

Predicting Recession Probabilities Using Term Spreads: New Evidence from a Machine Learning Approach

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in collaboration with

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Background: recession and yield-curve inversion

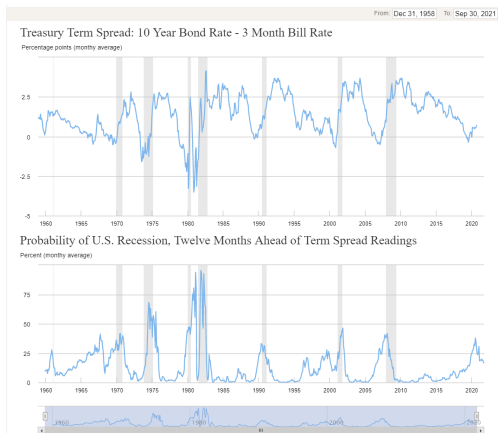
- Yield curve spread is one of the best recession predictors.
- In the past decades, the US recessions have been preceded by a yield curve inversion, i.e., short-term yield higher than long-term yield.
- *Monetary policy* channel:
 - Market expect a recession \Rightarrow lower policy rate \Rightarrow lower long-term rate.
 - Overheated economy (inflation) \Rightarrow FED's aggressive rate hike \Rightarrow recession



Source: FactSet, Federal Reserve, J.P. Morgan Asset Management. *From January 1962 to May 1976, short-term bond is U.S. 1-year note, and from June 1976 onwards the short-term bond is the 2-year note due to lack of data availability. Time to recession is calculated as the time between the final sustained inversion of the yield curve prior to recession and the onset of recession.
Guide to the Markets – U.S. Data are as of June 30, 2019.

Background: 10-year-minus-3-month spread

- In the literature and practice, 10-year-minus-3-month (10y/3m) spread with 1-year horizon is typically used for predicting recessions ([Estrella, 2005](#); [Estrella and Trubin, 2006](#); [Bauer and Mertens, 2018](#)).
- NY FED daily updates the recession probability based on the 10y/3m spread ([Estrella, 2005](#))
- The 10y/3m spread is a component of the US leading indicator (USSLIND) managed by St. Louis FED.



Summary of the result

Questions

- The use of the 10y/3m spread is not fully justified in the literature.
- Can we better predict recessions by relaxing the pair (i.e., 10y/3m) and the coefficient ratio (i.e., -1)?

Our findings

- We adopt a machine learning (ML) approach to search for the best maturity pair and the coefficients simultaneously.
- The ML algorithm finds a generalized spread: one long- and one short-term yield pair with the coefficients of the opposite signs and similar magnitudes.
- However, the out-of-sample prediction gain of the ML approach is not statistically significant.
- We justify the use of the simple 10y/3m spread.

Logistic Regression with the L_1 Regularization

- Logit model:

$$\hat{y}_{t+k} = \text{Prob}(y_{t+k} = 1 | \mathbf{x}_t) = \phi(-\beta_0 - \boldsymbol{\beta}^T \mathbf{x}_t), \quad \phi(z) = 1/(1 + e^{-z}),$$

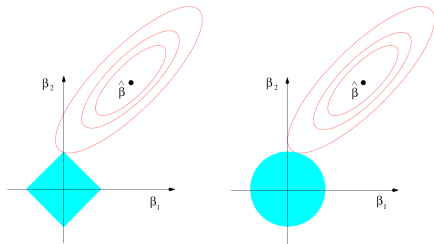
where \mathbf{x}_t is the Treasury yield vector at month t , $\boldsymbol{\beta}$ is the coefficient vector, and k is the forecasting horizon.

- We minimize the loss function with L_1 regularization:

$$J(\beta_0, \boldsymbol{\beta}) = -\log L(\beta_0, \boldsymbol{\beta}) + \lambda \|\boldsymbol{\beta}\|_1, \quad \|\boldsymbol{\beta}\|_1 = |\beta_1| + \cdots + |\beta_p|,$$

where $\log L$ is the log likelihood over the training period.

