In [2]:

```
import matplotlib
import matplotlib.pyplot as plt
matplotlib.style.use('ggplot')

from IPython.core.display import display, HTML
display(HTML("<style>.container { width:100% !important; }</style>"))

import numpy as np
import pandas as pd
```

Typesetting math: 100%

Constructing estimators

https://en.wikipedia.org/wiki/Autoregressive_model

Thomas Schmelzer

A very common estimator is based on AR models (autoregressive)

$$R_T = \sum_{i=1}^{n} w_i r_{T-i}$$

Predict the (unknown) return R_T using the last n previous returns. **Attention**: You may want to use volatility adjusted returns, apply filters etc.

How to pick the n free parameters in $\mathbf{w}$? (Partial) autocorrelations?

In [3]:

```python
def convolution(ts, weights):
    from statsmodels.tsa.filters.filtertools import convolution_filter
    return convolution_filter(ts, weights, nsides=1)
```

In [4]:

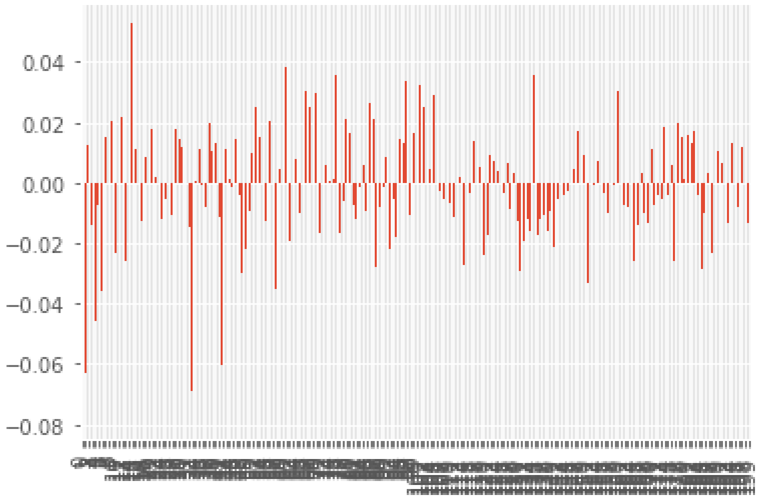
```python
r = pd.Series([1.0, -2.0, 1.0, 1.0, 1.5, 0.0, 2.0])
weights = [2.0, 1.0]
# trendfollowing == positive weights
x=pd.DataFrame()
x["r"] = r
x["pred"] = convolution(r, weights)
x["before"] = x["pred"].shift(1)
print(x)
print(x.corr())
```

```
     r  pred  before
0  1.0   NaN     NaN
1 -2.0  -3.0     NaN
2  1.0   0.0    -3.0
3  1.0   3.0     0.0
4  1.5   4.0     3.0
5  0.0   1.5     4.0
6  2.0   4.0     1.5
               r      pred    before
r       1.000000  0.895788 -0.190159
pred    0.895788  1.000000  0.538431
before -0.190159  0.538431  1.000000
```

Looking only at the last two returns might be a bit …

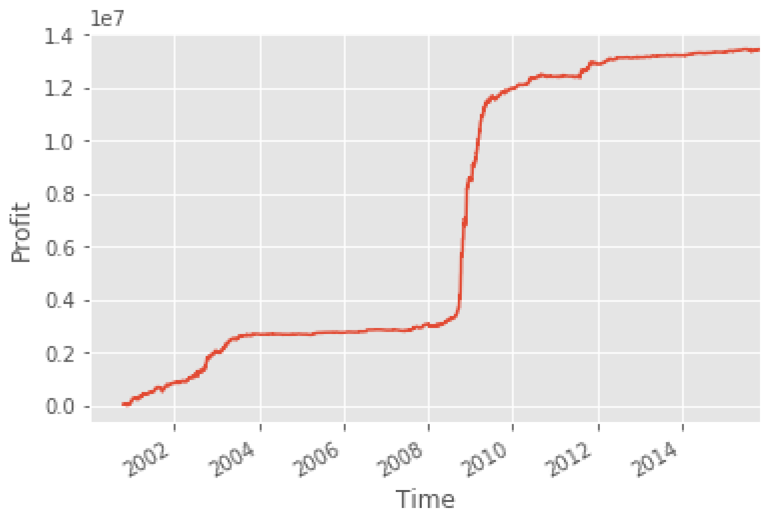
Is it a good idea to have $n = 200$ free parameters?

```
In [6]: import statsmodels.tsa.stattools as sts
        # generate random returns
        r = pd.read_csv("data/SPX_Index.csv", squeeze=True, index_col=0, parse_dates=True).pc
        # let's compute the optimal convolution!
        weights = sts.pacf(r, nlags=200)
        pd.Series(data=weights[1:]).plot(kind="bar")
        plt.show()
```



