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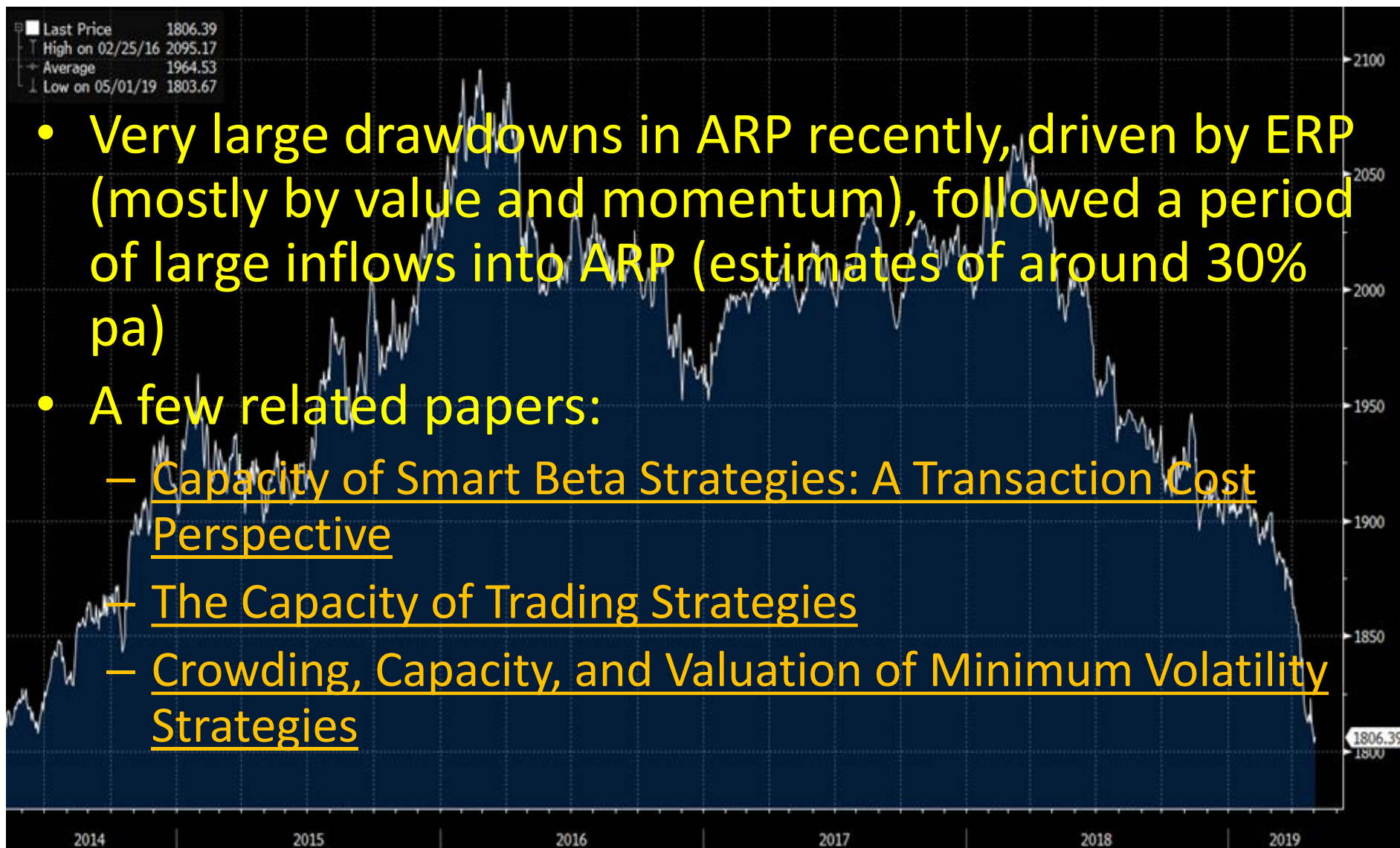
Factor risk premiums and invested capital: calculations with stochastic discount factors

by Ang, Hogan, and Shores (2017), Journal of Asset Management

- Methodology to assess “crowding” in equity factors
- By “crowding”, I mean here the increase in the weights of equity factor portfolios vs. market cap
- “Crowding” should decrease the excess returns of factor strategies
- Empirically estimate how “crowding” may impact expected returns of factor strategies: value, size, momentum, quality, and defensive (low vol)

Background on the paper

- Very large drawdowns in ARP recently, driven by ERP (mostly by value and momentum), followed a period of large inflows into ARP (estimates of around 30% pa)
- A few related papers:
 - Capacity of Smart Beta Strategies: A Transaction Cost Perspective
 - The Capacity of Trading Strategies
 - Crowding, Capacity, and Valuation of Minimum Volatility Strategies



