Seven years in Finance

Linear Algebra meets Wall Street

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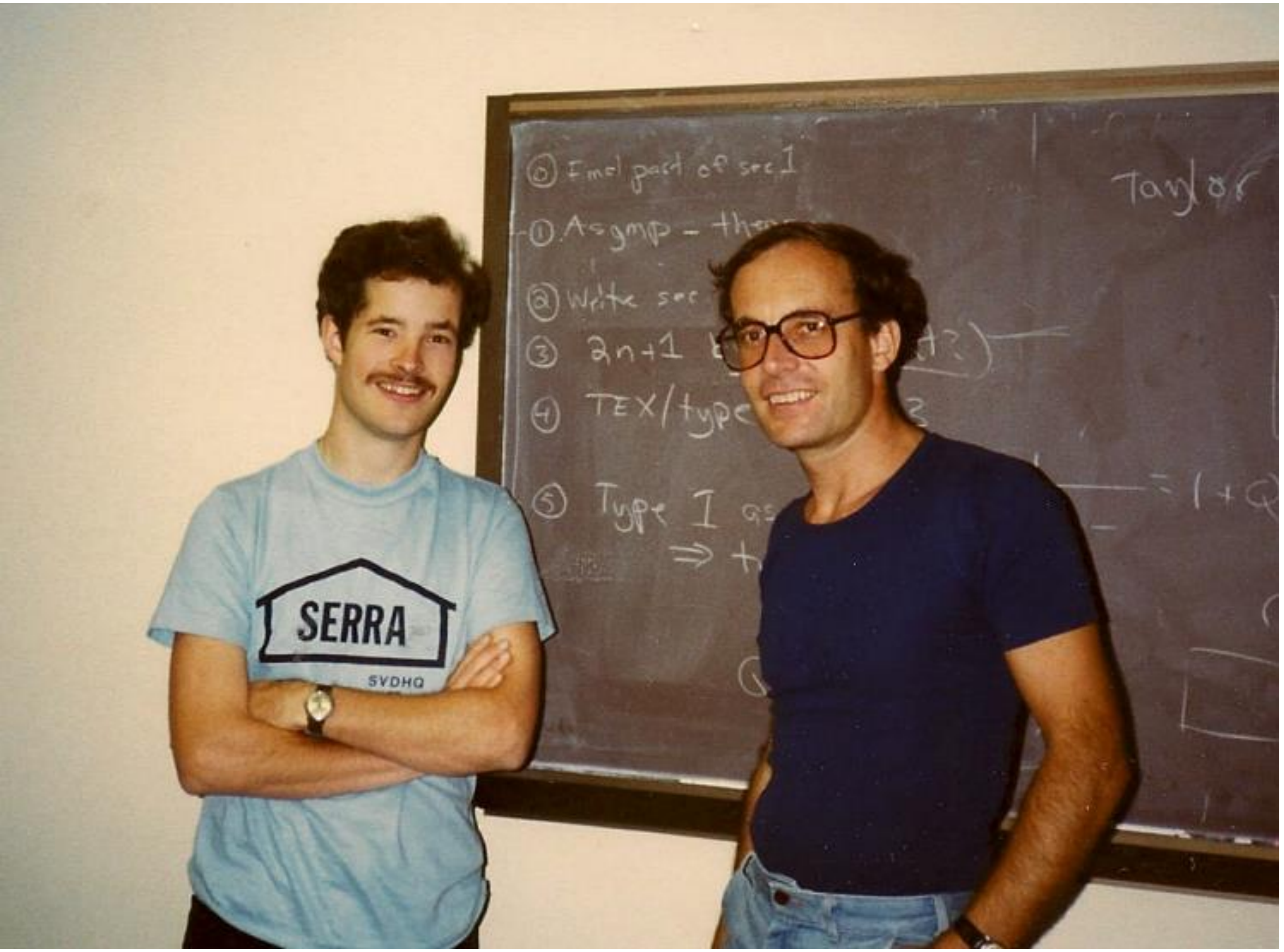
Congratulations!



Martin,
when I first met you you were half your age, and you seemed pretty old. Now you seem pretty young!

I wish I could be with you and Ursula and the family to share in the celebration.