In [7]:

```
# The trading system!
pos = convolution(r, weights[1:])
pos = 1e6*(pos/pos.std())
# profit = return[today] * position[yesterday]
(r*pos.shift(1)).cumsum().plot()
plt.xlabel('Time'), plt.ylabel('Profit')
plt.show()
```



Bias

We assume the weights are exponentially decaying, e.g.

$$w_i = \frac{1}{S}\lambda^i$$

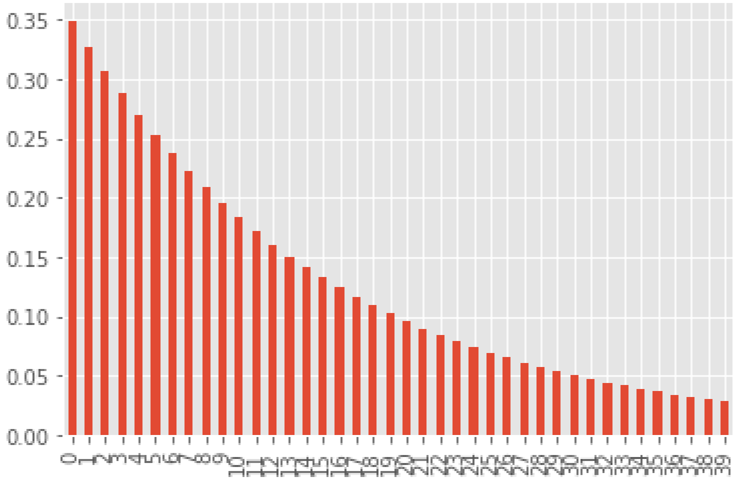
where S is a suitable scaling constant and $\lambda = 1 - 1/N$. Note that $N \neq n$.

Everything that is **not** an exponentially weighted moving average is **wrong**.

In [8]:

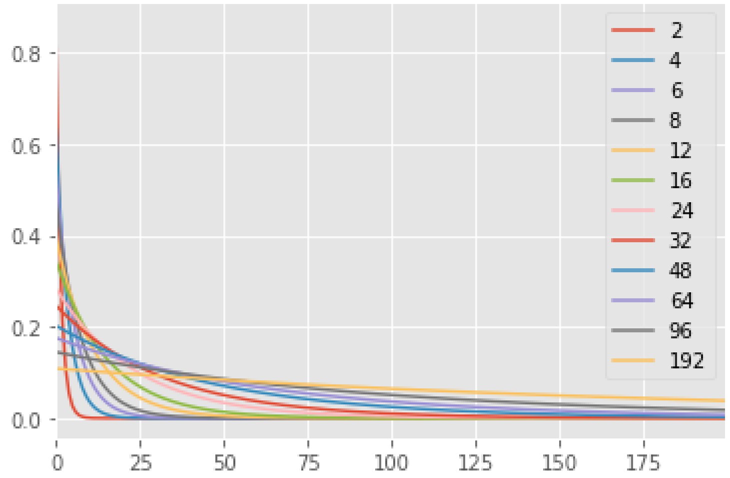
```python
def exp_weights(m, n=100):
    x = np.power(1.0 - 1.0/m, range(1,n+1))
    S = np.linalg.norm(x)
    return x/S

pd.Series(exp_weights(m=16,n=40)).plot(kind="bar")
plt.show()
```



In [9]:

```
periods = [2,4,6,8,12,16,24,32,48,64,96,192]
# matrix of weights
W = pd.DataFrame({period : exp_weights(m=period, n=200) for period in periods})
W.plot()
plt.show()
```

