MONASH University

MONASH BUSINESS SCHOOL

Free Lunch Multivariate Forecast Projection: reducing forecast variance using linear combinations

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Free lunch forecast projection

A model-independent post-forecast adjustment method that can reduce forecast variance without introducing new data or information.

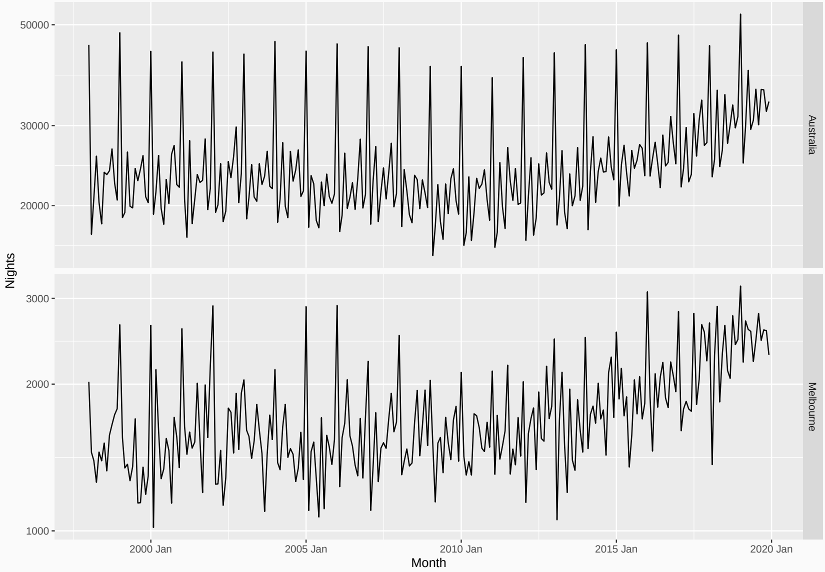
Australian tourism data

- The data include tourism information on seven states and territories which can be divided into 77 regions
  - For example, Melbourne, Sydney, East Coast

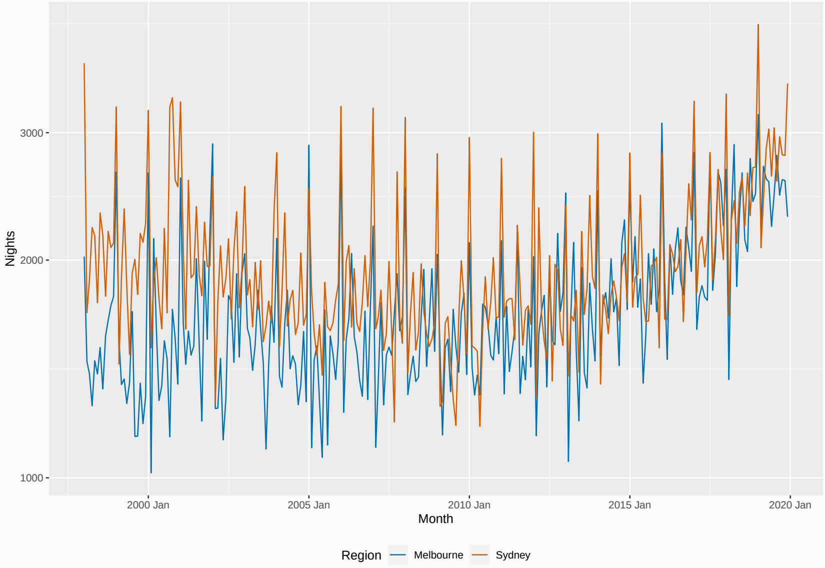
Visitor nights

The total number of nights spent by Australians away from home recorded monthly

Total and Region



Melbourne and Sydney



Intuition

Observation

1. Better signal-noise ratio in the linear combination.

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One step further

Finding components that have better signal-noise ratio:

1. Easy to forecast;
2. Capturing the common signals;
3. Improving forecast of original series.

Literature

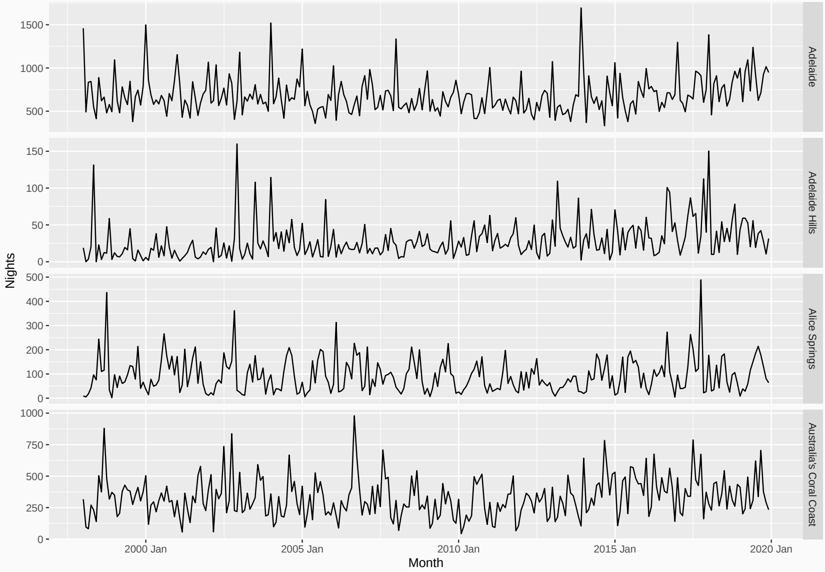
Forecast reconciliation

bagging

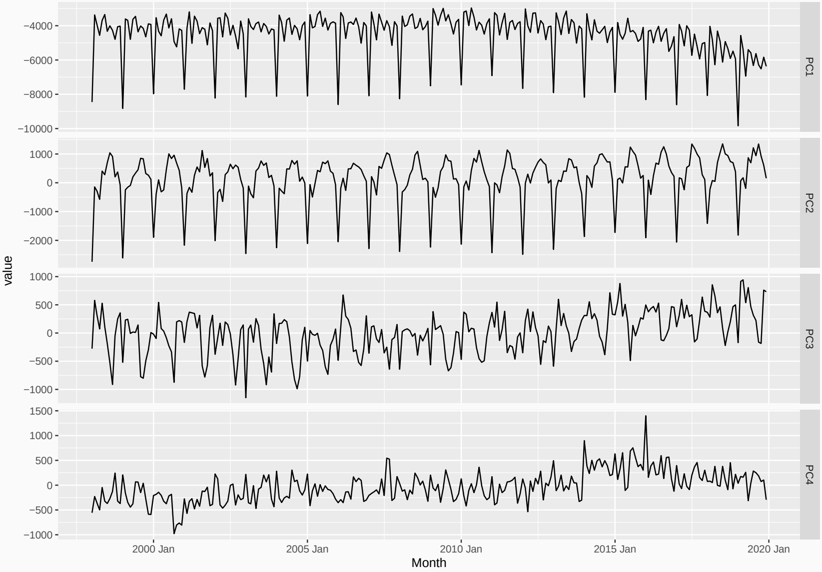
Forecast combination

Factor model

Series $z_t \in \mathbb{R}^m$



Components $\boldsymbol{c}_t = \Phi \boldsymbol{z}_t \in \mathbb{R}^p$



Free lunch forecast projection

$$\boldsymbol{y}_t = \begin{bmatrix} \boldsymbol{z}_t \\ \boldsymbol{c}_t \end{bmatrix} \qquad \tilde{\boldsymbol{y}}_{t+h} = \boldsymbol{M}\hat{\boldsymbol{y}}_{t+h}$$

$$\tilde{\boldsymbol{z}}_{t+h} = \boldsymbol{J}\tilde{\boldsymbol{y}}_{t+h} = \boldsymbol{J}\boldsymbol{M}\hat{\boldsymbol{y}}_{t+h} \qquad \boldsymbol{J} = \boldsymbol{J}_{m,p} = \begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{O}_{m\times p} \end{bmatrix}$$

$$\boldsymbol{M} = \boldsymbol{I}_{m+p} - \boldsymbol{W}_h\boldsymbol{C}'\left(\boldsymbol{C}\boldsymbol{W}_h\boldsymbol{C}'\right)^{-1}\boldsymbol{C},$$

$$\boldsymbol{C} = \begin{bmatrix} -\Phi & \boldsymbol{I}_p \end{bmatrix}$$

Properties

Lemma 2.1

The matrix $\boldsymbol{M}$ is a projection onto the space where the constraint $\boldsymbol{C}$ is satisfied.

