

# Algorithmic Trading : Risk Management

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# Papers

- Andrei Kirilenko, Albert S. Kyle, Mehrdad Samadi, and Tuckan Tuzun, 2017, The Flash Crash: High-Frequency Trading in an Electronic Market, *Journal of Finance*
  - David Easley, Marcos M. Lopez de Prado, and Maureen O'Hara, 2012, Flow Toxicity and Liquidity in a High-frequency World, *Review of Financial Studies*
  - Jangkoo Kang, Kyung Yoon Kwon, and Wooyeon Kim, 2019, Flow Toxicity of High-Frequency Trading and Its Impact on Price Volatility: Evidence from the KOSPI 200 Futures Market, *Journal of Futures Markets*
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# Flash Crash

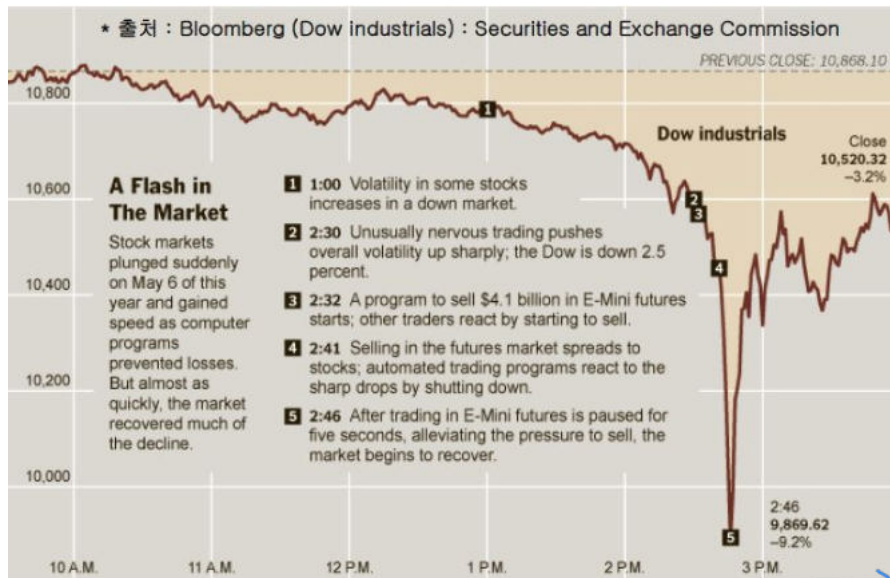


Figure 5-1. Prices of the Dow industrials on May 6, 2010

- On May 6, 2010, U.S. financial markets experienced a systemic intraday event known as the “Flash Crash.”
- The Dow Jones Industrial Average had its second biggest intraday point decline (from the opening) up to that point, plunging 998.5 points (about 9%), most **within minutes**, only to recover a large part of the loss

## Flash Crash 가 갖는 의미 / 중요성

- Fundamental한 shock 없이 증권시장의 systematic한 이유로 시장이 급락했음.
- Flash crash은 촉발시킨 주원은 알리리듬에 의해 결정되었고, 이후 거래를 역시 알리리듬 거래가 많이 참여되어 있었음.
- 시장의 9% 넘게 급락하는데 걸린 시간은 단 몇분 밖에 걸리지 않았고, 급락 후 반등도 매우 빠르게 일어남.

원가매 큰 반향음 불거 일으켰고, 약  
SEC-CFTC 합동 조사로 인해 권상파악에 나섰음.

# Flash Crash (cont'd)

Commodity Futures Trading Commission (상품선물거래위원회)  
Securities and Exchange Commission (증권거래위원회)

