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[Dynamic Trading with Predictable Returns and Transaction Costs - Gârleanu and Pedersen (2013)]

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- Closed-form optimal dynamic portfolio construction methodology under some assumptions:
  - mean-variance preferences
  - (convex) transaction costs
  - prices are predictable by signals of different strengths
  - signals have different mean-reversion speeds
- Methodology suggests some guiding principles:
  - signals give you an "optimal" portfolio under the assumption of no transaction costs
  - signals mean-reversion speeds and transaction costs give you a "will be optimal in the future" portfolio
  - "aim" in between the two portfolios above

Model

Factors (may include a constant)

- Price changes are predictable: $r_{t+1} = Bf_t + u_{t+1}$

$$r_{t+1} = p_{t+1} - (1 + r^f)p_t$$

Price changes, not returns!

- Factors are mean-reverting: $\Delta f_{t+1} = -\Phi f_t + \varepsilon_{t+1},$

- Convex transaction costs: $TC(\Delta x_t) = \frac{1}{2}\Delta x_t^\top \Lambda \Delta x_t.$
  - These are appropriate for a temporary market price impact which increases with traded notional but note that prices revert instantaneously after trading
  - The model omits costs which are linear in the amount of $ traded like broker and financing fees
  - Model does not assume different costs for long and short positions, effectively omitting the fact that shorting costs more

Mean-variance preferences

Risk aversion parameter but on price changes (will then vary by AUM)!

$$\max_{x_0, x_1, \ldots} \mathrm{E}_0 \left[\sum_t (1-\rho)^{t+1} \left(x_t^\top r_{t+1} - \frac{\gamma}{2} x_t^\top \Sigma x_t \right) - \frac{(1-\rho)^t}{2} \Delta x_t^\top \Lambda \Delta x_t \right]$$

Discount rate

Cost incurred at the start of the period to set up portfolio x_t

Bellman equation:

$$\begin{aligned} & V(x_{t-1}, f_t) \\ &= \max_{x_t} \left\{ -\frac{1}{2} \Delta x_t^\top \Lambda \Delta x_t + (1-\rho) \left(x_t^\top \mathrm{E}_t[r_{t+1}] - \frac{\gamma}{2} x_t^\top \Sigma x_t + \mathrm{E}_t[V(x_t, f_{t+1})] \right) \right\} \end{aligned}$$

Solution:

$$V(x_t, f_{t+1}) = -\frac{1}{2} x_t^\top A_{xx} x_t + x_t^\top A_{xf} f_{t+1} + \frac{1}{2} f_{t+1}^\top A_{ff} f_{t+1} + A_0.$$

Trade Partially Toward the Aim

The **$\mathbf{Aim}_t$** portfolio is the projection of weights onto the factors!

$$x_t = x_{t-1} + \Lambda^{-1} A_{xx} \left(A_{xx}^{-1} A_{xf} f_t - x_{t-1} \right)$$

Current portfolio

Trade a proportional rate given by the matrix $\Lambda^{-1} A_{xx}$ towards the aim portfolio

If TCs are proportional to risk $\Lambda = \lambda \Sigma$.

$$x_t = \left(1 - \frac{a}{\lambda}\right) x_{t-1} + \frac{a}{\lambda} aim_t$$

where: $$a = \frac{-(\gamma(1-\rho) + \lambda\rho) + \sqrt{(\gamma(1-\rho) + \lambda\rho)^2 + 4\gamma\lambda(1-\rho)^2}}{2(1-\rho)}$$

- trade slowly when TCs, λ, are high!
- trade fast if you are risk averse, γ!

