# NONEXPECTED UTILITY IN MACROECONOMICS*

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This paper introduces, within the context of an infinite horizon optimal consumption problem, a parametric class of Kreps-Porteus nonexpected utility preferences—generalized isoelastic utility—which distinguishes attitudes toward risk from behavior toward intertemporal substitution. Some of the theoretical and empirical implications for macroeconomics of these state- and time-nonseparable preferences are examined.

Aversion to risk and aversion to intertemporal substitution are two conceptually distinct attributes of a consumer's tastes, and should be parameterized independently of each other. Yet it is well-known that the class of time-additive Von Neumann-Morgenstern (VNM) preferences constrains the coefficient of relative risk aversion to be the reciprocal of the elasticity of intertemporal substitution. This purely mechanical restriction, which is devoid of any economic rationale, makes it impossible (i) to replicate the behavior of agents who, as most of the empirical evidence suggests, have both a considerable distaste for intertemporal substitution (a low elasticity of intertemporal substitution) and a moderate aversion to risk taking (a low coefficient of relative risk aversion), and (ii) to determine the channel, risk aversion or intertemporal substitution, through which macroeconomic policies may affect optimal consumption.[1]

While the separation between risk aversion and substitution across commodities can easily be achieved, in a static context, within the framework of expected utility preferences (it suffices to apply a concave transformation, à la Kihlstrom and Mirman [1974], to the original utility index), this solution, or the one that consists in abandoning time-additivity, often runs, in dynamic environments, into problems of either stationarity (in the sense of Koopmans [1960]) or time-consistency (in the sense of Johnsen and Donaldson [1985])—two properties of preferences that it might be thought desirable to preserve.

That one may look outside of the expected utility framework to achieve the desired separation between risk aversion and intertem-

    1. See Hall [1988].

poral substitution was first suggested by the two-period "ordinal certainty equivalent" preferences of Selden [1978, 1979], recently used by Barsky [1987] and Hall [1988], which indeed parameterize the two concepts of risk aversion and intertemporal substitution independently of each other. Although naive extensions of Selden's work to many-period contexts violate temporal consistency, as realized by Hall [1988] and Zin [1987], Kreps and Porteus [1978, 1979a, 1979b] have developed, in a series of articles that forms the theoretical basis of this paper, an axiomatic framework that, in essence, provides the appropriate time-consistent general-ization to multiperiod environments of Selden's two-period prefer-ences. The fact that the separation of risk aversion from intertem-poral substitution is possible within the context of Kreps-Porteus (KP) preferences is implicit in the recent contribution of Farmer [1990], who proposed a class of risk-neutral KP preferences with a not necessarily infinite elasticity of intertemporal substitution.[2]

I introduce in Section I a simple parametric class of KP preferences, *generalized isoelastic utility,* characterized by a con-stant coefficient of relative risk aversion and constant, but in general unrelated, elasticity of intertemporal substitution. In Sec-tion II, I present the explicit solution of a simple infinite-horizon stochastic consumption problem, to show that the well-known solution methods used in the time-additive VNM case carry over to these preferences. I examine in Section III some of the macroeco-nomic implications of this framework, particularly in connection with stochastic consumption theory. Section IV offers some con-cluding remarks by discussing extensions as well as limitations of this work.

## I. A CLASS OF NONEXPECTED UTILITY PREFERENCES

In this section I briefly explain why attitudes toward risk and behavior toward intertemporal substitution can be distinguished within the framework of KP preferences. I then introduce a new parametric class of KP preferences, which generalizes isoelastic utility by allowing for a constant coefficient of relative risk aversion and a constant, but in general unrelated, elasticity of intertemporal substitution.

2. After completing the first draft of this paper, I received a copy of closely related work by Epstein and Zin [1987a], in which they also propose using Kreps-Porteus preferences to separate risk aversion from intertemporal substitution. Their results and mine are similar, but independent.