Being a Quant

- Left academia (Canada, thank you!) and became a quant in 2007

- Applying maths (optimization, linear algebra, statistics, machine learning, etc.) to problems in finance

- Returned to Switzerland in 2010 and since February 2014:

  Head of Quantitative Research for a Swiss Wealth Manager @ Geneva

Last year

- Gardening leave at Maui, Hawaii
- Padi Open Water Diver, Cycling up Haleakala, …

- Wrote papers with Raphael Hauser, Erling Andersen and Joachim Dahl
- Only available on ArXiv.org

Motivation

- Great research out there, but ignored in industry.
- Numerous and often amazing mistakes.

The Sculptor method

$$\min_{\mathbf{w}\in\mathbb{R}^m} \|\mathbf{X}\mathbf{w}-\mathbf{y}\|_2^2$$

Such that:

$$w \geq 0$$
$$\sum w = 1$$

- Compute $\mathrm{C} = (X^T X)^{-1}$
- Compute $w' = C * X^T y$
- Delete negative entries from w′ (first constraint)
- Scale w′ (second constraint)

Seven Sins in Portfolio Optimization

- The mother of all problems in portfolio optimization:

$$x(t) = \arg\max_{x \in \mathbb{R}^n} x^{\mathrm{T}} \mu, \quad \text{s.t. } x^{\mathrm{T}} Q x \le \sigma_{\max}^2$$

- Where do we get μ and Q from?
  - Time series analysis
  - regression and extrapolation
  - The car analogy

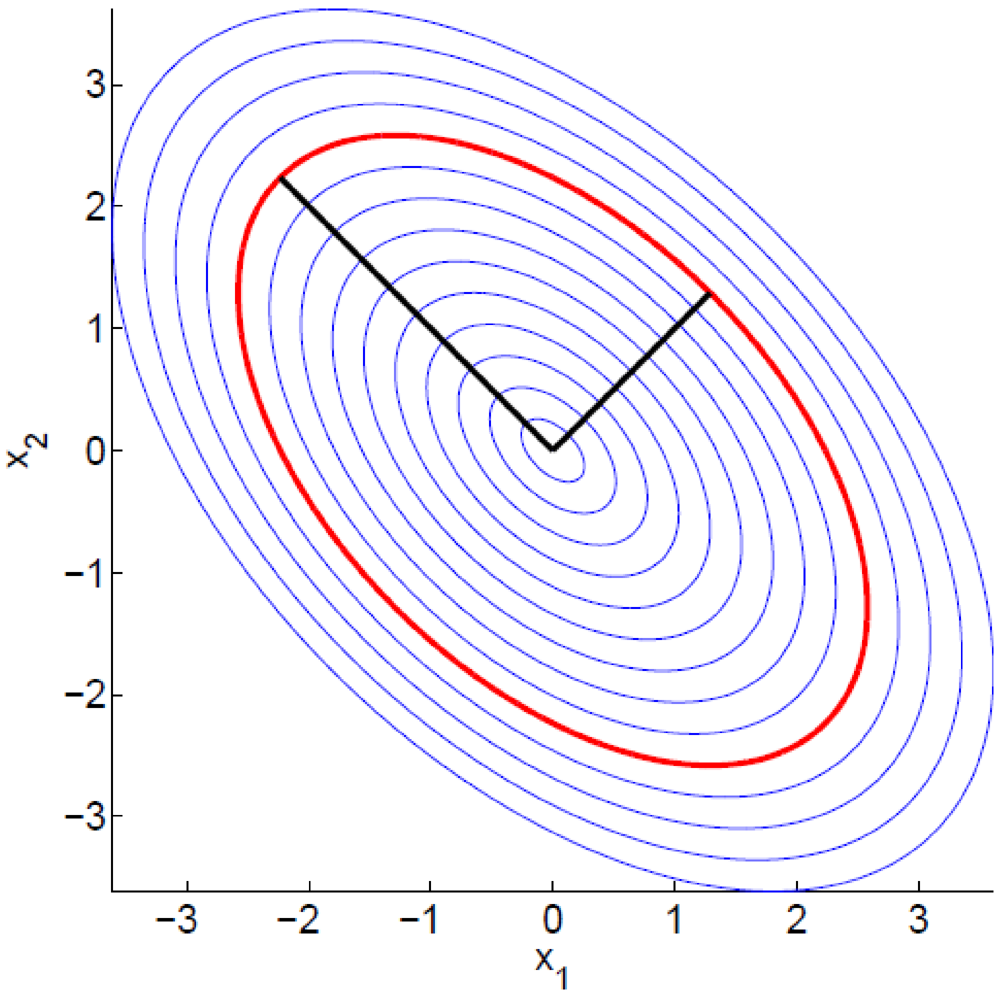


FIG. 2.1. *Contour lines of the expected variance* $x^{\mathrm{T}}Qx$ *for* $Q = \begin{pmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{pmatrix}$. *The red line marks the boundary of the ellipsoid* $x^{\mathrm{T}}Qx \leq 1$. *The eigenvectors of* Q *define the principal directions of the ellipsoid and the inverse of the square roots of the eigenvalues are the corresponding equatorial radii.*

