An Incomplete Overview of Strategic Models of Network Formation

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Overview

- Why study models of network formation?
  - equilibrium $\mathbf{D}$ may be inefficient.
  - planner may have preferences over $\mathbf{D}$ and hence is interested in policies which influence it.
  - we might view network manipulations as a mechanism for influencing other outcomes.
  - correct for "endogenous network formation bias" (cf., Auerbach, 2016)

Existing Approaches

- "Applied theory approach": posit generative models of network formation that match "stylized facts" (Albert and Barabasi, 2002).

- ERGM: directly write down likelihoods for $\Pr(\mathbf{D} = \mathbf{d})$ and try to maximize them (cf., Shalizi and Rinaldo, 2013, *Annals of Statistics*):

  - generally no micro-foundations...

  - ...but see Mele (2017, *Econometrica*)

Existing Approaches

- Random Utility Models (RUM): Sheng (2012), Christakis *et al.* (2012), Imbens and Goldsmith-Pinkham (2013), Graham (2013, 2016, 2017), de Paula et al. (2015).

- Specialized structural modes: Banerjee *et al.* (2012).

Random Utility Approach

- This approach is both natural, and familiar, to economists.

- Provides a framework for modeling the effect on link surplus (i.e., utility) of

  – observed agent/dyad covariates;

  – unobserved agent attributes (heterogeneity);

  – preference interdependencies (e.g., a taste for transitivity).

A Simple Model of Network Formation

- Consider a network of three agents: i, j, k
  - Link formation: $D_{ij} = \mathbf{1}\left(\alpha + \beta D_{ik} D_{jk} - U_{ij} \geq 0\right)$ with $\beta \geq 0$ (returns to transitivity).
  - Three "types" of U_{ij} draws: $\mathbb{U}_L = (-\infty, \alpha]$, $\mathbb{U}_M = (\alpha, \alpha + \beta]$ or $\mathbb{U}_H = (\alpha + \beta, \infty)$.
  - Positive measure on the subset of the support of $\mathbf{U} = \left(U_{ij}, U_{ik}, U_{jk}\right)'$ with multiple NE networks.
- The model is *incomplete* (cf., Bresnahan and Reiss, 1991; Tamer, 2003).

A Simple Model of Network Formation (continued)

- There are $3^3 = 27$ "configurations" of $\mathbf{U}$...

- ...but only $\frac{(3+3-1)!}{3!(3-1)!} = 10$ non-isomorphic ones.

- Two of these configurations admit multiple NE networks:
  - if $U_{ij} \in \mathbb{U}_L$, $U_{ik} \in \mathbb{U}_M$ and $U_{jk} \in \mathbb{U}_M$;
  - if $U_{ij} \in \mathbb{U}_M$, $U_{ik} \in \mathbb{U}_M$ and $U_{jk} \in \mathbb{U}_M$.

A Simple Model of Network Formation (continued)

- Only one realization (out of 4 possible realizations of $\mathbf{D}$) is uniquely predicted:
  - if $U_{ij} \in \mathbb{U}_L$, $U_{ik} \in \mathbb{U}_L$ and $U_{jk} \in \mathbb{U}_H$, then links $\{i, j\}$ and $\{i, k\}$ form and $\{j, k\}$ does not.

- cf., Ciliberto and Tamer (2009), Sheng (2012), de Paula et al. (2015) provide methods for analyzing incomplete models of network formation.

- serious challenges to implementation at scale.

Christakis, Fowler, Imbens and Kalyanaraman (2010)

- Dyads form links *sequentially* and *myopically.*

- If the linking order is ij, ik and jk we have:
  - $D_{ij} = \mathbf{1}\left(\alpha - U_{ij} \geq 0\right)$;
  - $D_{ik} = \mathbf{1}(\alpha - U_{ik} \geq 0)$;
  - $D_{jk} = \mathbf{1}\left(\alpha + \beta\mathbf{1}\left(\alpha - U_{ij} \geq 0\right)\mathbf{1}(\alpha - U_{ik} \geq 0) - U_{jk} \geq 0\right)$.

- Conditional on the ij, ik and jk the realization of $\mathbf{U}$ delivers a unique prediction of $\mathbf{D}$.

Christakis, Fowler, Imbens and Kalyanaraman (2010)

- Since we don't observe the order of link formation we
  - assign a (prior) distribution to it;
  - work with an integrated likelihood.

- With three dyads there are $3! = 6$ possible link orderings. Let $O \in \mathbb{O} = \{1, 2, 3, 4, 5, 6\}$ be the possible orderings. Our integrated likelihood is

$$\Pr(\mathbf{D} = \mathbf{d}) = \sum_{o \in \mathbb{O}} \Pr(\mathbf{D} = \mathbf{d} | O = o) \Pr(O = o).$$

Christakis, Fowler, Imbens and Kalyanaraman (2010)

- Christakis et al. (2010) use Bayesian MCMC methods
  - provides a method of inference as well;
  - (no large sample theory for their estimator).

- Simulation methods, and assumptions about the timing of link formation, are also central to work by Goldsmith-Imbens and Pinkham (2013), Hsieh & Lee (2016), and Mele (2017).