*Kreps-Porteus Preferences, Risk Aversion, and Intertemporal Substitution*

Although a detailed expression of the axiomatic foundation of the preferences introduced by Kreps and Porteus [1978] is outside the scope of this paper,[3] a simple (and classic) example is useful to illustrate their essential characteristic, namely the nonindifference of KP consumers to the timing of resolution of uncertainty over temporal lotteries.[4] Consider the two temporal lotteries presented in Figure I. While the compound probability of each consumption prize is identical, from the point of view of the first period, for the two lotteries, lottery A differs from lottery B in that uncertainty resolves earlier in lottery B than in lottery A. As Kreps and Porteus show, the application of the originally timeless VNM utility theory to temporal lotteries amounts to assuming that agents are indifferent between these temporal lotteries and thus indifferent to the timing of resolution of uncertainty. This is because VNM utility theory imposes on temporal gambles the axiom of reduction of compound lotteries (which postulates that, when faced with compound lotteries whose prizes are tickets to other lotteries, a consumer cares only about the compound probability of each prize).

Kreps and Porteus [1978] establish that when that axiom is abandoned to allow for nonindifference toward the timing of resolution of uncertainty over temporal lotteries (the other axioms of VNM theory, in particular the so-called "independence" axiom, are maintained, while the temporal consistency of optimal plans is imposed axiomatically), preferences can be represented recursively by

$$(1) \qquad V_t = U[c_t, E_t, V_{t+1}],$$

where $U(.,.)$ is, in the terminology of Koopmans [1960], the aggregator function (through which current consumption and future utility are aggregated),[5] and $E_t$ denotes mathematical expectation conditional on information available at time $t$.

As shown by Kreps and Porteus [1978], agents exhibit a preference for early (respectively, late) resolution of uncertainty

3. The interested reader is referred to the original papers of Kreps and Porteus, or the more recent presentation of Kreps-Porteus theory in Farmer [1990].
4. It is important to note that one is dealing here with lotteries over consumption, not income. Early resolution of uncertainty on income lotteries always improves planning.
5. By writing that the aggregator function is time-independent, I implicitly assume that preferences exhibit "payoff history independence."
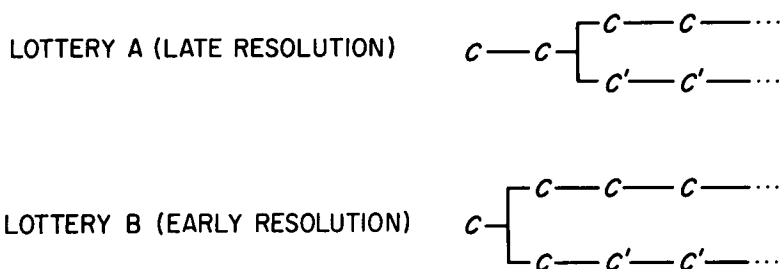
LOTTERY A (LATE RESOLUTION)

LOTTERY B (EARLY RESOLUTION)

FIGURE I
Late Versus Early Resolution of Uncertainty

over temporal lotteries depending on whether $U(c, .)$ is convex
(respectively, concave) in its second argument. The VNM time-
additive expected utility case of indifference toward the timing of
resolution of uncertainty emerges as the borderline case in which
the aggregator function is linear in its second argument, for the
recursive formulation in (1) can then be iterated forward to obtain
the "standard" time- and state-additive representation. The de-
rivative of the aggregator function with respect to its second
argument, $U_2$, has the interpretation of a subjective discount rate:
Kreps-Porteus preferences can thus be viewed as a stochastic
generalization of endogenous time preference.[6] Finally, note that
Kreps and Porteus preferences are in general both time- and
state-nonseparable.[7]

　　Perhaps more importantly for the purpose of this paper,
attitudes toward intertemporal substitution and risk aversion can
be distinguished within the context of KP preferences—precisely
because these preferences do not impose indifference toward the
timing of resolution of uncertainty over temporal lotteries. Heuris-
tically, one can show that in utility terms (which is the appropriate
metric when the timing of resolution matters), lotteries in which
uncertainty resolves early (our lottery B) are less risky than late
resolution lotteries with the same distribution of prizes (lottery A).
Early resolution lotteries, however, feature certainty equivalent
fluctuations of utility over time which are of larger amplitude.
There is therefore, in general, a trade-off between safety and
stability of utility. Therefore, agents who dislike risk "more" than