CAPM

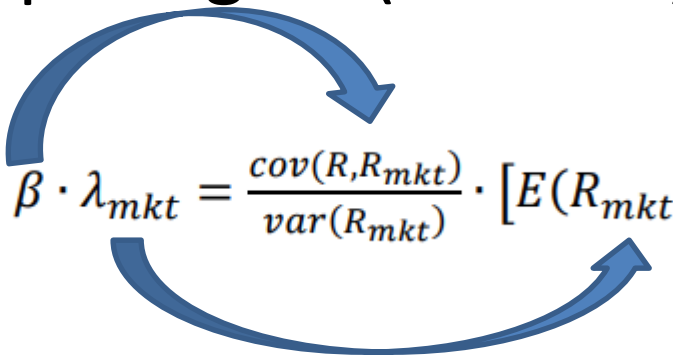
- Start with the market portfolio:

$$R_{mkt} = w_{mkt} \cdot R$$

where R is the vector of returns and w_{mkt} is the vector of market cap weights (sum = 1)

- CAPM:**

Risk-free rate

$$E(R) - R_f = \beta \cdot \lambda_{mkt} = \frac{\text{cov}(R, R_{mkt})}{\text{var}(R_{mkt})} \cdot [E(R_{mkt}) - R_f]$$


- Empirically:**

$$\bar{R} - R_f = a + \beta \cdot [\bar{R}_{mkt} - R_f]$$

Avg. historical returns of test portfolios

Alpha is often positive and statistically significant for factor portfolios

Stochastic Discount Factor (SDF)

- SDF generalizes CAPM to:

$$E(R) - R_f = \underbrace{\gamma}_{\text{Exposure or loading}} \cdot \underbrace{\lambda}_{\text{Price of risk}} = \frac{\text{cov}(R, m)}{\text{var}(m)} \cdot \left[- \frac{\text{var}(m)}{E(m)} \right]$$

m: pricing kernel or SDF

- [Hansen and Jagannathan \(1991\)](#): there exists an optimal SDF, m^* , for which that equation holds exactly, i.e., there are no alphas:

$$m^* = \underbrace{\left[\frac{1}{R_f} + (R - \bar{R})' \Sigma_R^{-1} \left(1 - \frac{1}{R_f} \right) \right]}_{w^*} \cdot R$$

w^* ← Optimal tangent portfolio

Completion portfolio

$$w^* = w_{mkt} + \underbrace{(w^* - w_{mkt})}_{\text{zero cost completion portfolio}} \leftarrow \text{zero cost completion portfolio}$$

Interpretation: the completion portfolio is the “*flow of capital*” from the market cap portfolio to the optimal portfolio that would “wipe out” all the alphas of factor strategies

One can think of Fama-French type of models as forms of completion portfolios which are “added” to the market cap portfolio to bring it closer to the efficient portfolio

Application to the Value factor

Pricing kernel	Portfolio C APM Alphas			
	Growth	Neutral	Value	Value–growth
Cap-weighted market	– 1.04%	0.43%	2.77%	3.81%

	Weights		
	Market	Efficient	Completion
Growth	0.522	– 0.305	– 0.827
Neutral	0.334	– 0.576	– 0.910
Value	0.144	1.881	1.737
Sum	1.000	1.000	0.000

The efficient portfolio requires a high degree of leverage towards the value portfolio.

Value factor: economic interpretation

- Why is the value premium so large? Why don't investors allocate more capital to value strategies?
- Perhaps investors do not want the extra risk associated with firms with less flexible capital structure
- Perhaps investors irrationally believe growth stocks will continue to grow at rapid rates in the future
- The value factor can even come from macro risks, such as shocks to labor income or endowments, associated with changes in the investment opportunity set

Capital flows from growth to value

	Excess return (%)	Candidate pricing kernels					
		Market CAPM		Transfer capital from low-return to high-return portfolio		Efficient portfolio	
		Alpha (%)	Weights	Alpha (%)	Weights	Alpha (%)	Weights
Value-growth portfolios							
Growth	5.97	− 1.04	0.522	− 1.40	0.000	0.00	− 0.305
Neutral	6.80	0.43	0.334	− 0.96	0.334	0.00	− 0.576
Value	9.57	2.77	0.144	0.48	0.666	0.00	1.881
Value premium		3.81		1.88		0.00	

- The CAPM alpha associated with value, the value premium, is 3.81% historically
- If all the capital in growth stocks moved to value stocks, this methodology suggests the value premium would shrink to 1.88%
- Not that the capital flow reduces the alpha associated with value stocks but turns the alpha associated with growth stocks more negative

Capital flows from small to large

	Excess return (%)	Candidate pricing kernels					
		Market CAPM		Transfer capital from low-return to high-return portfolio		Efficient portfolio	
		Alpha (%)	Weights	Alpha (%)	Weights	Alpha (%)	Weights
Size portfolios							
Large	9.07	– 0.30	0.810	0.84	0.000	0.00	0.472
Neutral	6.80	0.43	0.334	– 0.96	0.334	0.00	– 0.576
Small	5.96	1.43	0.051	– 0.15	0.861	0.00	– 0.439
Size premium		1.73		– 0.99		0.00	

