MONASH University

MONASH BUSINESS SCHOOL

Dimension Reduction in Stochastic Optimal Control

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Outline

1 Setting and Goals

2 Background, Literature and Motivations

3 Methodology

4 Preliminary Result

5 Future Research Direction

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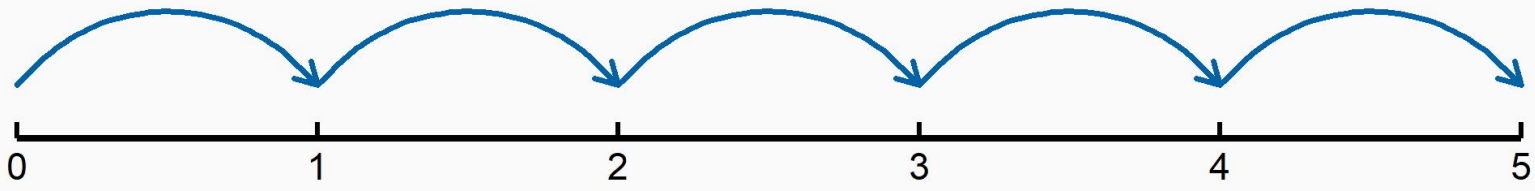
Monopoly

Monopoly

Elements in Monopoly:

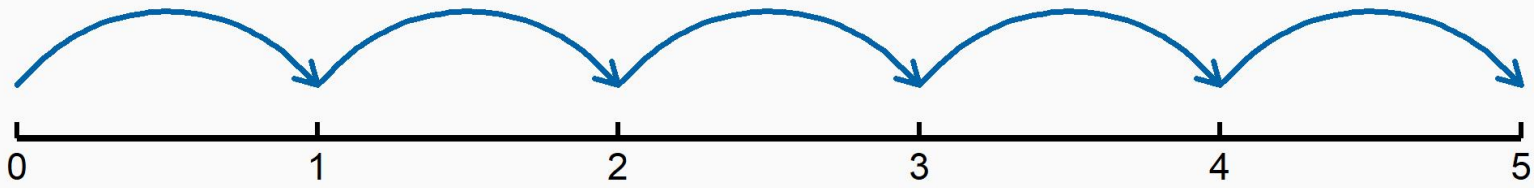
- Randomness from the dice
- Moving in each round
- Decision to buy or not to buy the land
- Money to save
- Decision on whether to consume



Our setting

Elements in our setting:

- Randomness from ~~the dice~~ returns of risky assets
- Moving ~~in each round~~ forward in time
- Decision to ~~buy or not~~ how much to buy
- Money to ~~save~~ invest in risk-free asset
- Decision on ~~whether~~ how much to consume



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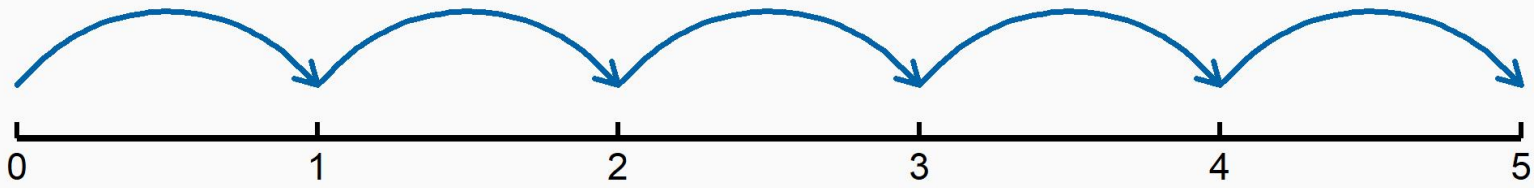

Difficult in high dimension!

Our setting

Goals

- To develop an algorithm that achieves the **optimal portfolio selection** w.r.t. the objective utility function in an **optimal control setting** using **dimension reduction**, where the risky assets are projected into one risky portfolio using linear regression.