　　6. KP preferences also provide the $n$-period $(n > 2)$ temporally consistent
generalization of Selden's [1978] two-period "OCE" preferences long sought for by
macroeconomists (see Barsky [1987] or Hall [1988]).
　　7. However, they rank, timeless (but not temporal) lotteries according to
expected utility.

intertemporal fluctuations prefer, ceteris paribus, early resolution; but consumers who have a stronger distaste for intertemporal fluctuations than for risk prefer late resolution. For VNM consumers with time-additive preferences, the cost and benefit of each lottery in terms of safety and stability of utility balance out, for the very same factor that leads a time-additive VNM agent to dislike risk is necessarily conducive, in the presence of the parametric restriction on tastes implied by the combination of time- and state-separability, to a strong distaste for intertemporal fluctuations in utility. VNM consumers with time-additive utility are, therefore, indifferent to the timing of resolution of uncertainty over temporal lotteries. The separation of attitudes toward risk from behavior toward intertemporal substitution is thus made possible, with KP preferences, by the nonindifference of consumers toward the timing of resolution of uncertainty over consumption lotteries.[8]

## *Generalized Isoelastic Preferences*

Generalized isoelastic preferences, characterized by a constant coefficient of relative risk aversion, and a constant but distinct elasticity of intertemporal substitution, are defined by the following CES-like aggregator function:

$$V_t = U[c_t, E_t V_{t+1}]$$
$$(2) \quad \equiv \frac{\{(1 - \beta)c_t^{1-\rho} + \beta[1 + (1 - \beta)(1 - \gamma)E_t V_{t+1}]^{(1-\rho)/(1-\gamma)}\}^{(1-\gamma)/(1-\rho)} - 1}{(1 - \beta)(1 - \gamma)},$$

where $\beta \in (0,1)$, $\rho > 0$, and $\gamma > 0$.[9]

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8. One may conjecture that it is the (axiomatic) recursivity of tastes that constrains preference for late or early resolution to be governed by attitudes toward risk and intertemporal substitution. Less tightly axiomatized preferences should make it possible to disentangle those three concepts—very much in the same way that Kreps-Porteus preferences make it possible to separate risk aversion from intertemporal substitution.

9. This formulation ensures that current utility be increasing and concave in current consumption, and increasing in future utility. It also makes it possible to deal with the special cases $\gamma = 1$ and $\rho = 1$ (whose interpretation will be given below) by simply applying L'Hospital's rule to (2). A more cumbersome alternative, which is not pursued here for the sake of brevity, is instead to write a separate utility function for each special case. This route is followed by Epstein and Zin [1987a] and Epstein [1988]; their aggregator function, which must deal with special cases separately, obtains by setting $v_t = [1 + (1 - \beta)(1 - \gamma)V_t]^{1/(1-\gamma)} (1 - \beta)^{\rho-1}$ in (2), yielding the equivalent representation of tastes $v_t = [c_t^{1-\rho} + \beta\{(E_t v_{t+1}^{1-\gamma})^{1/(1-\gamma)}\}^{1-\rho}]^{1/(1-\rho)}$. This last representation points out that, with KP preferences, current utility is the aggregate of current consumption and the certainty equivalent of future *utility*. In a two-period setting (and only in a two-period setting), this is equivalent to aggregating current consumption and the certainty equivalent of future consumption, which is Selden's [1978] formulation.

The three parameters, $\beta$, $\gamma$, and $\rho$, which characterize this representation of tastes, have the following interpretation:

1. $\beta$ is, under certainty, the subjective discount factor. Under uncertainty, however, time preference is in general endogenous (unless $\gamma = \rho$, in which case the subjective discount factor is constant and equal to $\beta$), and measured by $U_2$.

2. $\gamma$ is the (constant) Arrow-Pratt coefficient of relative risk aversion for timeless gambles. As a consequence, it is also, as will be shown below, the elasticity of the (indirect) marginal utility of wealth.[10] Under certainty, $\gamma$ can be eliminated from the representation of tastes by a monotone increasing transformation: risk aversion must obviously play no role in the ranking of deterministic consumption sequences.