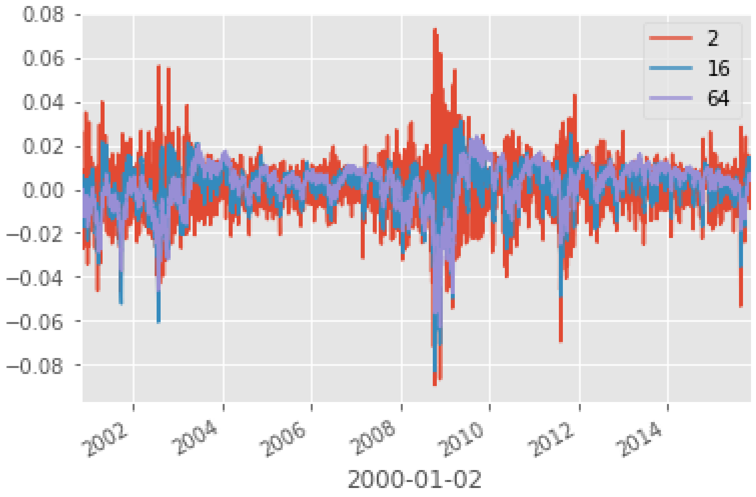


In [10]:

```
# each column of A is a convoluted return time series
A = pd.DataFrame({period : convolution(r, W[period]).shift(1) for period in periods})

A = A.dropna(axis=0)
r = r[A.index].dropna()

A[[2,16,64]].plot()
plt.show()
```



(Naive) regression

$$\mathbf{w}^* = \arg\min_{\mathbf{w}\in\mathbb{R}^m} \|\mathbf{A}\mathbf{w} - \mathbf{r}\|_2$$

(Naive) regression

$$\mathbf{w}^* = \arg\min_{\mathbf{w}\in\mathbb{R}^m} \|\mathbf{A}\mathbf{w} - \mathbf{r}\|_2$$

In [11]:

```
from numpy.linalg import lstsq
# sometimes you don't need to use MOSEK :-)
weights = pd.Series(index=periods, data=lstsq(A.values, r.values)[0])
print(weights)
(W*weights).sum(axis=1).plot(kind="bar")
(W*weights).sum(axis=1).plot()
plt.show()
```

```
/opt/conda/envs/beakerx/lib/python3.6/site-packages/ipykernel_launcher.py:3: Fut
ureWarning: `rcond` parameter will change to the default of machine precision ti
mes ``max(M, N)`` where M and N are the input matrix dimensions.
To use the future default and silence this warning we advise to pass `rcond=None
`, to keep using the old, explicitly pass `rcond=-1`.
  This is separate from the ipykernel package so we can avoid doing imports unti
l
```

Mean variation

We provide a few indicators. Avoid fast indicators. Prefer slower indicators as they induce less trading costs. Use the mean variation of the signal (convoluted returns here)

$$f(\mathbf{x}) = \frac{1}{n}\sum |x_i - x_{i-1}| = \frac{1}{n}\|\Delta \mathbf{x}\|_1$$

The ith column of $\mathbf{A}$ has a mean variation d_i. We introduce the diagonal penalty matrix $\mathbf{D}$ with $D_{i,i} = d_i$.