Corollary 2.1

1. The projected forecast $\tilde{\boldsymbol{y}}_{t+h}$ satisfies the constraint $\boldsymbol{C}$.
2. For $\boldsymbol{y}_{t+h}$ that already satisfies the constraint, the projection does not change its value: $\boldsymbol{M}\boldsymbol{y}_{t+h} = \boldsymbol{y}_{t+h}$
3. If the base forecasts are unbiased, then the projected forecasts are also unbiased.

Variance reduction

Theorem 2.1

The variance reduction is **positive semi-definite**:

$$\mathrm{Var}(\hat{\boldsymbol{y}}_{t+h} - \boldsymbol{y}_{t+h}) - \mathrm{Var}(\tilde{\boldsymbol{y}}_{t+h} - \boldsymbol{y}_{t+h}) \\ = \boldsymbol{W}_h\boldsymbol{C}'(\boldsymbol{C}\boldsymbol{W}_h\boldsymbol{C}')^{-1}\boldsymbol{C}\boldsymbol{W}_h,$$

$$\mathrm{Var}(\hat{\boldsymbol{z}}_{t+h} - \boldsymbol{z}_{t+h}) - \mathrm{Var}(\tilde{\boldsymbol{z}}_{t+h} - \boldsymbol{z}_{t+h}) \\ = \boldsymbol{J}\boldsymbol{W}_h\boldsymbol{C}'(\boldsymbol{C}\boldsymbol{W}_h\boldsymbol{C}')^{-1}\boldsymbol{C}\boldsymbol{W}_h\boldsymbol{J}'$$

Example

$$\boldsymbol{W}_h = \boldsymbol{I}_{m+p}$$

$$\Phi\Phi' = \boldsymbol{I}_p \text{ when } p \leq m$$
$$\Phi'\Phi = \boldsymbol{I}_m \text{ when } p = m.$$

$$\mathrm{Var}(\hat{\boldsymbol{y}}_{t+h} - \boldsymbol{y}_{t+h}) - \mathrm{Var}(\tilde{\boldsymbol{y}}_{t+h} - \boldsymbol{y}_{t+h}) = \frac{1}{2}\begin{bmatrix} \Phi'\Phi & -\Phi' \\ -\Phi & \boldsymbol{I}_p \end{bmatrix}.$$