3. $1/\rho$ is the (constant) elasticity of intertemporal of substitution for *deterministic*[11] consumption paths. This is most easily seen by noticing that the preferences specified by (2) are equivalent under certainty, through a monotone increasing transformation to $\sum_{s=0}^{\infty} \beta^s [c_{t+s}^{1-\rho} - 1]/(1 - \rho)$.

Consumers endowed with our generalized isoelastic preferences prefer early (late) resolution when $\gamma$ is larger (smaller) than $\rho$[12]—a result that confirms the explanation given above of the link between timing preference, risk aversion, and intertemporal substitution. VNM time- and state-separable isoelastic preferences, to which (2) specializes when $\gamma = \rho$, are thus characterized by the equality of the inverse, $\rho$, of the elasticity of intertemporal substitution and of the coefficient of relative risk aversion, $\gamma$.

Logarithmic risk preferences and logarithmic intertemporal substitution preferences obtain, respectively, when $\gamma \rightarrow 1$ or $\rho \rightarrow 1$.

## II. Optimum Consumption

In this section I show that the methods commonly used to solve for the optimal infinite-horizon consumption program when preferences are representable by a VNM isoelastic utility index can be easily adapted to deal with the class of generalized isoelastic preferences introduced above.

10. Another interpretation is that the proportional premium that a consumer is willing to pay to avoid facing a small and fair lottery on *permanent* consumption is proportional to $\gamma$, with the constant of proportionality reflecting, as usual, the variance of that lottery.

11. The elasticity of intertemporal substitution is not a well-defined concept in a stochastic environment.

12. The aggregator function is then convex (concave) in its second argument.

As in the VNM case, an explicit solution can in general be derived, when preferences are isoelastic, only for the case of random interest rates and deterministic noninterest income.[13] To illustrate the solution procedure, assume, for simplicity, that noninterest income is zero in every period, and that the interest rate $\tilde{R}$ is identically and independently distributed.[14] Letting $w_t$ denote wealth at the beginning of period $t$, the budget constraint facing our consumer in period $t$ is simply

$$(3) \qquad w_{t+1} = \tilde{R}(w_t - c_t).$$

In complete parallel with the VNM case, one must find, for this infinite horizon problem, a value function $V(.)$ which solves the following functional equation, derived from (1):[15]

$$(4) \qquad V(w_t) = \max_{c_t} U[c_t, E_t V(w_{t+1})]$$

such that (3) holds. Exploiting the fact that the solution of the functional equation (4) is well-known in the VNM case, i.e., when $\gamma = \rho$, and that utility is homothetic, conjecture that

$$(5) \qquad V(w) = \frac{(\psi w)^{1-\gamma} - 1}{(1 - \beta)(1 - \gamma)}$$

and that

$$(6) \qquad c_t = \mu w_t,$$

where $\psi$ and $\mu$ are yet undetermined constants. One can show, by performing the maximization called for by (4), identifying the left- and right-hand sides of (4) term by term using the resulting expression, and after a few substitutions, that $\psi$ and $\mu$ are given by[16]

$$(7) \qquad \mu = 1 - \beta^{1/\rho}[(E\tilde{R}^{1-\gamma})^{1/(1-\gamma)}]^{(1-\rho)/\rho},$$

and

$$(8) \qquad \psi = [(1 - \beta)\mu^{-\rho}]^{1/(1-\rho)}.$$

13. In the special case of risk neutrality ($\gamma = 0$), however, Farmer [1990] shows that it is possible to derive an explicit solution for random interest rate *and* income.

14. Note that this (counterfactual) i.i.d. assumption raises, as shown by Kocherlakota [1987], observational equivalence issues. Much more serious observational equivalence problems arise in the presence of incomplete markets, as shown by Polemarchakis and Selden [1980].

15. The current state of nature does not affect the valuation of wealth because of the i.i.d. assumption.

16. A restriction on the distribution of $\tilde{R}$ is, as usual, required to ensure the existence of an optimal consumption plan since, for given $\beta$, $\gamma$ and $\rho$, we need $\mu > 0$.