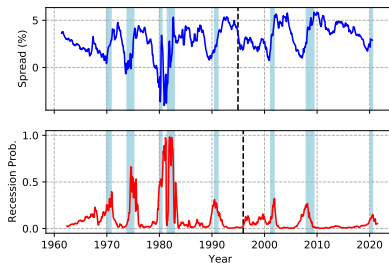
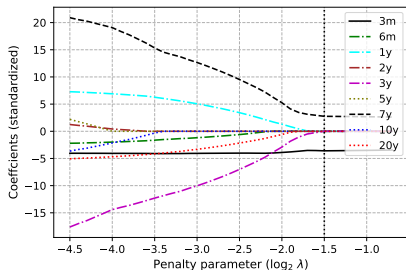
- L_1 penalty sets the coefficients of not-so-important variables zero, performing feature selection. (cf. ridge regression with L_2 penalty)
- In continuous variable regression, it is well-known as the least absolute shrinkage and selection operator (LASSO) ([Hastie et al., 2009](#)).



- Monthly Treasury yields from Federal reserve H.15 website:
 - Sample period: June 1961 to July 2020. Training-vs-test split at 1995.
 - Maturities: 3- and 6-month, and 1-, 2-, 3-, 5-, 7-, 10-, and 20-year.
 - Some missing series in early period are from other sources: secondary market rate and [Gürkaynak et al. \(2006\)](#)
- Monthly recession defined by NBER.
- For robustness check, the US leading indicator from St. Louis FED website and 30-year Treasury yield (since 1982).

ML algorithm: searching the maturity pair and coefficients

- We increase $\lambda = 2^{k/10}$ until only two non-zero coefficients survive.
- An example (training period: 1961–1995):
 - 7y and 3m are selected.
 - Generalize spread: 1.04 (7y yield) $- 1.23$ (3m yield) $+ 2.57$
 - The black dotted line (left) is the value of $\lambda = 0.3536$.



Model specifications

- Panel A: \mathcal{M} (generalized spread of the ML pair)

$$\hat{y}_{t+k} = \phi(-\beta_0 - \beta_i x_{i,t} - \beta_j x_{j,t}).$$

- Panel B: \mathcal{M} (simple spread of the ML pair)

$$\hat{y}_{t+k} = \phi(-\beta_0 - \beta_i(x_{i,t} - x_{j,t})), \quad (\beta_i = -\beta_j)$$

- Panel C: \mathcal{M} (generalized spread of the conventional pair)

$$\hat{y}_{t+k} = \phi(-\beta_0 - \beta_i l_t - \beta_j s_t),$$

- Panel D (Benchmark): \mathcal{M} (simple spread of the conventional pair)

$$\hat{y}_{t+k} = \phi(-\beta_0 - \beta_i(l_t - s_t)), \quad (\beta_i = -\beta_j).$$

where l_t and s_t are the 10-year and 3-month yields, respectively.

Performance measure

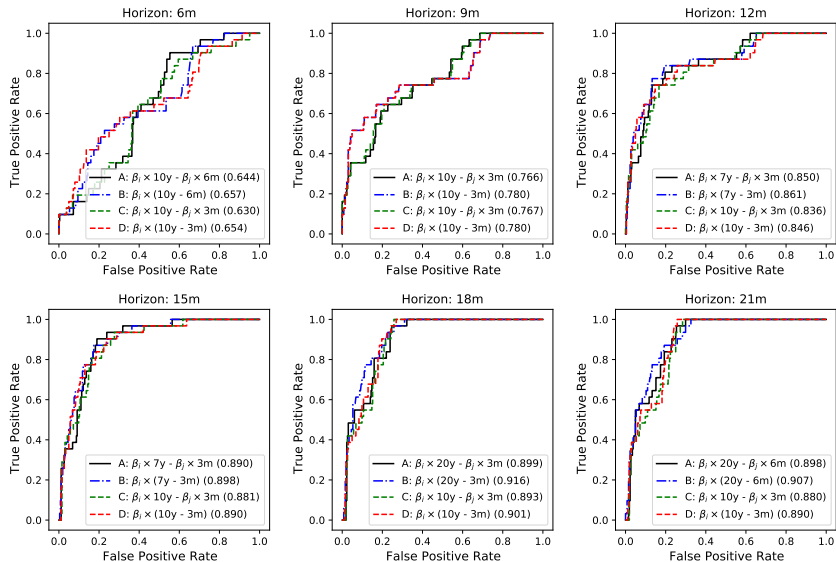
- (Log) posterior predictive likelihood (PPL)
 - (Log) likelihood for out-of-sample (test) period.
- Empirical Bayes factor (EBF)
 - The ratio of the PPL of an alternative model to that of the benchmark model.
 - The outperformance of an alternative model is statistically significant if $EBF > \sqrt{10} \approx 3.3$ (Jeffreys' criterion).
- The area under the receiver operating characteristic curve (ROC-AUC)
 - ROC is a collection of the (false positive rate, true positive rate) coordinates
 - The predictive power of the model without any specific decision threshold
 - Range: 0.5 (random guess) to 1.0 (perfect classification)
 - Popular measure in machine learning (Bradley, 1997) and recession prediction (Bauer and Mertens, 2018; Tsang and Wu, 2019).