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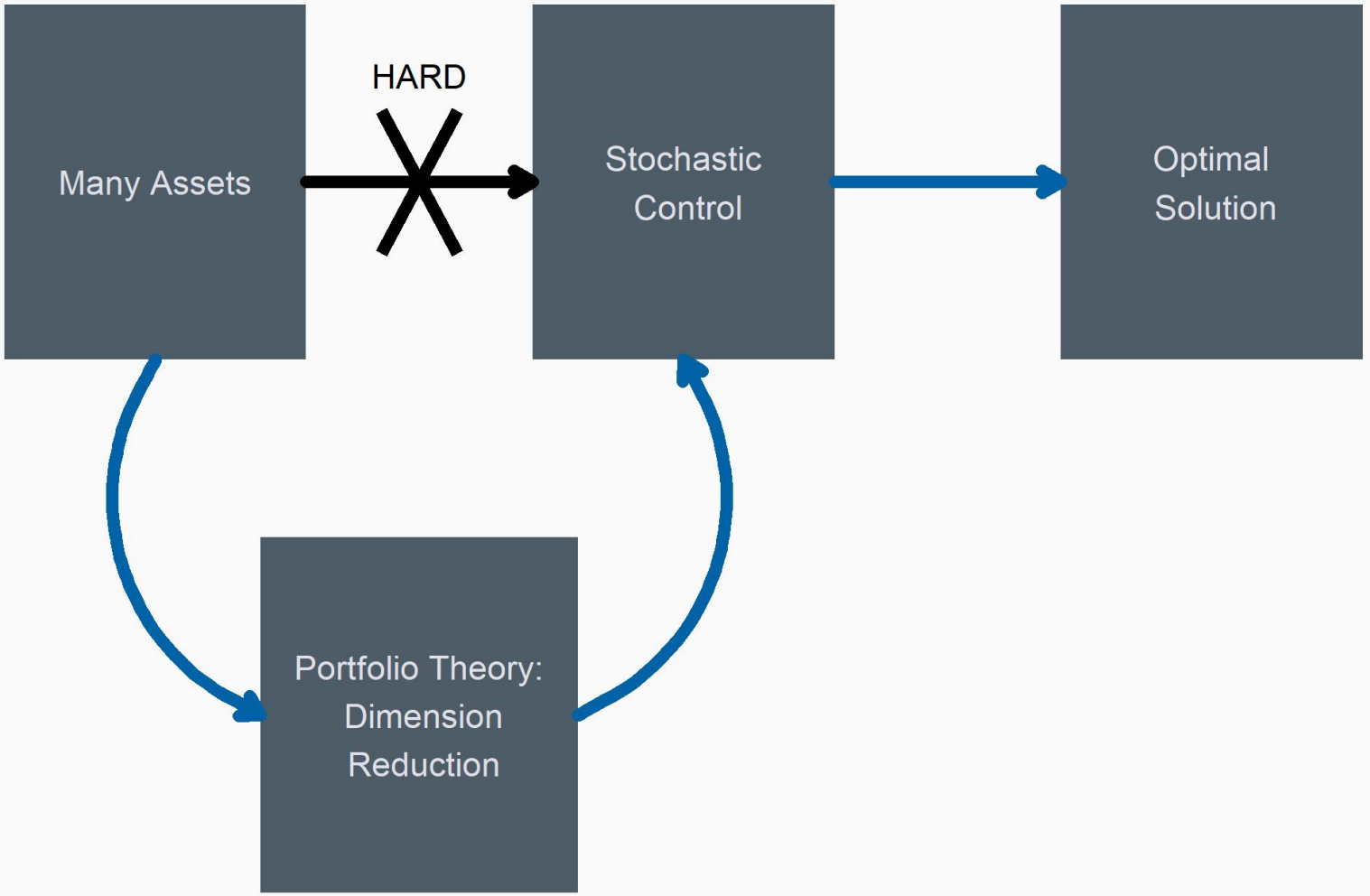
Goals

- To develop an algorithm that achieves the **optimal portfolio selection** w.r.t. the objective utility function in an **optimal control setting** using **dimension reduction**, where the risky assets are projected into one risky portfolio using linear regression.
- To understand the convergence property of the algorithm.