Notice that the Arrow-Pratt measure of the curvature of the value function, $[-wV''(w)/V'(w)]$, is simply $\gamma$—a property that confirms the interpretation, suggested above, of $\gamma$ as a coefficient of relative risk aversion for static gambles.[17] We thus generalize and make more precise Merton's [1973] result: for generalized isoelastic preferences, the value function is a power function of wealth, and its curvature is governed by attitudes toward *risk*.

As for the propensity to consume out of wealth, $\mu$, its constancy of course reflects the two assumptions of infinite lifespan (so that it is age-independent) and of i.i.d. returns (so that it is time- and state-independent). Notice that in the special case of a unit elasticity of intertemporal substitution ($\rho = 1$), the marginal propensity to consume is equal to $1 - \beta$ independently of the value of the risk aversion coefficient and of the distribution of the asset return. One can therefore conclude that the rational "myopia" in consumption and savings decisions exhibited by VNM logarithmic utility stems from a unit elasticity of intertemporal substitution rather than from a unit coefficient of relative risk aversion.[18]

### III. MACROECONOMIC IMPLICATIONS

I now analyze two macroeconomic implications of the foregoing results: the relation between interest rates and consumption, and the effect of uncertainty on the marginal propensity to save.

### *Consumption and Interest Rates*

While Breeden [1979] has shown that, within the context of time-additive isoelastic expected utility and log-normally distributed returns, the parameter that measures both relative risk aversion and intertemporal substitution plays a crucial role in determining the response of the optimal consumption profile to changes in the mean interest rate, there has been substantial debate in the literature as to whether this role is in fact played by risk aversion or intertemporal substitution.

Equation (7) makes clear that the marginal propensity to consume, $\mu$, and thus the shape of the optimal consumption profile, depends in general both on the elasticity of intertemporal substitution $1/\rho$ and on the degree of risk aversion $\gamma$. Therefore, one cannot

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17. The value function is linear in wealth when $\gamma = 0$, so that our consumer is then risk neutral for static gambles. This case is studied by Farmer [1988].
18. See Giovannini and Weil [1989] for a general derivation of this result, and its implications for the capital asset pricing model.

*in general* characterize the response of the consumption profile to changes in the mean rate distribution solely by the magnitude of the elasticity of intertemporal substitution. Hall's [1988] results are thus model specific.

Formally, using (7), one finds that

$$
(9) \quad
\begin{aligned}
E \ln \left( \frac{c_{t+1}}{c_t} \right) &= \ln (1 - \mu) + E \ln \tilde{R} \\
&= \frac{1}{\rho} \left[ \ln \beta + \frac{1 - \rho}{1 - \gamma} \ln (E\tilde{R}^{1-\gamma}) \right] + E \ln \tilde{R},
\end{aligned}
$$

an expression that, given the distribution of $\tilde{R}$, generally depends in a complex way on all the parameters of the utility function.

A notable exception emerges, however, when the interest rate is log-normal—a distributional assumption often made in the empirical literature. Suppose, for instance, that $\ln \tilde{R} \sim N(r, 2\sigma^2)$. Then it is immediate, from (9) and the properties of the log-normal distribution, that

$$
(10) \quad E \ln \left[ \frac{c_{t+1}}{c_t} \right] = \frac{1}{\rho} [r + \ln \beta] + \frac{(1 - \rho)(1 - \gamma)}{\rho} \sigma^2.
$$

Therefore,

$$
(11) \quad \frac{\partial (E \ln[c_{t+1}/c_t])}{\partial r} = \frac{1}{\rho},
$$

and it is unambiguously the elasticity of intertemporal substitution and not the coefficient of risk aversion that governs the response of the optimum consumption profile to changes in the mean (log) interest rate, $r$. Hall's [1988] intuition, based on Selden's two-period OCE preferences, hence carries over to multiperiod economies in the special case of log-normal returns.