Main result: training period, 1961–1995

- The ML algorithm choose the spread of one long- and one short-term yields.
- The pair is different from (10y, 3m) and the coefficient ratio different from -1.
- The performance gain is not significant compared to the benchmark (Panel D).

| Horizon | Pair | β | λ | AUC _{train} | AUC _{test} | log L | log PPL | EBF |
|---|-----------|-----------------|-----------|----------------------|---------------------|--------|---------|-------|
| Panel A. Generalized spread of the ML pair | | | | | | | | |
| 6 | (10y, 6m) | (0.773, -1.069) | 0.933 | 0.937 | 0.644 | -0.255 | -0.413 | 0.931 |
| 12 | (7y, 3m) | (1.039, -1.231) | 0.354 | 0.919 | 0.850 | -0.270 | -0.277 | 0.981 |
| 18 | (20y, 3m) | (0.428, -0.538) | 2.639 | 0.806 | 0.899 | -0.343 | -0.271 | 0.985 |
| 24 | (20y, 1y) | (0.383, -0.409) | 1.231 | 0.671 | 0.892 | -0.391 | -0.283 | 1.016 |
| Panel B. Simple spread of the ML pair | | | | | | | | |
| 6 | (10y, 6m) | (1.274, -1.274) | | 0.879 | 0.657 | -0.279 | -0.353 | 0.989 |
| 12 | (7y, 3m) | (1.342, -1.342) | | 0.892 | 0.861 | -0.280 | -0.250 | 1.008 |
| 18 | (20y, 3m) | (0.759, -0.759) | | 0.791 | 0.916 | -0.341 | -0.252 | 1.004 |
| 24 | (20y, 1y) | (0.518, -0.518) | | 0.669 | 0.903 | -0.390 | -0.273 | 1.026 |
| Panel C. Generalized spread of the conventional pair | | | | | | | | |
| 6 | (10y, 3m) | (0.762, -1.094) | | 0.940 | 0.630 | -0.258 | -0.416 | 0.929 |
| 12 | (10y, 3m) | (0.983, -1.162) | | 0.915 | 0.836 | -0.271 | -0.284 | 0.974 |
| 18 | (10y, 3m) | (0.646, -0.738) | | 0.796 | 0.893 | -0.341 | -0.263 | 0.993 |
| 24 | (10y, 3m) | (0.245, -0.292) | | 0.619 | 0.877 | -0.403 | -0.297 | 1.002 |
| Panel D. Simple spread of the conventional pair | | | | | | | | |
| 6 | (10y, 3m) | (1.158, -1.158) | | 0.871 | 0.654 | -0.290 | -0.342 | 1.000 |
| 12 | (10y, 3m) | (1.258, -1.258) | | 0.894 | 0.846 | -0.280 | -0.258 | 1.000 |
| 18 | (10y, 3m) | (0.786, -0.786) | | 0.783 | 0.901 | -0.344 | -0.256 | 1.000 |
| 24 | (10y, 3m) | (0.305, -0.305) | | 0.610 | 0.892 | -0.404 | -0.299 | 1.000 |

Receiver operating characteristic (ROC) curve



Robustness check 1: training period, 1961–2005

- The performance gain is not significant compared to the benchmark (Panel D).

