Assignment 2017/18

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Instructions:

This is an individual assignment worth 50% of the total marks available for this module and will assess learning outcomes 1.4, 1.5, 2.1, 2.3, 2.4 as specified in the module handbook.

You may hand in a hand-written assignment or a word-processed assignment, or a combination of both. Questions will indicate when spreadsheets and graphs are required. Where spreadsheets are used, your work should show both the solution obtained and an indication of the formulae used. For each question below, all solutions must contain full details of all the steps in your working out. It is not sufficient to simply submit the final answers. Unless otherwise stated you should work to the full accuracy of your calculator.

The hand-out and hand-in dates for your completed work, together with the date for returning marked work and feedback to you are those indicated on the module assessment timetable.

Late submission will be penalised as per the University's regulations. Your assignment must be submitted together with a fully completed and signed cover sheet – which is available from the student intranet.

Please note that this is an individual assignment. You should ensure that you are fully aware of the University's regulations on academic conduct.

Questions:

1. Ever wondered how a computer actually calculates $\sin(x)$, $\ln(x)$ or e^x? One way is to approximate their behaviour by using simpler forms such as polynomials. However, when making approximations, it is important to know what limits on accuracy (if any) such approximations might have. This question allows you to form a polynomial approximation to a known continuous function and to explore any limits on accuracy.

   a) Working in radians, obtain the Taylor polynomial $P_5(x)$, of degree 5, about $x_0 = 0$, of the function $f(x) = 0.5\cos(2x)$.

   b) Use this polynomial to estimate the value of $f(0.2)$ to 8 decimal places.

   c) Construct an upper bound ε for the error obtained in b) and hence obtain an interval in which the true value of $0.5\cos(0.4)$ is guaranteed to lie.

   d) Find the actual error in your estimate obtained in b) and compare the magnitude of the actual error with the magnitude of the upper bound ε.

[20 marks]

2. Often, solving equations of a single variable can be a difficult task. One approach is to re-cast the problem into a root finding problem. Performing an iterative approach to determine a sequence of successively more accurate approximations to the true solution can be efficient and effective in generating answers to a specified tolerance. For example, if we want to know where an object travelling along a sinusoidal path will crash into an object travelling along a cubic path we could set the two functions equal to each other and solve as below

   a) Working in radians, write a spreadsheet that performs and fully records five iterations of the bisection method to obtain an estimate for the solution of the equation

   $$\sin(x) = \frac{x^3}{4} - 1, \quad \text{in the interval } [1.75, 2.25].$$

   b) What is the maximum error in your answer for part a)?

   c) How many iterations would be required for the error bound on the estimate to be no greater than 0.000001?

   d) Recalculate a spreadsheet estimate to the above problem using 5 iterations of the fixed point iteration scheme x = g(x) where: $x = \sqrt[3]{(4\sin(x)+4)}$ and 5 iterations of the Newton-Raphson scheme where $f(x) = \sin(x) - \frac{x^3}{4} + 1$ and $f'(x) = \cos(x) - \frac{3x^2}{4}$. For both methods use x = 2 as your starting value.

   e) Given that the solution, corrected to 8 decimal places, is x ≈ 1.97303383, Compare and comment on the rate of convergence of the three schemes.

[20 marks]

3. Graph paper is advised for this question. You may use a spreadsheet if you wish but it should additionally show the formulae used for the calculation of the matrix products.

   The flag of the Philippines is constructed from a single triangle and two trapezia. A simple 2D drawing can be described using the vertex coordinates

   Triangle: (2, 4), (2, 8), (5, 6)
   Trapezium 1: (5, 6), (8, 6), (2, 8), (8, 8)
   Trapezium 2: (5, 6), (8, 6), (2, 4), (8, 4)

   a) Plot the three shapes on the same, appropriately labelled, set of axes and join the vertices with straight lines so that the Flag's shape can be seen.

   b) Show that the points for the three shapes can be optimally combined and represented as a single matrix of homogeneous coordinates.

   c) Construct a 2D translation matrix that will slide your flag 12 units to the left and 3 units down. Use matrix multiplication to translate your shape's coordinates (show all of your working out) and then plot the resulting points on the same graph.

(Question 3 continued)

d) Construct a 2D rotation matrix that will rotate your translated shape 180 degrees anti-clockwise. Use matrix multiplication to rotate your translated shape and then plot the resulting points on the same graph.

e) Describe how these two transformations may be combined into a single transformation? Demonstrate your answer using your results from parts c) and d).

[20 marks]

4. The velocity of Super Mario's cart after gaining a Power-up (in meters per second) is given by a modified exponential function (shown below). The area under the curve gives the total distance Mario can travel during the Power-up period (in *x* meters). A fast and accurate method for determining this distance is required to ensure that any bombs released by opponents reach their target.

   In parts a) and b) of this question you are required to construct a spreadsheet that will order, calculate and record all of the steps involved in accurately finding Mario's Power-up distance. Your spreadsheets should clearly show all index values, sub-interval values and ordinates used in the calculations as well as the summations and final answer.

   a) Use the composite Simpson's rule with 8 subintervals to estimate the integral (distance Mario has travelled) during the first 1.6 seconds

   $$\int_0^{1.6} (2e^x + x^3)\ dx.$$

   b) Recalculate your answer to part a) using 16 subintervals. You may use the same worksheet to display your results if you wish.

   c) Evaluate, by hand, to 9 decimal places the definite integral in part a) using exact integration.

   d) Given that the composite Simpson's rule is of order $O(h^4)$, where h is the subinterval width, use your answer from part c) to comment on the rate of convergence observed in your answer to part b) when compared with your answer to part a).

[20 marks]

5. Replacing discrete data values with a continuous function is extremely useful. For example, a set of discrete way-points (x, y) in a game map, which controls the direction of movement for non-playing characters (NPCs) can be replaced by a continuous curve so that the NPCs move in a smoother, more realistic way. We will explore this using the example below.

You are given the following tabular data:

x_i	0.6	0.8	1.0	1.4
$f(x_i)$	-0.5108	-0.2231	0.0000	0.3365

a) Use the Lagrange interpolation polynomial of degree 2 which interpolates $f(x)$ at the three tabular points

$$x = 0.8, \quad x = 1.0 \text{ and } \quad x = 1.4$$

to estimate a value of $f(0.9)$.

b) Construct a divided difference table, using all four map points in the table, and hence obtain the Newton divided difference interpolating polynomial of degree 3, that passes through all four tabulated points. Use this polynomial to estimate a value of $f(0.9)$. You are advised to set out your answer in a spreadsheet

c) Given that continuous functions allow for smoother NPC motion, an underlying smooth partial-path for NPCs was chosen to be the natural logarithm function. Therefore, given that $f(x) = \ln x$, calculate the actual error:
i) in your answer to a) ii) in your answer to b).

d) To what extent has the increase in degree of polynomial affected the accuracy of your approximation to $\ln(0.9)$?

[20 marks]

END OF ASSIGNMENT