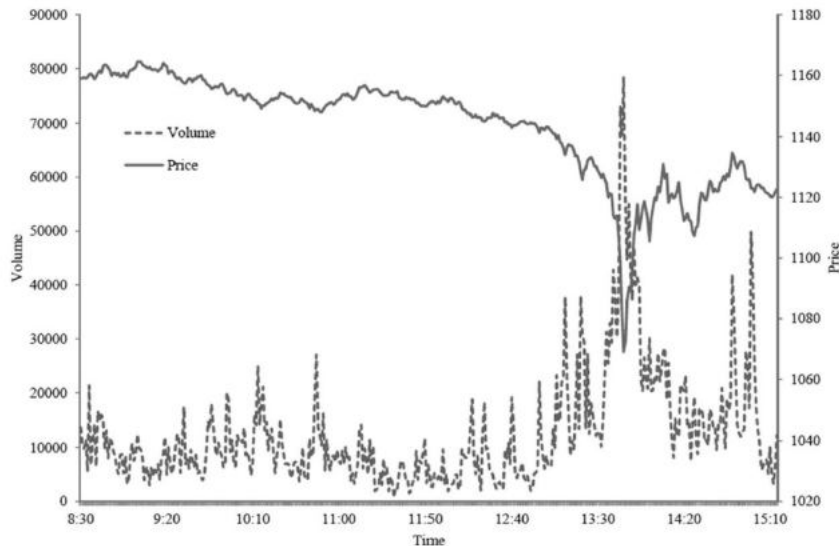


Figure 5-2. Prices and trading volume of the E-mini S&P 500 stock index futures contract on May 6, 2010

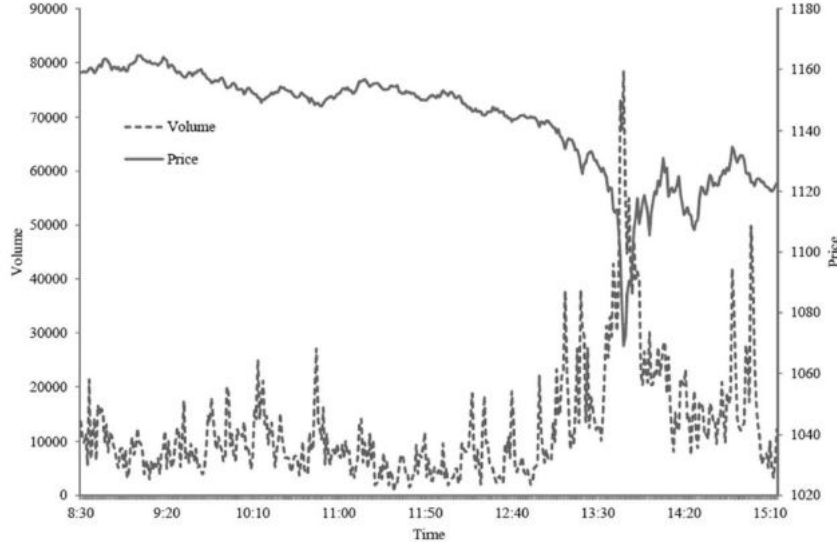
According to the joint report of the CFTC-SEC,

- At 2:32 [ET] p.m., a large fundamental trader (a mutual fund) initiate a sell program to sell a total 75,000 E-mini S&P 500 futures contracts (~\$4.1 billion) as a hedge to an existing equity position
- This sell pressure was initially absorbed by HFTs, other intermediaries, fundamental buyers, and cross-market arbitrageurs

- 단기적 목적이나 시장교란행위 목적 등이 불분명한 의도를 가진 대량 애도 주문을 집중한 것이 확인, 해지목적의 대량거래였음.  
- 이 거래편의의 대량 애도 주문이 쏟아진 초기에는 다른 시장참여자들, 특히 AT/HFT 등은 이 애도 주문을 받아줄것임

# Flash Crash (cont'd)

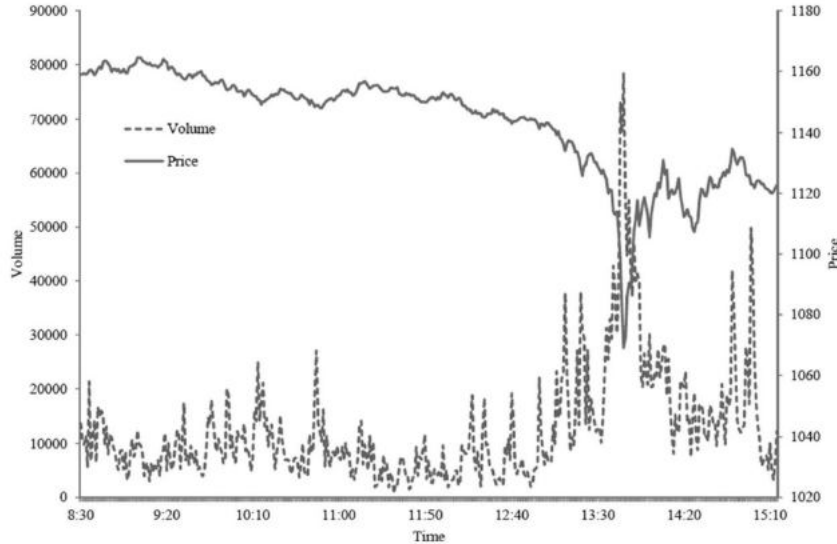
- 하지만 지속적으로 가격 하락에 걸음 대량 매도 주문을  
발사하던 AT/HFT들도 위험 관리를 위해 매수포지션을  
청산, 즉 매도주문을 제출하게 되었고 결국가 매수주문을  
매수 모두 취소했음 => 가격하락 심화