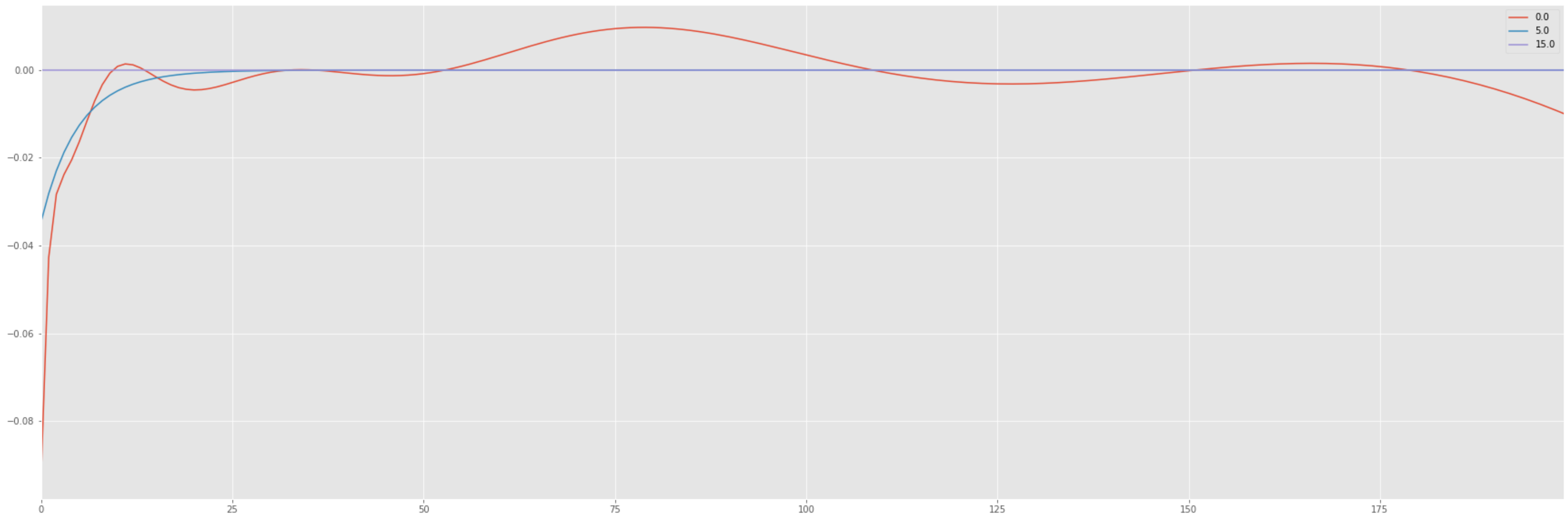
$$\mathbf{w}^* = \arg\min_{\mathbf{w}\in\mathbb{R}^m} \|\mathbf{A}\mathbf{w} - \mathbf{r}\|_2 + \lambda\|\mathbf{D}\mathbf{w}\|_1$$

In [15]:
```
from cvx.util import cvx, minimize

def mean_variation(ts):
    return ts.diff().abs().mean()

def ar(A, r, lamb=0.0):
    # introduce the variable for the var
    x = cvx.Variable(A.shape[1])
    D = np.diag(A.apply(mean_variation))
    minimize(objective=cvx.norm(A.values*x-r,2) + lamb*cvx.norm(D*x,1))
    return pd.Series(index=A.keys(), data=x.value)
```

In [16]:
```
t_weight = pd.DataFrame({lamb : (W*ar(A, r.values, lamb=lamb)).sum(axis=1) for lamb :
t_weight[[0.0,5.0,15.0]].plot(figsize=(30,10))
plt.show()
```

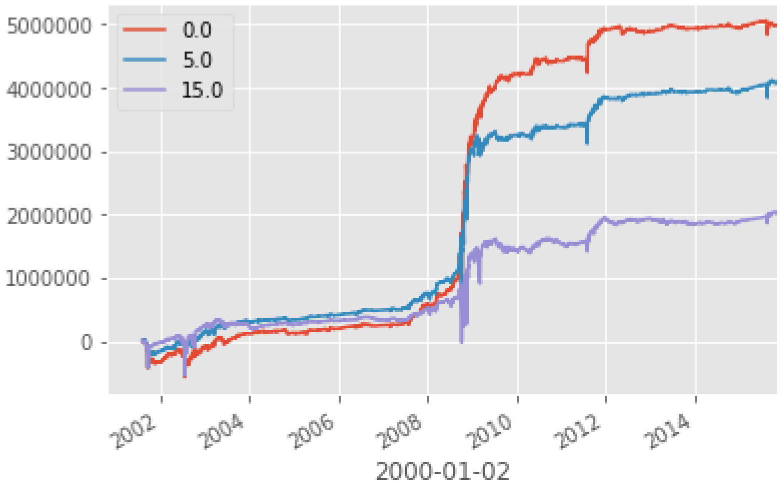


In [17]:

```python
#for lamb in sorted(t_weight.keys()):

pos = pd.DataFrame({lamb : convolution(r, t_weight[lamb]) for lamb in t_weight.keys()
pos = 1e6*(pos/pos.std())

profit = pd.DataFrame({lamb : (r*pos[lamb].shift(1)).cumsum() for lamb in pos.keys()
profit[[0.0, 5.0, 15.0]].plot()
plt.show()
```



Summary

- The problem of constructing an estimator is corresponds to tracking an index. The index is here a historic return time series. The **assets** are standard estimators.
- Using the (mean) total variation of the signals can help to prefer slower signals rather than expensive fast signals.