equity premium puzzle explanations
  - General: DeLong and Magin (2009), Farhi and Gourio (2019)
  - Prospect theory by Kahneman and Tversky
  - the role of personal debt
  - the importance of credit risk and liquidity: Gourio (2013)
  - the impact of government regulation
  - consideration of taxes
  - rare events/disasters: see references in Tsai and Wachter (2015), including Barro (2009), Gourio (2012)
- Our current calibration does not match the NBER paper because I am not sure I like the way we calculated the median (see last paragraph of Technical Appendix T-1.).