

# Inventory Reduction Benefits from Good Estimates of Regional Correlations

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The literature often talks about the benefits of risk-pooling by reducing warehouses and demonstrating the inventory reduction for the case of uncorrelated demand. However, we know that in practice, the correlations are not exactly zero, but would typically be closer to zero than to one. In this note, we derive the dependence of inventory-reduction benefits on not just the number of warehouses but also on the correlation. In particular, we show how significant inventory-reduction benefits can be obtained as we go from say correlation of 20%  $\rightarrow$  10%  $\rightarrow$  0% for our business.

We have  $m$  stores and  $n$  warehouses. Assume the stores have demand  $N(\mu, \sigma)$  per day with pairwise-correlation of  $\rho$  among stores.

Each warehouse experiences withdrawal from  $\frac{m}{n}$  stores. Assume a lead time of  $L$  days at a warehouse and we require a service level of  $\alpha$  at each warehouse.

Let  $\Phi(\cdot)$  be the standard normal cumulative distribution function.

The safety stock at a warehouse is:

$$\Phi^{-1}(\alpha) \cdot \sqrt{L} \cdot \sqrt{\frac{m}{n} \cdot \sigma^2 + \left(\frac{m}{n}\right) \cdot \left(\frac{m}{n} - 1\right) \cdot \sigma^2 \cdot \rho}$$

The total safety stock across all the warehouses is:

$$\begin{aligned} & n \cdot \Phi^{-1}(\alpha) \cdot \sqrt{L} \cdot \sqrt{\frac{m}{n} \cdot \sigma^2 + \left(\frac{m}{n}\right) \cdot \left(\frac{m}{n} - 1\right) \cdot \sigma^2 \cdot \rho} \\ &= \Phi^{-1}(\alpha) \cdot \sqrt{L} \cdot \sigma \cdot \sqrt{m \cdot n \cdot \left(1 + \left(\frac{m}{n} - 1\right) \cdot \rho\right)} \\ &= \Phi^{-1}(\alpha) \cdot \sqrt{L} \cdot \sigma \cdot \sqrt{m \cdot (n + (m - n) \cdot \rho)} \\ &= \Phi^{-1}(\alpha) \cdot \sqrt{L} \cdot \sigma \cdot \sqrt{m \cdot (m \cdot \rho + n \cdot (1 - \rho))} \end{aligned}$$

$$= \Phi^{-1}(\alpha) \cdot \sqrt{L} \cdot \sigma \cdot m \cdot \sqrt{\rho + \frac{n}{m} \cdot (1 - \rho)}$$

Denoting the warehouses to stores ratio  $\frac{n}{m}$  as  $\gamma$ , the total safety stock across all the warehouses can be written as:

$$= \Phi^{-1}(\alpha) \cdot \sqrt{L} \cdot \sigma \cdot m \cdot \sqrt{\rho + \gamma \cdot (1 - \rho)}$$

Therefore, going from a higher correlation of  $\rho_1$  to a lower correlation of  $\rho_2$  gives us a percentage reduction in the total safety stock across all the warehouses

$$= 1 - \sqrt{\frac{\rho_2 + \gamma \cdot (1 - \rho_2)}{\rho_1 + \gamma \cdot (1 - \rho_1)}}$$

As a practical example, assume stores  $m = 2000$ , warehouses  $n = 20$  ( $\gamma = 0.01$ ), and  $\rho$  reducing from 0.2 to 0.1.

Then, percentage reduction in total safety stock across all the warehouses is:

$$1 - \sqrt{\frac{0.1 + 0.01 \cdot 0.9}{0.2 + 0.01 \cdot 0.8}} \approx 28\%$$

And if  $\rho$  reduces from 0.2 to 0, the percentage reduction shoots up to:

$$1 - \sqrt{\frac{0.0 + 0.01 \cdot 1.0}{0.2 + 0.01 \cdot 0.8}} \approx 78\%$$

Further, if we assumed deeper risk-pooling with warehouses  $n = 10$ , we get even greater inventory reduction benefits. The percentage reduction goes even higher to:

$$1 - \sqrt{\frac{0.0 + 0.005 \cdot 1.0}{0.2 + 0.005 \cdot 0.8}} \approx 84\%$$

This non-linear dependence on  $\rho$  can be taken advantage of by obtaining good estimates of the correlation. The lower the upper-bound on the correlation, the more-accelerated the inventory reduction benefits.