1. (1) 由于

$$^0J = \begin{bmatrix} ^0R_n & 0 \\ 0 & ^0R_n \end{bmatrix} {}^nJ$$

其中 ^0R_n 为正交矩阵，故

$$\begin{aligned} ^nJ &= \begin{bmatrix} ^0R_n & 0 \\ 0 & ^0R_n \end{bmatrix}^{-1} {}^0J \\ &= \begin{bmatrix} ^0R_n^{-1} & 0 \\ 0 & ^0R_n^{-1} \end{bmatrix} {}^0J \quad = \begin{bmatrix} ^0R_n^T & 0 \\ 0 & ^0R_n^T \end{bmatrix} {}^0J \end{aligned}$$

下面求解基座雅可比矩阵 ^0J

基于 D-H 参数，求得各关节齐次变换矩阵如下：

$$^0T_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1\cos\alpha_1 & \sin\theta_1\sin\alpha_1 & a_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1\cos\alpha_1 & -\cos\theta_1\sin\alpha_1 & a_1\sin\theta_1 \\ 0 & \sin\alpha_1 & \cos\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1T_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2\cos\alpha_2 & \sin\theta_2\sin\alpha_2 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2\cos\alpha_2 & -\cos\theta_2\sin\alpha_2 & a_2\sin\theta_2 \\ 0 & \sin\alpha_2 & \cos\alpha_2 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2T_3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3\cos\alpha_3 & \sin\theta_3\sin\alpha_3 & a_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3\cos\alpha_3 & -\cos\theta_3\sin\alpha_3 & a_3\sin\theta_3 \\ 0 & \sin\alpha_3 & \cos\alpha_3 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

故可构造得雅可比矩阵各分块为

$$
\begin{aligned}
{}^0J_1 &= \begin{bmatrix} \xi_1 \times ({}^0p_3 - {}^0P_0) \\ \xi_1 \end{bmatrix} \\
&= \begin{bmatrix} \begin{bmatrix} -\sin\theta_1 \\ \cos\theta_1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0.5\cos\theta_2 + 0.5\cos(\theta_2+\theta_3) \\ 0.5\sin\theta_2 + 0.5\sin(\theta_2+\theta_3) \\ 0.3 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \\
{}^0J_2 &= \begin{bmatrix} \xi_2 \times ({}^0p_3 - {}^0P_1) \\ \xi_2 \end{bmatrix} \\
{}^0J_3 &= \begin{bmatrix} \xi_3 \times ({}^0p_3 - {}^0P_2) \\ \xi_3 \end{bmatrix}
\end{aligned}
$$