| Horizon | Pair | β | λ | AUC _{train} | AUC _{test} | log L | log PPL | EBF |
|---|-----------|-----------------|-----------|----------------------|---------------------|--------|---------|-------|
| <i>Panel A. Generalized spread of the ML pair</i> | | | | | | | | |
| 6 | (10y, 3m) | (0.597, -0.901) | 2.297 | 0.925 | 0.655 | -0.248 | -0.518 | 0.945 |
| 12 | (20y, 3m) | (0.486, -0.695) | 5.278 | 0.924 | 0.844 | -0.262 | -0.381 | 0.953 |
| 18 | (20y, 6m) | (0.573, -0.663) | 1.741 | 0.822 | 0.967 | -0.310 | -0.303 | 0.970 |
| 24 | (20y, 1y) | (0.397, -0.434) | 1.414 | 0.679 | 0.985 | -0.356 | -0.310 | 1.013 |
| <i>Panel B. Simple spread of the ML pair</i> | | | | | | | | |
| 6 | (10y, 3m) | (1.240, -1.240) | | 0.870 | 0.582 | -0.266 | -0.461 | 1.000 |
| 12 | (20y, 3m) | (1.268, -1.268) | | 0.903 | 0.802 | -0.253 | -0.368 | 0.965 |
| 18 | (20y, 6m) | (0.825, -0.825) | | 0.809 | 0.966 | -0.309 | -0.280 | 0.992 |
| 24 | (20y, 1y) | (0.547, -0.547) | | 0.677 | 0.988 | -0.355 | -0.294 | 1.029 |
| <i>Panel C. Generalized spread of the conventional pair</i> | | | | | | | | |
| 6 | (10y, 3m) | (0.839, -1.142) | | 0.926 | 0.637 | -0.244 | -0.551 | 0.914 |
| 12 | (10y, 3m) | (1.105, -1.290) | | 0.926 | 0.838 | -0.244 | -0.377 | 0.957 |
| 18 | (10y, 3m) | (0.628, -0.746) | | 0.799 | 0.969 | -0.315 | -0.296 | 0.977 |
| 24 | (10y, 3m) | (0.243, -0.315) | | 0.634 | 0.977 | -0.367 | -0.329 | 0.994 |
| <i>Panel D. Simple spread of the conventional pair</i> | | | | | | | | |
| 6 | (10y, 3m) | (1.240, -1.240) | | 0.870 | 0.582 | -0.266 | -0.461 | 1.000 |
| 12 | (10y, 3m) | (1.407, -1.407) | | 0.905 | 0.823 | -0.251 | -0.333 | 1.000 |
| 18 | (10y, 3m) | (0.811, -0.811) | | 0.781 | 0.971 | -0.319 | -0.272 | 1.000 |
| 24 | (10y, 3m) | (0.342, -0.342) | | 0.617 | 0.978 | -0.369 | -0.322 | 1.000 |

Robustness check 2: training period, 1961–2015

- The performance gain is not significant compared to the benchmark (Panel D).

| Horizon | Pair | β | λ | AUC _{train} | AUC _{test} | log L | log PPL | EBF |
|---|-----------|-----------------|-----------|----------------------|---------------------|--------|---------|-------|
| <i>Panel A. Generalized spread of the ML pair</i> | | | | | | | | |
| 6 | (10y, 3m) | (0.302, -0.494) | 4.595 | 0.799 | 0.832 | -0.313 | -0.282 | 0.967 |
| 12 | (7y, 3m) | (1.033, -1.147) | 1.231 | 0.883 | 1.000 | -0.273 | -0.199 | 0.987 |
| 18 | (20y, 3m) | (0.481, -0.556) | 4.925 | 0.832 | 0.996 | -0.315 | -0.253 | 0.984 |
| 24 | (20y, 1y) | (0.631, -0.611) | 1.625 | 0.747 | 0.996 | -0.344 | -0.249 | 1.023 |
| <i>Panel B. Simple spread of the ML pair</i> | | | | | | | | |
| 6 | (10y, 3m) | (0.883, -0.883) | | 0.791 | 0.902 | -0.314 | -0.248 | 1.000 |
| 12 | (7y, 3m) | (1.380, -1.380) | | 0.876 | 1.000 | -0.274 | -0.177 | 1.009 |
| 18 | (20y, 3m) | (0.874, -0.874) | | 0.828 | 0.988 | -0.306 | -0.232 | 1.004 |
| 24 | (20y, 1y) | (0.698, -0.698) | | 0.748 | 0.996 | -0.344 | -0.244 | 1.028 |
| <i>Panel C. Generalized spread of the conventional pair</i> | | | | | | | | |
| 6 | (10y, 3m) | (0.632, -0.795) | | 0.809 | 0.864 | -0.306 | -0.260 | 0.988 |
| 12 | (10y, 3m) | (1.075, -1.159) | | 0.875 | 1.000 | -0.276 | -0.197 | 0.989 |
| 18 | (10y, 3m) | (0.859, -0.910) | | 0.828 | 0.992 | -0.309 | -0.240 | 0.996 |
| 24 | (10y, 3m) | (0.484, -0.518) | | 0.714 | 0.996 | -0.359 | -0.271 | 1.000 |
| <i>Panel D. Simple spread of the conventional pair</i> | | | | | | | | |
| 6 | (10y, 3m) | (0.883, -0.883) | | 0.791 | 0.902 | -0.314 | -0.248 | 1.000 |
| 12 | (10y, 3m) | (1.230, -1.230) | | 0.872 | 1.000 | -0.277 | -0.186 | 1.000 |
| 18 | (10y, 3m) | (0.953, -0.953) | | 0.821 | 0.992 | -0.310 | -0.237 | 1.000 |
| 24 | (10y, 3m) | (0.542, -0.542) | | 0.710 | 0.984 | -0.359 | -0.271 | 1.000 |