Aim in Front of the Target

Under no TC ($\Lambda = 0$): $A_{xx}^{-1} A_{xf} f_t$ becomes:

$$Markowitz_t = (\gamma \Sigma)^{-1} B f_t \longleftarrow \textit{Definition!}$$

If TCs are proportional to risk $\Lambda = \lambda \Sigma$.

$$aim_t = z\, Markowitz_t + (1 - z)\, E_t(aim_{t+1}).$$

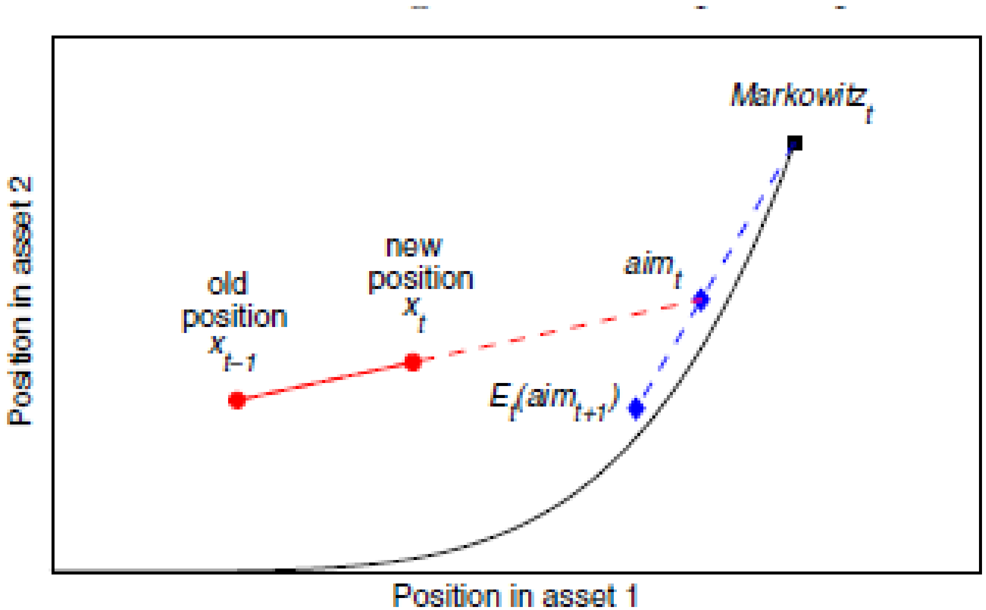
$$z = \gamma / (\gamma + a).$$

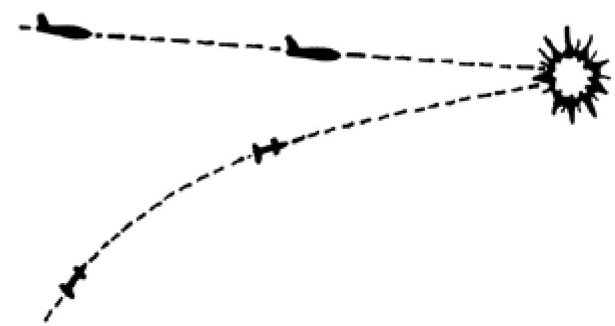
and

$$aim_t = \sum_{\tau=t}^{\infty} z(1 - z)^{\tau - t} E_t \left(Markowitz_\tau \right)$$

The weight z decreases with the transaction costs (λ) and increases in risk aversion (γ)

Aim in Front of the Target





"A great hockey player skates to where the puck is going to be, not where it is." — Wayne Gretzky

Weight Signals Based on Alpha Decay

The aim portfolio is the Markowitz portfolio built as if the signals f were scaled down based on their mean-reversion:

$$aim_t = (\gamma\,\Sigma)^{-1}\,B\left(I + \frac{a}{\gamma}\Phi\right)^{-1} f_t$$

scaled down based on TC (*a*), risk aversion (*γ*), and, mean-reversion speed (Φ)

If Φ is diagonal:

$$aim_t = (\gamma\,\Sigma)^{-1} B\left(\frac{f_t^1}{1+\phi^1 a/\gamma}, \ldots, \frac{f_t^K}{1+\phi^K a/\gamma}\right)^{\top}$$

Each factor scaled down by its alpha decay speed

- Give more weight to more persistent factors
- The larger the transaction costs, the more important it is to do that as the ratio of optimal weights between increases in λ

Position Homing In

Then the current portfolio is an EWMA of past **aim** portfolios:

$$x_t = \sum_{\tau=-\infty}^{t} \frac{a}{\lambda}\left(1-\frac{a}{\lambda}\right)^{t-\tau} aim_\tau \longleftarrow$$