The Big Picture



Our setting

Significance

- Overcoming the curse of dimensionality.
- Relaxing the requirement on the computational power.
- Filling the gap to utilise the dynamics of portfolio selection in the stochastic control theory.

Outline

Background: Stochastic Control

The classical stochastic optimal control problem has been limited to a small number of states.

$$V_t(\mathbf{x}_t) = \inf_{\{c_j\}_{j=t}^{T-1}} E\left[\sum_{j=t}^{T-1} U(\mathbf{X_j}, \mathbf{c_j}) + U(\mathbf{X}_T)\right]$$

where V_t is the value function at time t; U is the utility function; $\mathbf{X}_t$ is a vector of states variable at time t; c_t is the control variable at time t.

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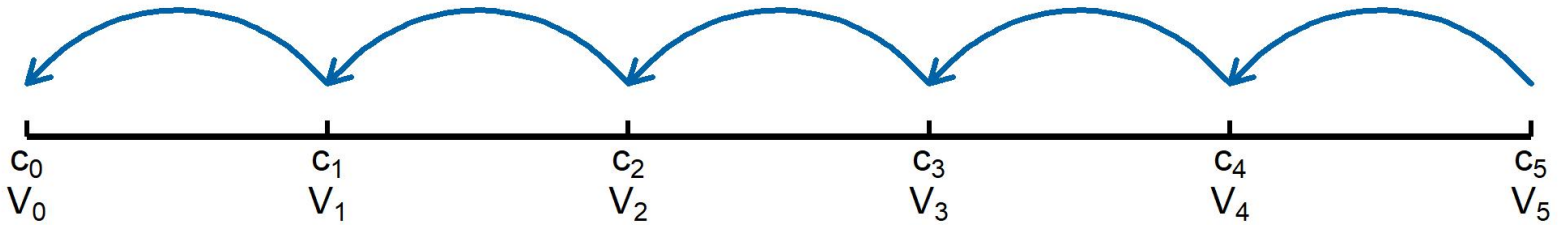
Bellman Equation

$$V_t(\mathbf{x}) = \inf_{c_t} \left(U(\mathbf{x}, c_t) + E[V_{t+1} | \mathcal{F}_t]\right)$$

Literature: EM algorithm

E step: Simulate forward in time

M step: Optimise backward in time



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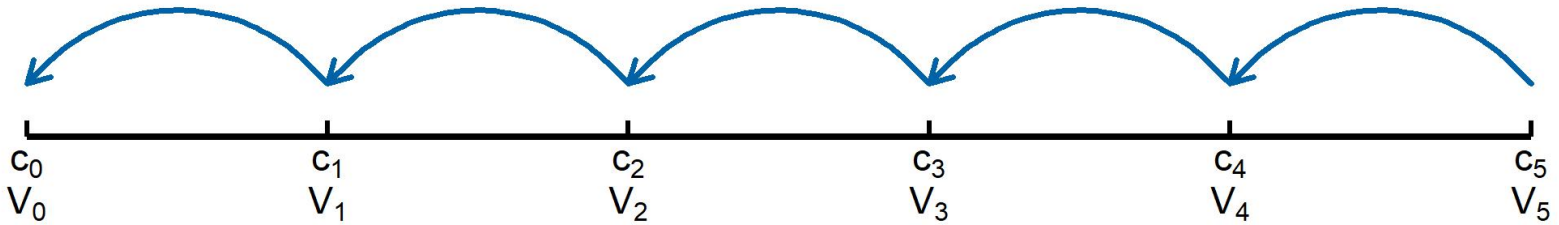

Neglecting the content specific structure

Outline

Methodology: Portfolio Selection

The focus of portfolio selection has been on risk minimization.

$$\min_{\omega^T \mathbf{1}=1} Var(\omega^T \mathbf{R})$$

where $\mathbf{R}$ is the return vector; ω is its portfolio allocation vector.