- The CAPM alpha associated with size, the size premium, is 1.73% historically
- If all the capital in large stocks moved to small stocks, this methodology suggests the size premium would go to -0.99%
- Not that the magnitude of capital flow required to do that is still very large, moving 81% of the entire market cap size from large to small stocks

Capital flows from winners to losers

	Excess return (%)	Candidate pricing kernels					
		Market CAPM		Transfer capital from low-return to high-return portfolio		Efficient portfolio	
		Alpha (%)	Weights	Alpha (%)	Weights	Alpha (%)	Weights
Momentum portfolios							
Losers	3.53	− 4.22	0.212	− 4.67	0.000	0.00	− 0.841
Neutral	5.58	− 0.43	0.449	− 1.50	0.449	0.00	− 0.254
Winners	9.77	3.22	0.339	1.22	0.551	0.00	1.096
Momentum premium		7.44		5.89		0.00	

- The CAPM alpha associated with momentum, the momentum premium, is 7.44% historically
- If all the capital in winners stocks moved to losers stocks, this methodology suggests the size premium would reduce to 5.89%
- Again, the alpha zero, efficient portfolio, is a highly levered portfolio

Capital flows from junk to quality

	Excess return (%)	Candidate pricing kernels					
		Market CAPM		Transfer capital from low-return to high-return portfolio		Efficient portfolio	
		Alpha (%)	Weights	Alpha (%)	Weights	Alpha (%)	Weights
Profitability portfolios							
Low profitability	7.03	− 2.24	0.167	− 2.56	0.000	0.00	1.337
Neutral	7.93	− 0.28	0.386	− 0.73	0.386	0.00	0.046
High profitability	9.35	1.08	0.447	0.46	0.614	0.00	1.291
Quality premium		3.32		3.02		0.00	

- The CAPM alpha associated with quality, the momentum premium, is 3.32% historically
- If all the capital in winners stocks moved to losers stocks, this methodology suggests the size premium would reduce to 3.02%
- Note that there is very little capital deployed on low profitability, junk, and therefore this capital flow methodology suggests only a small reduction in the quality premium as a result of “crowding”

Capital flows from low to high vol

	Excess return (%)	Candidate pricing kernels					
		Market CAPM		Transfer capital from low-return to high-return portfolio		Efficient portfolio	
		Alpha (%)	Weights	Alpha (%)	Weights	Alpha (%)	Weights
Idiosyncratic volatility portfolios							
Volatile	7.35	− 4.70	0.167	− 5.02	0.000	0.00	− 0.668
Neutral	0.09	0.30	0.342	− 0.69	0.342	0.00	1.018
Stable	8.45	1.38	0.492	0.36	0.658	0.00	0.650
Volatility premium		6.08		5.38		0.00	

- The CAPM alpha associated with defensive strategies is 6.08% historically
- If all the capital in winners stocks moved to losers stocks, this methodology suggests the defensive premium would reduce to 5.38%
- Note that there is very little capital deployed on high vol stocks and therefore this capital flow methodology suggests only a small reduction in the defensive premium as a result of “crowding”

Conclusion

- Under this methodology, it would require an extreme amount of capital flows to completely “wipe out” the premia associated with value, momentum and low vol factors
- Even moving all capital from growth to value, from losers to winners, from junk to quality, and from high vol to low vol, premiums would decrease by at most 50%
- A large amount of leverage is required for “crowding” to have a material impact on ERP
- The size factor is more sensitive to “crowding” and it could significantly decrease over time if flows into ERP continue to increase

Related literature

Capacity of Smart Beta Strategies: A Transaction Cost Perspective

- **Capacity:** breakeven AUM for which its transaction costs (mostly related to turnover) and market impact (based on Blackrock's model) would “wipe out” the factor premia
- Estimated transaction costs based capacity is huge (1.6 tn for the multi-factor strategy)
- Momentum has high turnover and therefore the least amount of capacity while size has the largest capacity

Related literature

The Capacity of Trading Strategies

- Use a famous trading model to link the net-of-transaction costs Sharpe ratio with the gross-risk-AUM deployed in the strategy and build a “capacity frontier”
- For US stocks: value and momentum are the ones with lower capacity while quality and defensive strategies have larger capacity
- There are large gains in “slowing down” trading in classic factor strategies
 - **My comment:** This result suggests we should apply “regularization” to “cost penalization” parameters to improve out-of-sample performance of factor strategies

Related literature

Crowding, Capacity, and Valuation of Minimum Volatility Strategies

- Separates concepts of crowding (more linked to the tail risk of investors rushing to exit) and valuations (more linked to alpha erosion)
- Both the min vol inflows and AUM are still tiny. A shift of approximately \$600 billion to minimum volatility strategies would be required for the market not to be underweight low vol
- In decomposing returns into earnings growth and change in PE, seems like min vol strategies has benefited from earnings growth and not from higher valuations
- Min vol strategies tend to outperform the market when the market is volatile, which also tend to be periods of market drawdowns. This increases its unconditional Sharpe ratio relative to standard market exposure