The common market practice is to have the Markowitz portfolio here, not the aim portfolio!

$$aim_t = (\gamma\,\Sigma)^{-1} B\left(I + \frac{a}{\gamma}\Phi\right)^{-1} f_t$$

VS

$$Markowitz_t = (\gamma\,\Sigma)^{-1} Bf_t$$

Closed form solutions

Timing (time-series):

$$x_t = \left(1 - \frac{a}{\lambda}\right) x_{t-1} + \frac{a}{\lambda} \frac{1}{\gamma \sigma^2} \sum_{i=1}^{K} \frac{\beta^i}{1 + \phi^i a/\gamma} f_t^i .$$

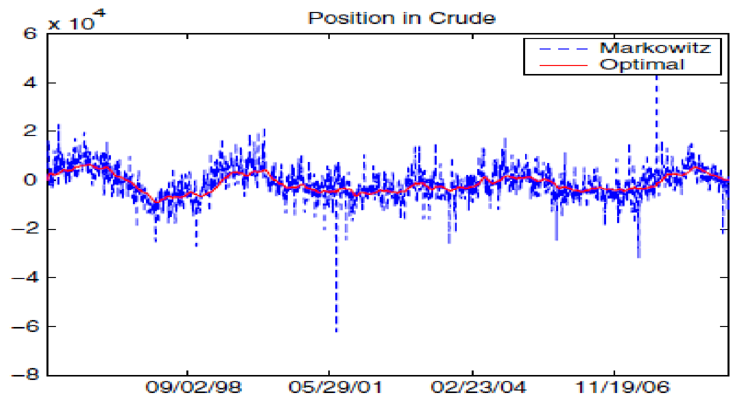
Tilting/relative-value (cross-section):

$$x_t = \left(1 - \frac{a}{\lambda}\right) x_{t-1} + \frac{a}{\lambda} (\gamma \Sigma)^{-1} \sum_{i=1}^{I} \frac{1}{1 + \phi^i a/\gamma} \beta^i f_t^i .$$

The major difference vs. the static portfolio construction problem solution is that this dynamic solution gives more weight to factors with slower alpha decays while the static portfolio treats them equally

Empirical application to commodities

$$\Delta f_{t+1}^{5D,s} = -0.2519\, f_t^{5D,s} + \varepsilon_{t+1}^{5D,s}$$
$$\Delta f_{t+1}^{1Y,s} = -0.0034\, f_t^{1Y,s} + \varepsilon_{t+1}^{1Y,s}$$
$$\Delta f_{t+1}^{5Y,s} = -0.0010\, f_t^{5Y,s} + \varepsilon_{t+1}^{5Y,s}.$$



	Gross SR	Net SR
Markowitz	0.83	−9.84
Dynamic optimization	**0.62**	**0.58**
Static optimization		
Weight on Markowitz = 10%	0.63	−0.41
Weight on Markowitz = 9%	0.62	−0.24
Weight on Markowitz = 8%	0.62	−0.08
Weight on Markowitz = 7%	0.62	0.07
Weight on Markowitz = 6%	0.62	0.20
Weight on Markowitz = 5%	0.61	0.31
Weight on Markowitz = 4%	0.60	0.40
Weight on Markowitz = 3%	0.58	0.46
Weight on Markowitz = 2%	**0.52**	**0.46**
Weight on Markowitz = 1%	0.36	0.33

- Markowitz trades too much
- Static solution, which is also a weighted average between the current and the Markowitz portfolio, can work well but for a particular set of parameters
- Dynamic solution is better than the best static solution

Conclusions

- Tractable closed form solution for dynamic mean-variance portfolios in the presence of several “return” predictors, risk and correlation considerations, as well as transaction costs
- Corroborates the market practice of “slowing down” trading via smoothing of the optimal weights
- Suggests a change to the standard market practice which is to moves in the direction of an “aim” portfolio and not the “optimal” portfolio like it is typically done
- This “aim” portfolio differs from the “optimal” portfolio by weighting more heavily return predictors with slow alpha decay or slow mean-reversion