After simple algebra …

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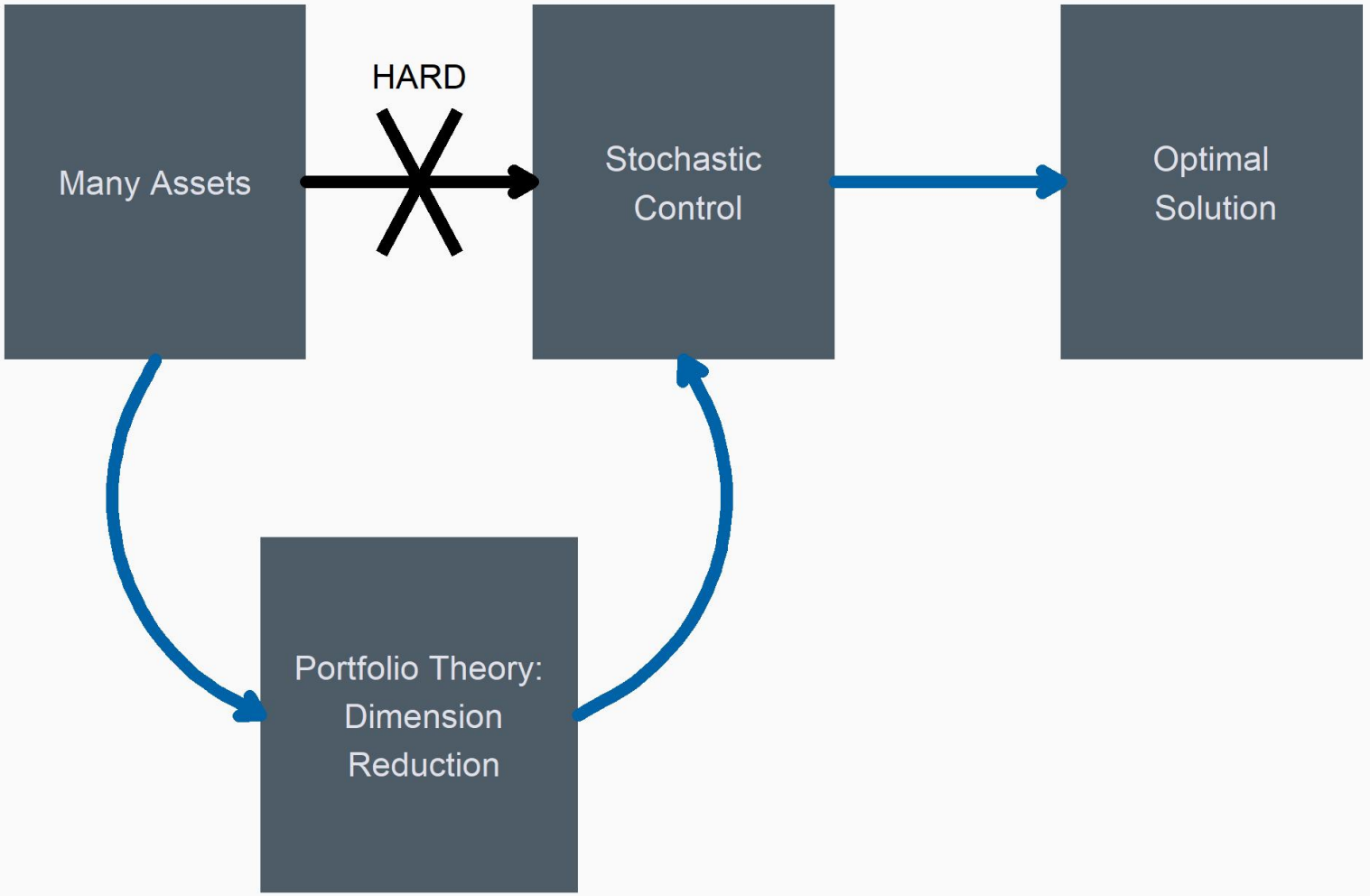
After simple algebra … **OLS solvable!**