## The Effects of Interest Rate Risk

The traditional (expected utility) analysis of the effects of interest rate risk, based on the work of Leland [1968] and Sandmo [1970], has established that whether the marginal propensity to save rises or falls in response to a Rothschild-Stiglitz mean-preserving increase in interest rate risk depends only on whether the elasticity of intertemporal substitution (or the inverse of the coefficient of relative risk aversion) is smaller or greater than 1.

This result is difficult to explain within the context of time-additive expected utility, precisely because the traditional analysis does not distinguish between attitudes toward risk and attitudes

toward intertemporal substitution. Once this distinction is made, the condition under which increased uncertainty may lead increased savings becomes obvious—as I now show for the case of our multiperiod generalized isoelastic preferences.[19]

To determine whether increased rate risk leads, for a given mean interest and a given level of wealth, to increased savings, it suffices to examine whether increased uncertainty raises or lowers the marginal propensity to consume, given in (7), which can be rewritten as

$$(12) \qquad\qquad \mu = 1 - \beta^{1/\rho}\hat{R}^{(1-\rho)/\rho},$$

where

$$(13) \qquad\qquad \hat{R} \equiv (E\tilde{R}^{1-\gamma})^{1/(1-\gamma)}$$

is simply the *certainty equivalent* of the interest rate distribution.

Now, for any given $\gamma$, a mean-preserving spread of the interest rate distribution lowers, for risk-averse consumers ($\gamma > 0$), the certainty equivalent interest rate $\hat{R}$. Asking whether more uncertainty leads to larger savings is thus equivalent to determining whether a lower certainty equivalent interest rate leads to a lower marginal propensity to consume.

The answer to this question, as is well-known and immediate from (12), depends only on the relative strength of income and substitution effects, i.e., on the magnitude of the elasticity of intertemporal substitution $1/\rho$. When income effects are relatively small ($\rho < 1$), the interest elasticity of savings is positive and a lower certainty equivalent interest rate leads to decreased savings. When income and substitution effects balance each other out ($\rho = 1$), the marginal propensity to consume is unaffected by changes in $\hat{R}$, and thus does not depend (whatever the value of $\gamma$) on the interest rate distribution (as already argued above). It is only when the elasticity of intertemporal substitution is small ($\rho > 1$), and the interest elasticity of savings is negative, that a mean-preserving increase interest rate risk—and, as a consequence, a lower certainty equivalent interest rate—leads to increased savings.

This certainty-equivalent analysis clarifies the respective role of risk aversion and intertemporal substitution. The substitution effect depresses the marginal propensity to save as soon as agents

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19. Selden [1979] presents a related explanation within the context of his two-period OCE preferences, whose market equilibrium implications are explored by Barsky [1987]. Epstein [1988] provides the multiperiod general equilibrium asset pricing counterpart to the following results.

are risk averse, as the optimum way to maintain the original utility level when interest rate risk increases is to consume more today (and thus avoid facing the increased risk). The income effect is simply a precautionary savings effect, whose magnitude depends on the intertemporal elasticity of substitution: increased interest rate risk implies a higher probability of low consumption tomorrow, against which consumers will protect themselves the more, by consuming less, the more averse they are to intertemporal fluctuations of consumption. Which of these two conflicting effects dominates depends on the strength of the precautionary motive, i.e., on the magnitude of the intertemporal elasticity of substitution.

Attitudes toward intertemporal substitution are thus the sole determinant of the *sign* of the effect of increased uncertainty on savings. Behavior toward risk, by determining the amplitude of the associated reduction in the certainty equivalent interest rate, affects only the *magnitude* of the effect.

## IV. Concluding Remarks

I now briefly address the broader question of whether these new preferences, or variations upon them, have a potential for explaining the puzzles in permanent income theory and the asset pricing anomalies that cannot be accounted for by more traditional representations of preferences.

From a theoretical standpoint, it is clear that the misspecification of preferences will lead both theorist and econometrician to predictions not likely to be supported by facts. Consider, for instance, Hall's [1978] random walk hypothesis for consumption. The most general characterization of the behavior of an optimizing consumer is that, with a constant safe interest rate and random income, the marginal (indirect) utility of *wealth* should follow a geometric random walk with a constant drift. Along an optimum path, the marginal utility of wealth is equalized to the marginal utility of consumption.[20] Whether one can translate this statement into Hall's claim that *consumption,* or a function thereof, should follow a random walk obviously depends on whether current consumption is a sufficient statistic for wealth.