User feedback

- It’s broken
- My problems are far too complicated for this
- We have developed a proprietary method far superior
- Some are rediscovering familiar concepts…

(The solvers) overuse statistically estimated information and magnify the impact of estimation errors. It is not simply a matter of garbage in, garbage out, but, rather, **a molehill of garbage in, a mountain of garbage out**” (Michaud 1998)

An analytic solution exists, but…

$$x_* = \sigma_{\max} \frac{Q^{-1}\mu}{\sqrt{\mu^{\mathrm{T}} Q^{-1} \mu}}$$

Constraints on x

- Positivity
- Constraints on the sum of (some) entries of x
- Constraints on the 1-Norm of x
- Bounds on individual entries
- Dependency on a previous state

Analytic solution rather useless here…. (as often)

Any negative eigenvalues here?

- Solution blows up.
- Problem is unbounded.
- Constraints to avoid this?!

$$x(t) = \arg\max_{x \in \mathbb{R}^n} x^{\mathrm{T}} \mu,$$
$$\text{s.t. } x^{\mathrm{T}} Q x \leq \sigma_{\max}^2$$

Eigenvalues too small?

$$x_* = \sigma_{\max} \frac{Q^{-1}\mu}{\sqrt{\mu^{\mathrm{T}} Q^{-1} \mu}} \qquad Q^{-1} = \sum_{i=1}^{n} \frac{1}{\lambda_i} v_i v_i^{\mathrm{T}}$$

- Small eigenvalues in a covariance often correspond to statistical noise. Solver will prefer the allocation in principal portfolios (e.g. eigenvectors) corresponding too small eigenvalues

- Regularization, Shrinkage estimators, etc.

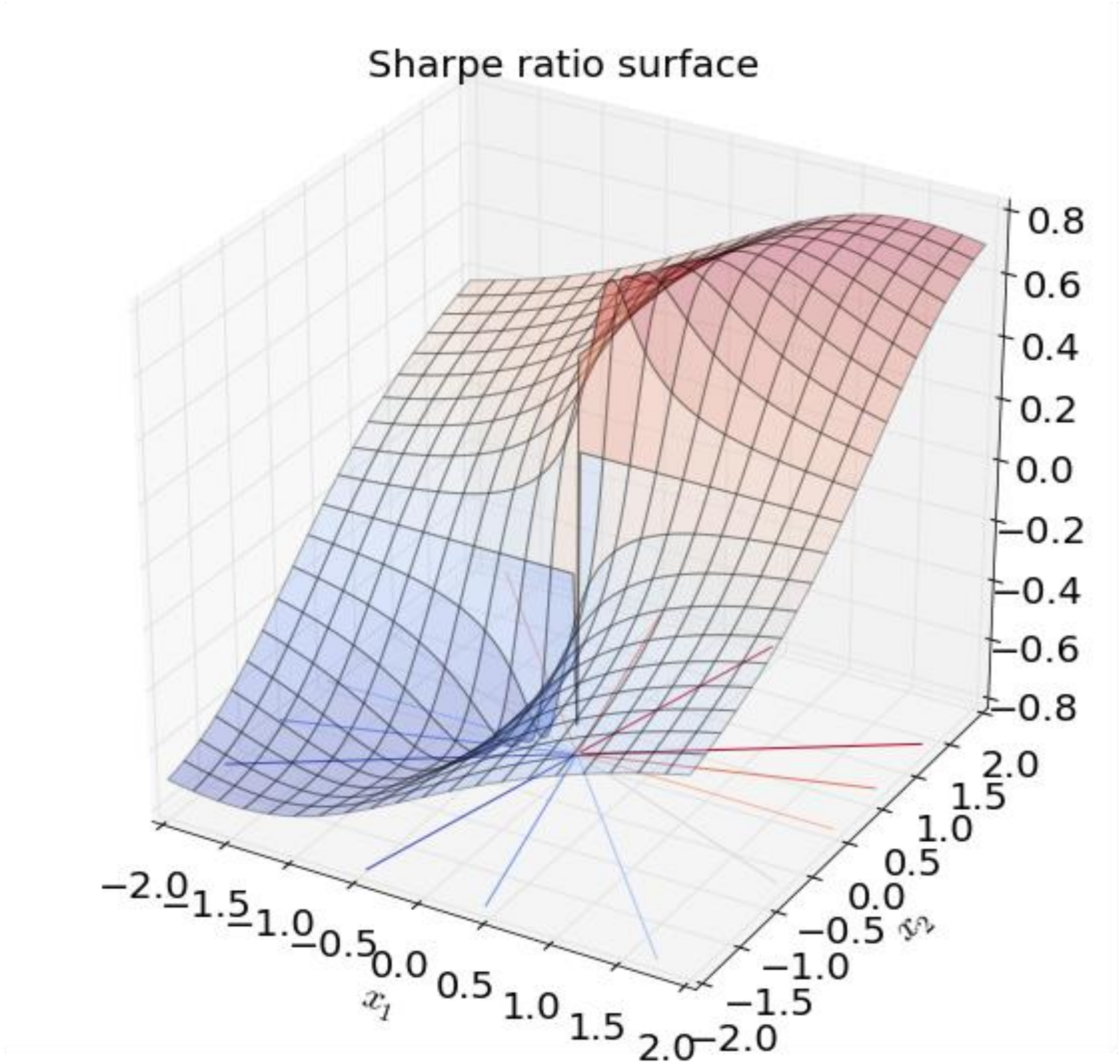
- Spectral transformation of Q required.

