



Free Lunch Multivariate Forecast Projection: reducing forecast variance using linear combinations

Yangzhuoran Fin Yang

Rob J Hyndman

George Athanasopoulos

Anastasios Panagiotelis

Free lunch forecast projection

A model-independent post-forecast adjustment method that can reduce forecast variance without introducing new data or information.

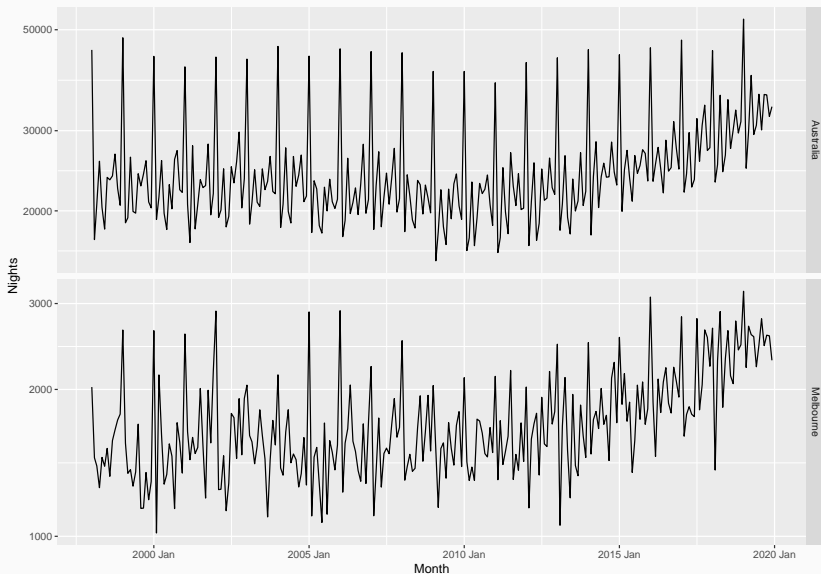
Australian tourism data

- The data include tourism information on seven states and territories which can be divided into 77 regions
 - ▶ For example, Melbourne, Sydney, East Coast

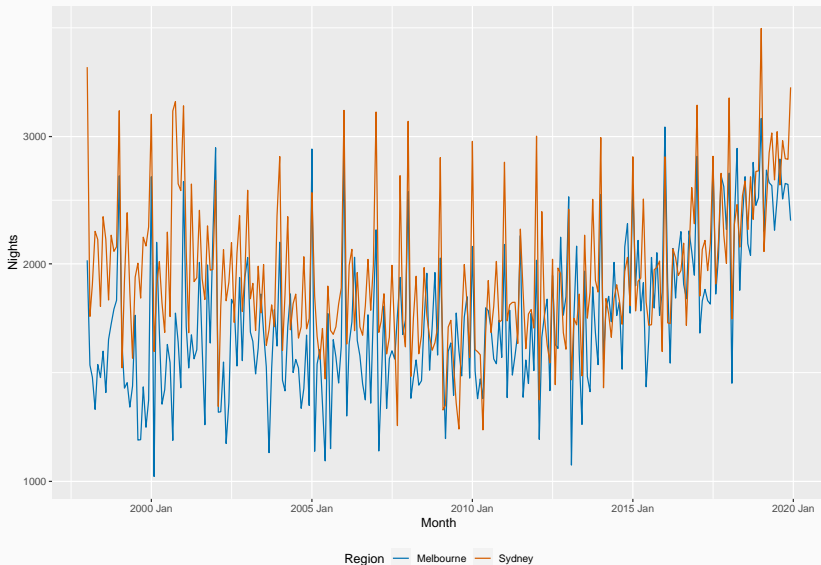
Visitor nights

The total number of nights spent by Australians away from home recorded monthly

Total and Region



Melbourne and Sydney



Intuition

Observation

1. Better signal-noise ratio in the linear combination.

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One step further

Finding components that have better signal-noise ratio:

1. Easy to forecast;
2. Capturing the common signals;
3. Improving forecast of original series.

Literature

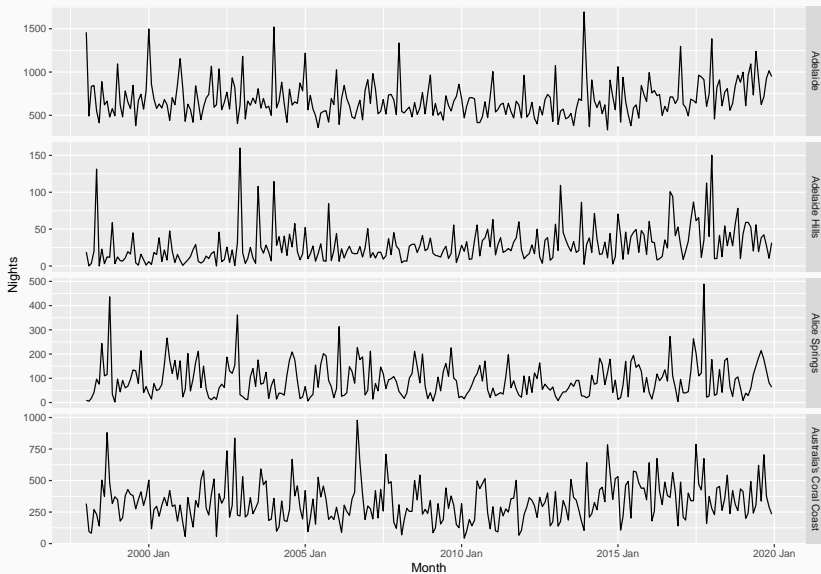
Forecast reconciliation

bagging

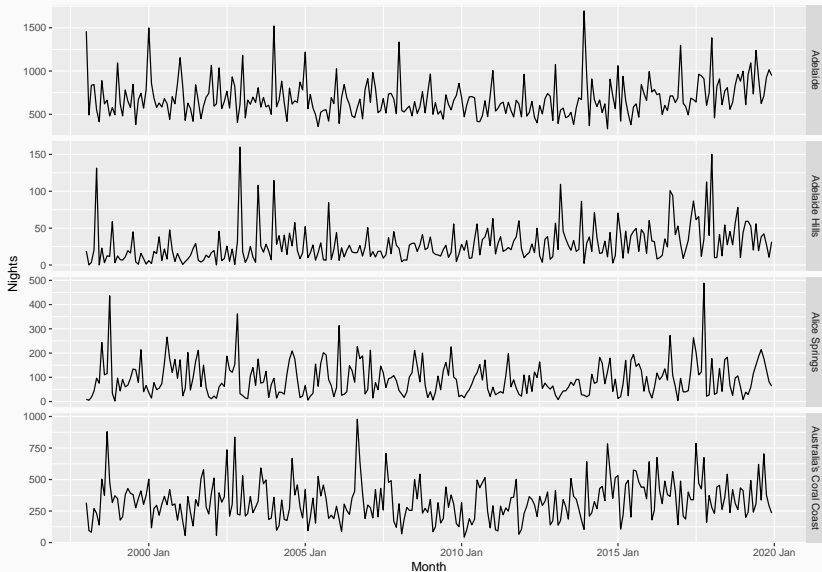
Forecast combination

Factor model

Series $\mathbf{z}_t \in \mathbb{R}^m$



Components $\mathbf{c}_t = \Phi \mathbf{z}_t \in \mathbb{R}^p$



Free lunch forecast projection

$$\mathbf{y}_t = \begin{bmatrix} \mathbf{z}_t \\ \mathbf{c}_t \end{bmatrix} \quad \tilde{\mathbf{y}}_{t+h} = \mathbf{M} \hat{\mathbf{y}}_{t+h}$$

$$\tilde{\mathbf{z}}_{t+h} = \mathbf{J} \tilde{\mathbf{y}}_{t+h} = \mathbf{J} \mathbf{M} \hat{\mathbf{y}}_{t+h} \quad \mathbf{J} = \mathbf{J}_{m,p} = \begin{bmatrix} \mathbf{I}_m & \mathbf{O}_{m \times p} \end{bmatrix}$$

$$\mathbf{M} = \mathbf{I}_{m+p} - \mathbf{W}_h \mathbf{C}' (\mathbf{C} \mathbf{W}_h \mathbf{C}')^{-1} \mathbf{C},$$

$$\mathbf{C} = \begin{bmatrix} -\Phi & \mathbf{I}_p \end{bmatrix}$$

Properties

Lemma 2.1

The matrix \mathbf{M} is a projection onto the space where the constraint \mathbf{C} is satisfied.

Corollary 2.1

- 1 The projected forecast $\tilde{\mathbf{y}}_{t+h}$ satisfies the constraint \mathbf{C} .
- 2 For \mathbf{y}_{t+h} that already satisfies the constraint, the projection does not change its value:
$$\mathbf{M}\mathbf{y}_{t+h} = \mathbf{y}_{t+h}$$
- 3 If the base forecasts are unbiased, then the projected forecasts are also unbiased.