Robustness check 3: oversampling of recessions

- Oversample the recession observations to *balance* the class:

$$\log L(\beta_0, \beta) = \sum w_{t+k} (y_{t+k} \ln(\hat{y}_{t+k}) + (1 - y_{t+k}) \ln(1 - \hat{y}_{t+k})),$$

where $w_t = \frac{1}{2r}$ if $y_t = 1$ or $\frac{1}{2(1-r)}$ if $y_t = 0$

| Horizon | Pair | β | λ | AUC _{train} | AUC _{test} | log L | log PPL | EBF |
|---|-----------|-----------------|-----------|----------------------|---------------------|--------|---------|-------|
| <i>Panel A. Generalized spread of the ML pair</i> | | | | | | | | |
| 6 | (20y, 3m) | (0.576, -1.037) | 6.063 | 0.938 | 0.626 | -0.370 | -1.232 | 0.618 |
| 12 | (20y, 3m) | (0.361, -0.630) | 12.126 | 0.901 | 0.807 | -0.465 | -0.632 | 0.877 |
| 18 | (20y, 6m) | (0.720, -0.814) | 1.866 | 0.815 | 0.916 | -0.538 | -0.467 | 0.997 |
| 24 | (20y, 1y) | (0.413, -0.435) | 2.639 | 0.671 | 0.895 | -0.642 | -0.531 | 1.055 |
| <i>Panel B. Simple spread of the ML pair</i> | | | | | | | | |
| 6 | (20y, 3m) | (1.208, -1.208) | | 0.871 | 0.640 | -0.457 | -0.866 | 0.891 |
| 12 | (20y, 3m) | (1.372, -1.372) | | 0.886 | 0.844 | -0.432 | -0.562 | 0.940 |
| 18 | (20y, 6m) | (0.922, -0.922) | | 0.801 | 0.924 | -0.540 | -0.436 | 1.029 |
| 24 | (20y, 1y) | (0.568, -0.568) | | 0.669 | 0.904 | -0.639 | -0.501 | 1.087 |
| <i>Panel C. Generalized spread of the conventional pair</i> | | | | | | | | |
| 6 | (10y, 3m) | (1.102, -1.655) | | 0.940 | 0.631 | -0.346 | -1.443 | 0.501 |
| 12 | (10y, 3m) | (1.296, -1.599) | | 0.916 | 0.832 | -0.387 | -0.655 | 0.857 |
| 18 | (10y, 3m) | (0.735, -0.845) | | 0.796 | 0.893 | -0.554 | -0.467 | 0.997 |
| 24 | (10y, 3m) | (0.269, -0.310) | | 0.616 | 0.880 | -0.668 | -0.576 | 1.009 |
| <i>Panel D. Simple spread of the conventional pair</i> | | | | | | | | |
| 6 | (10y, 3m) | (1.270, -1.270) | | 0.870 | 0.653 | -0.458 | -0.750 | 1.000 |
| 12 | (10y, 3m) | (1.513, -1.513) | | 0.894 | 0.846 | -0.421 | -0.500 | 1.000 |
| 18 | (10y, 3m) | (0.851, -0.851) | | 0.783 | 0.901 | -0.560 | -0.464 | 1.000 |
| 24 | (10y, 3m) | (0.320, -0.320) | | 0.610 | 0.892 | -0.670 | -0.585 | 1.000 |