Monotonicity

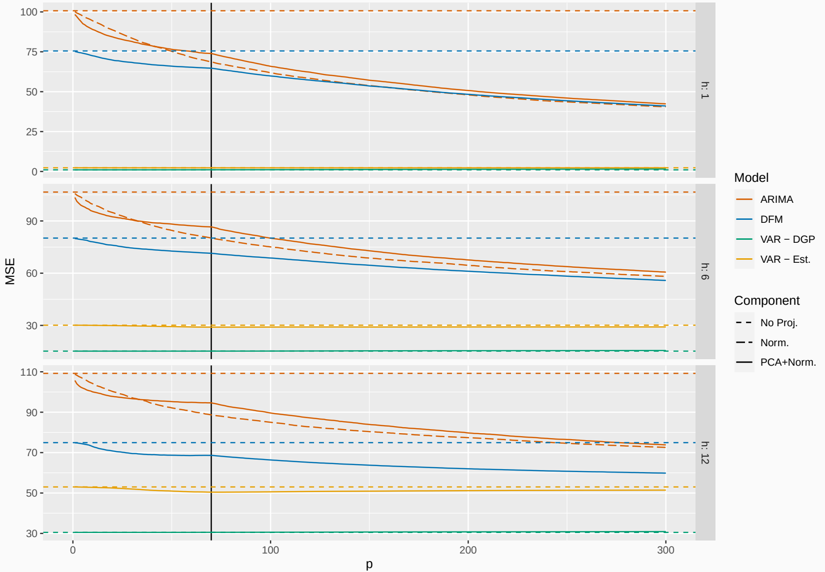
Theorem 2.2

Minimum variance of individual series

Theorem 2.4

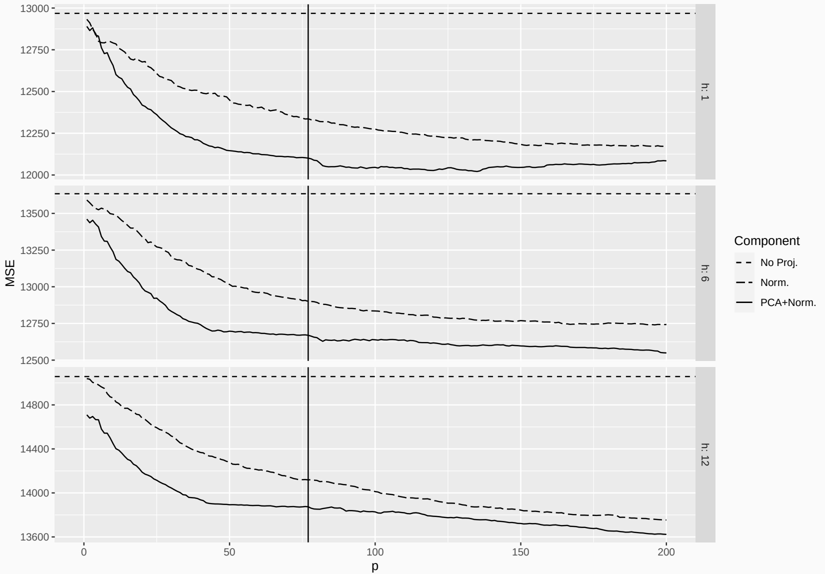
Estimation of W_h

Simulation



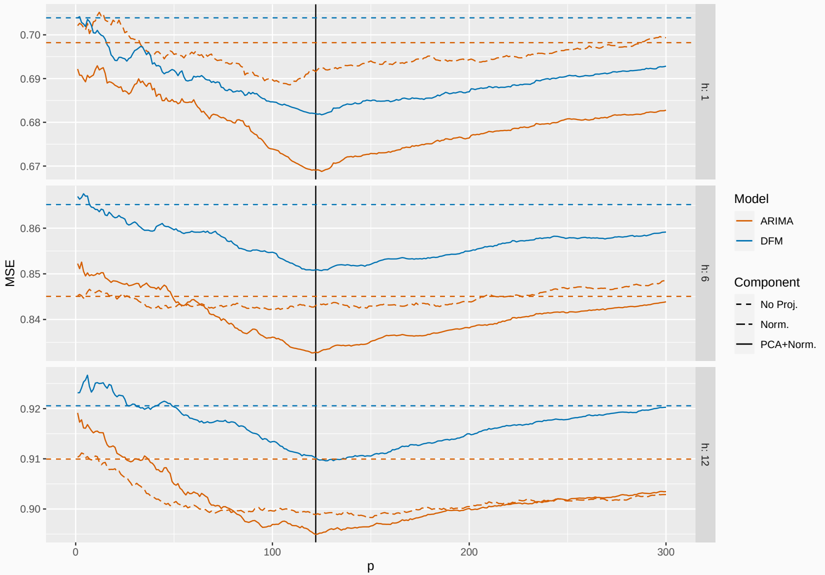
MSE
p
h: 1
h: 6
h: 12
0
100
200
300
Model
ARIMA
DFM
VAR − DGP
VAR − Est.
Component
No Proj.
Norm.
PCA+Norm.

Tourism



13000
12750
12500
12250
12000
13500
13250
13000
12750
12500
14800
14400
14000
13600
MSE
0
50
100
150
200
p
h: 1
h: 6
h: 12
Component
No Proj.
Norm.
PCA+Norm.

FRED-MD



0.70
0.69
0.68
0.67
0.86
0.85
0.84
0.92
0.91
0.90
MSE
0
100
200
300
p
h: 1
h: 6
h: 12
Model
ARIMA
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