Variance reduction

Theorem 2.1

The variance reduction is **positive semi-definite**:

$$\begin{aligned}\text{Var}(\hat{\mathbf{y}}_{t+h} - \mathbf{y}_{t+h}) - \text{Var}(\tilde{\mathbf{y}}_{t+h} - \mathbf{y}_{t+h}) \\ = \mathbf{W}_h \mathbf{C}' (\mathbf{C} \mathbf{W}_h \mathbf{C}')^{-1} \mathbf{C} \mathbf{W}_h,\end{aligned}$$

$$\begin{aligned}\text{Var}(\hat{\mathbf{z}}_{t+h} - \mathbf{z}_{t+h}) - \text{Var}(\tilde{\mathbf{z}}_{t+h} - \mathbf{z}_{t+h}) \\ = \mathbf{J} \mathbf{W}_h \mathbf{C}' (\mathbf{C} \mathbf{W}_h \mathbf{C}')^{-1} \mathbf{C} \mathbf{W}_h \mathbf{J}'\end{aligned}$$

Example

Monotonicity

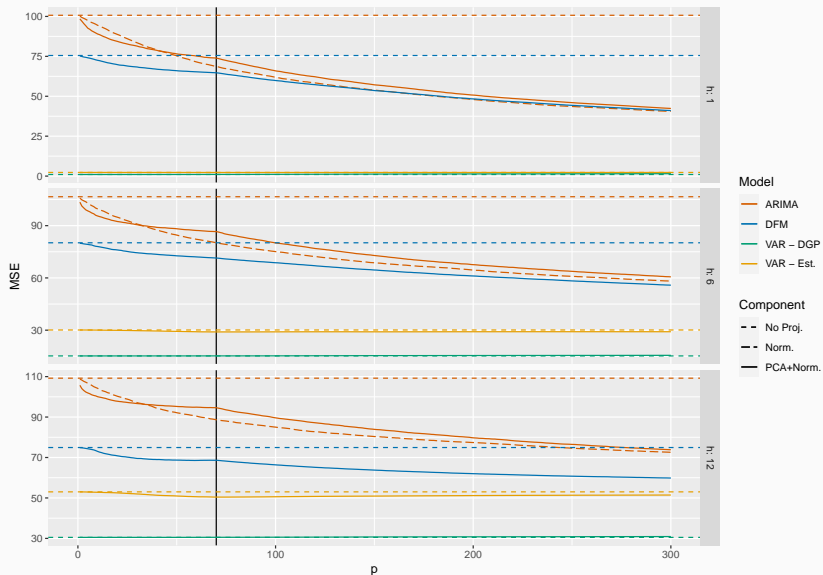
Theorem 2.2

Minimum variance of individual series

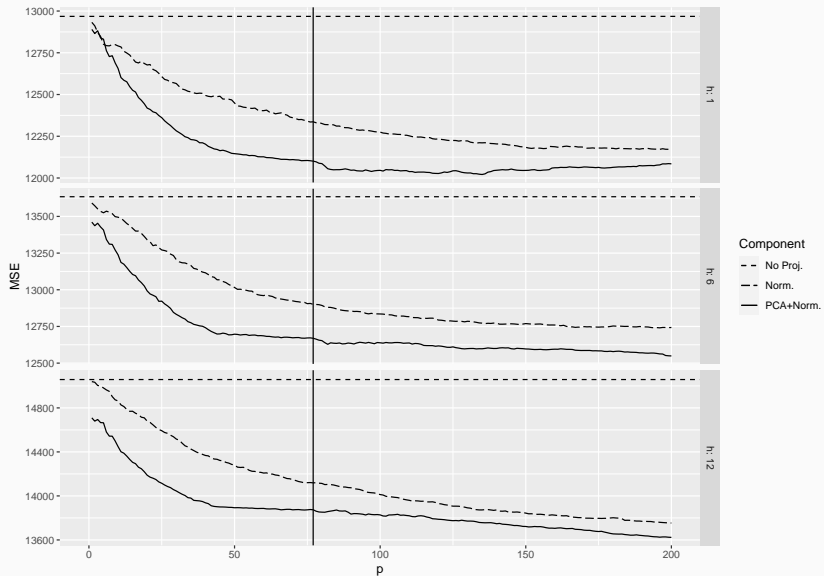
Theorem 2.4

Estimation of W_h

Simulation



Tourism



FRED-MD