Two steps?

$$y^* = \eta y = \sigma_{\max} \frac{\mu}{\sqrt{\mu^T Q \mu}}$$

Maximize the expected return (on a unit sphere) and then lengthen the vector

The Sharpe Ratio



- Solution is maximizing the Sharpe Ratio
- But maximizing the Sharpe ratio isn't a good idea!

$$S(x) = \frac{x^{\mathrm{T}}\mu - r_f}{\sqrt{x^{\mathrm{T}}Qx}}$$

Assume r_f = 0

The wrong solver

- Create your own solver!?
- Simulated Annealing!
- Matlab’s quadprog

- Dedicated solver for SOCP (Second Order Cone Programs):
- Mosek, Sedumi, SDPT3, …

$$x(t) = \arg \max_{(x,t)\in\mathbb{R}^{2n}} \left[x^{\mathrm{T}}\mu - \sum_{i=1}^{n} p_i t_i \right]$$

$$\begin{aligned} \text{s.t.}\ \ x^{\mathrm{T}} C x &\le \sigma_{\max}^2, \\ x_i - x_i^0 &\le t_i, \\ x_i^0 - x_i &\le t_i. \end{aligned}$$

Lifting may help

$$x(t) = \arg \max_{x\in\mathbb{R}^n} x^{\mathrm{T}}\mu - \sum_{i=1}^{n} p_i |x_i - x_i^0|$$

$$\text{s.t.}\ \ x^{\mathrm{T}} C x \le \sigma_{\max}^2\ .$$

Solving the impossible

$$x(t) = \arg\max_{x \in \mathbb{R}^n} x^{\mathrm{T}} E[R]$$
$$\text{s.t.} \;\; x^{\mathrm{T}} \operatorname{Cov}(R, R) x \leq \sigma^2_{\max}$$
$$e^{\mathrm{T}} x = 1$$

What’s the common source of problems?

- People underestimate the complexity of optimization problems.
- The freedom to formulate problems has resulted in various poor choices.
- Optimization is often considered as some sort of IT problem.

How to do it then?

Applications of second-order cone programming [1]

Miguel Sousa Lobo [a,2], Lieven Vandenberghe [b,*], Stephen Boyd [c,3], Hervé Lebret [d,4]

We consider the *second-order cone program* (SOCP)

$$\begin{array}{ll} \text{minimize} & f^{\mathrm{T}}x \\ \text{subject to} & \|A_i x + b_i\| \leqslant c_i^{\mathrm{T}} x + d_i, \quad i = 1, \ldots, N, \end{array}$$

mosek inside

Thank you

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http://arxiv.org/abs/1310.3396

http://arxiv.org/abs/1310.3397

or

git clone https://github.com/tschm/mhg70