Many cases in which consumption is not a sufficient statistic for wealth, because of various nonseparabilities in the utility function, have already been studied in the literature: nonseparabil-

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20. However, see Grossman and Laroque [1987] for an economy in which this equality, derived from the envelope theorem, does not obtain.

ity of leisure [Mankiw, Rotemberg, and Summers, 1985; Eichenbaum, Hansen, and Singleton, 1987] or of consumption durables [Bernanke, 1985]. Kreps-Porteus preferences, because they are not in general time-separable, also make it impossible to translate statements about wealth into simple propositions about the temporal behavior of consumption. Formally, from (1), the marginal utility of wealth is simply, along an optimal path,

$$ (14) \qquad U_1[c_t, E_t \, V_{t+1}]. $$

Consumption is not, therefore, a sufficient statistic for wealth, unless the aggregator function is separable in its second argument. Hence one cannot in general formulate a random walk hypothesis for consumption when preferences are KP.[21]

It is not likely, however, that the simple device of modifying preferences will in the end suffice to account for the temporal behavior of consumption and asset returns. Nonseparabilities, across goods, states, or dates[22] are of course relevant (as the foregoing analysis shows); yet the assumption that preferences are separable is probably *not* the most serious misspecification of most macroeconomic models.

More generally, it is obvious that a description of the economy based upon the Kreps-Porteus representation of preferences cannot replicate the temporal behavior of consumption and asset returns less accurately than time-additive expected utility (the latter being a special case of the former). But it is doubtful that it will do much better empirically as long as it maintains the "standard" simplifying assumption of complete markets and the paradigm of the representative agent.[23]

What, therefore, are the contributions of this paper? From a methodological standpoint it seems indubitable that the class of

21. More precise results, outside the scope of this paper, require solving explicitly a problem with random income and safe interest rate. This can only be done (see Kimball [1987]) for preferences exhibiting "constant absolute prudence," among which quadratic and exponential utility. A Kreps-Porteus version of exponential utility can be constructed by using the aggregator function $U[c, \zeta] = [e^{\rho c} + \beta \zeta^{\rho/\gamma}]^{\gamma/\rho}$, where $\rho < 0$ parameterizes attitudes toward intertemporal substitution and $\gamma$ behavior toward risk.

22. Constantinides [1988] studies time-nonseparable, but state-separable, preferences with "habit formation." These preferences match first moments of rates of return better than time- and state-separable preferences; yet, they seem to predict a counterfactually large variability of the risk-free rate.

23. In a related [1989] paper I show, for instance, that the Mehra-Prescott [1985] equity premium puzzle does not miraculously vanish when generalized isoelastic preferences are adopted. Quite the contrary, the mystery deepens: for "plausible" parameterizations of tastes, one can explain neither why the risk premium is so high, nor why the risk-free rate is so low.

preferences introduced here represents a convenient, flexible, and almost natural parameterization of tastes exhibiting the properties of constant elasticity of substitution and constant coefficient of relative risk aversion. One would even hazard the conjecture that the preferences introduced in (2) are the only ones with these two properties. From a theoretical perspective, separating risk aversion from attitudes toward intertemporal substitution yields new insights into the determinants of consumption and savings under uncertainty. Finally, from an empirical point of view, these preferences provide the simple framework required, under appropriate distributional assumptions, to derive unconstrained estimates of the coefficient of relative risk aversion and of the elasticity of intertemporal substitution.[24]

That these preferences, however, are only a tool, and not a panacea, is hardly surprising: in production theory the generalization of production functions from Cobb-Douglas to CES did not solve, for instance, the problems of investment theory. It "only" provided a more flexible and general analytical framework in which to conduct theoretical and empirical research.

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24. See Attanasio and Weber [1989], Epstein and Zin [1987b], or Giovannini and Weil [1989] for empirical evidence based on generalized isoelastic preferences.

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