$$R_t^r = \omega_t' \mathbf{R}_t$$

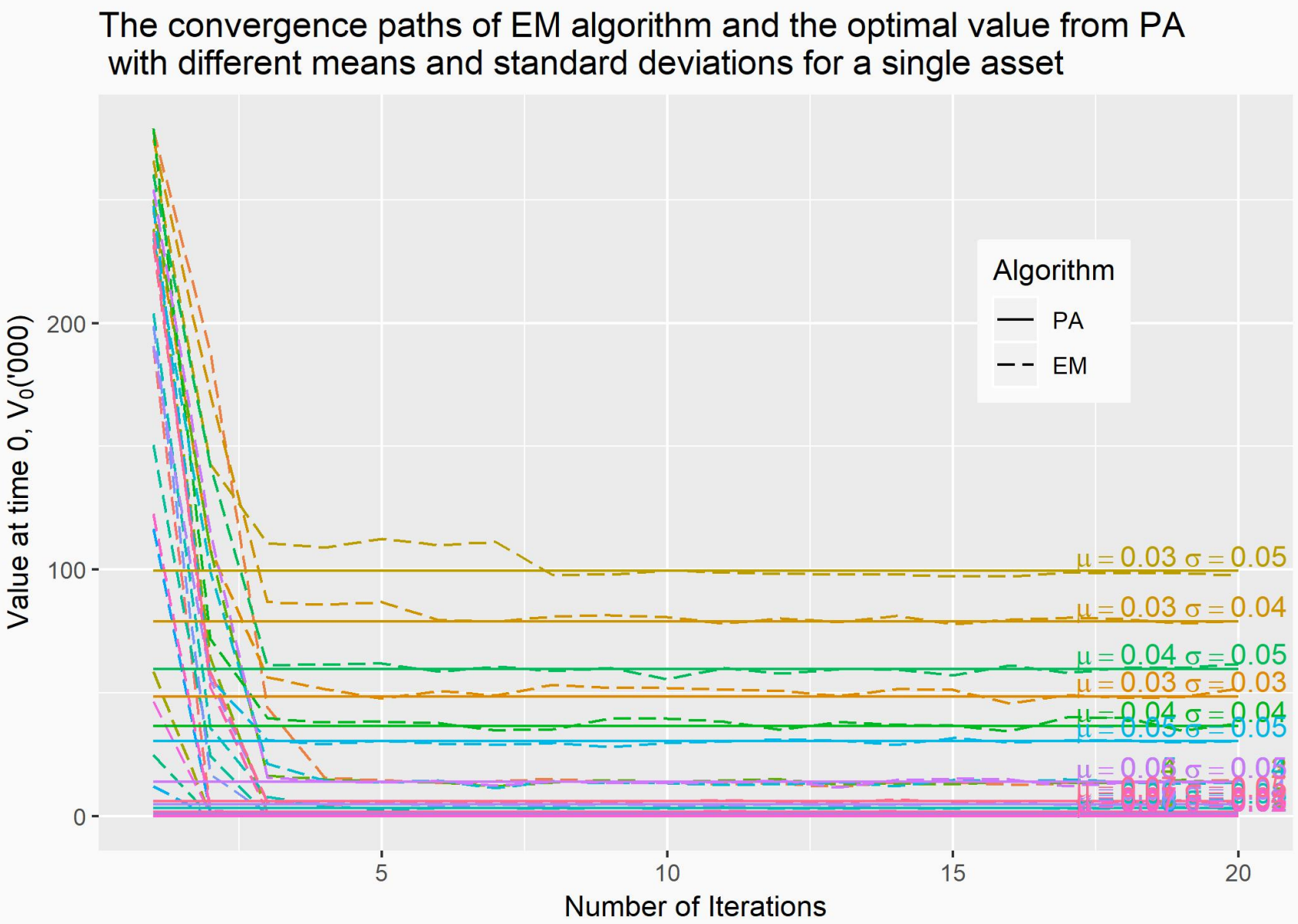
Dimension reduced.

The Big Picture



Outline

Single asset without estimation error



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Future Research Direction

- Exploring projection options
- Relaxing the independence assumption
- Applying restriction and constraint
- Considering the bound for optimal solution

R package `stocon`

The package is in development. You can install the development version

```
devtools::install_github(FinYang/stocon)
```

The documentation can be found at

`https://pkg.yangzhuoranyang.com/stocon`

References i