Robustness check 4: missing variable

- Includes the US leading indicator and always selects it by default.
- Also includes 30y Treasury yield (training period: 1982-1995).

| Horizon | Pair | β | β_{LL} | λ | AUC _{train} | AUC _{test} | log L | log PPL | EBF |
|---|-----------|-----------------|--------------|-----------|----------------------|---------------------|--------|---------|-------|
| <i>Panel A. Generalized spread of the ML pair</i> | | | | | | | | | |
| 6 | (30y, 3m) | (0.364, -0.627) | 1.020 | 1.414 | 0.931 | 0.859 | -0.165 | -0.311 | 0.934 |
| 12 | (30y, 3m) | (1.091, -0.831) | 0.013 | 1.516 | 0.977 | 0.845 | -0.126 | -0.280 | 0.996 |
| 18 | (30y, 3m) | (1.375, -1.216) | -0.150 | 1.072 | 0.986 | 0.915 | -0.111 | -0.219 | 1.013 |
| 24 | (30y, 6m) | (0.688, -0.606) | -0.862 | 1.414 | 0.924 | 0.888 | -0.167 | -0.265 | 0.983 |
| <i>Panel B. Simple spread of the ML pair</i> | | | | | | | | | |
| 6 | (30y, 3m) | (1.326, -1.326) | 1.262 | | 0.939 | 0.848 | -0.150 | -0.268 | 0.975 |
| 12 | (30y, 3m) | (2.008, -2.008) | -0.499 | | 0.964 | 0.821 | -0.101 | -0.300 | 0.977 |
| 18 | (30y, 3m) | (2.335, -2.335) | -0.505 | | 0.990 | 0.903 | -0.084 | -0.235 | 0.998 |
| 24 | (30y, 6m) | (1.558, -1.558) | -1.291 | | 0.916 | 0.897 | -0.139 | -0.244 | 1.003 |
| <i>Panel C. Generalized spread of the conventional pair</i> | | | | | | | | | |
| 6 | (10y, 3m) | (0.926, -1.278) | 0.968 | | 0.938 | 0.792 | -0.148 | -0.350 | 0.898 |
| 12 | (10y, 3m) | (2.013, -1.708) | -0.132 | | 0.981 | 0.832 | -0.096 | -0.323 | 0.954 |
| 18 | (10y, 3m) | (1.877, -1.838) | -0.224 | | 0.984 | 0.894 | -0.097 | -0.228 | 1.005 |
| 24 | (10y, 3m) | (1.148, -1.188) | -1.057 | | 0.895 | 0.878 | -0.157 | -0.259 | 0.989 |
| <i>Panel D. Simple spread of the conventional pair</i> | | | | | | | | | |
| 6 | (10y, 3m) | (1.310, -1.310) | 1.345 | | 0.934 | 0.893 | -0.157 | -0.242 | 1.000 |
| 12 | (10y, 3m) | (2.222, -2.222) | -0.426 | | 0.973 | 0.825 | -0.093 | -0.276 | 1.000 |
| 18 | (10y, 3m) | (2.371, -2.371) | -0.370 | | 0.984 | 0.893 | -0.087 | -0.233 | 1.000 |
| 24 | (10y, 3m) | (1.366, -1.366) | -1.202 | | 0.895 | 0.884 | -0.153 | -0.247 | 1.000 |

Conclusion

Our findings

- We adopt a machine learning (ML) approach to search for the best maturity pair and the coefficients simultaneously.
- The ML algorithm finds a generalized spread: one long- and one short-term yield pair with the coefficients of the opposite signs and similar magnitudes.
- However, the out-of-sample prediction gain of the ML approach is not statistically significant.
- We justify the use of the simple 10y/3m spread.

Thank you for your attention.

The working paper is available in

SSRN: 3723717.

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