**Figure 5-2.** Prices and trading volume of the E-mini S&P 500 stock index futures contract on May 6, 2010

- Between 2:32 p.m. and 2:45 p.m., as prices of the E-mini rapidly declined, the Sell Algorithm sold about 47% of intended
- By 2:45:28 there were less than 1,050 contracts of buy-side resting orders in the E-mini, less than 1% of buy-side market depth at the beginning of the day
- At 2:45:28 p.m., trading on the E-mini was paused for 5 seconds when CME Stop Logic Functionality was triggered in order to prevent a cascade of further price declines

# Flash Crash (cont'd)

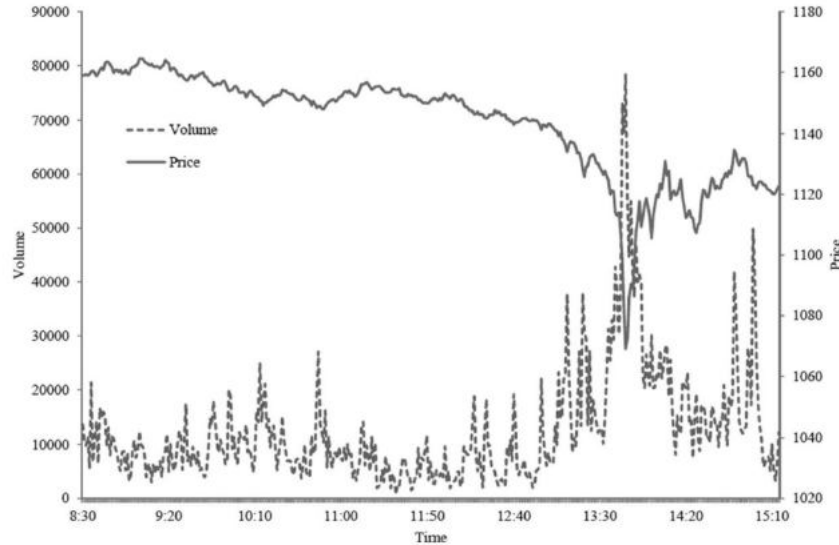


**Figure 5-2.** Prices and trading volume of the E-mini S&P 500 stock index futures contract on May 6, 2010

- When trading resumed at 2:45:33 p.m., prices stabilized and shortly thereafter, the E-mini began to recover, followed by the SPY
- Even though after 2:45 p.m. prices in the E-mini and SPY were recovering from their severe declines, sell orders placed for some individual securities and ETFs founded reduced buying interest, which led to further price declines in those securities

→ 몇몇 개별주식과 ETF들의 경우  
하락이 지속되기도 했음.

# Flash Crash (cont'd)



- Between 2:40 p.m. And 3:00 p.m., over 20,000 trades across more than 300 separate securities were executed at prices 60% or more away from their 2:40 p.m. prices
- By 3:08 p.m., the E-mini prices were back to nearly their pre-drop level and most securities had reverted back to trading at prices reflecting true consensus values

**Figure 5-2.** Prices and trading volume of the E-mini S&P 500 stock index futures contract on May 6, 2010

# Flash Crash (cont'd)

As a **market maker**, how to avoid this toxic event?

Putting it differently, how to measure order flow toxicity?

- If we are able to construct a measure of **order flow toxicity**, then we monitor market environment in a real-time basis and make trading decision accordingly

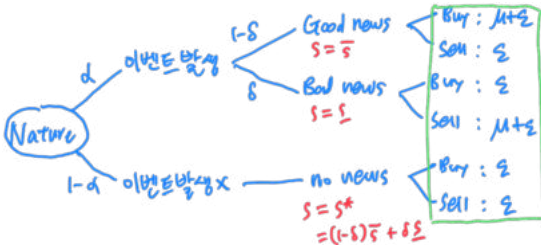
Easley, Lopez de Prado, and O'Hara (2012) suggest the use of **VPIN** as a **useful indicator of short-term, toxicity-induced volatility**

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# Volume-Synchronized Probability of Informed Trading (VPIN)

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• 주가  $S$ 가 어떤 값으로 실현될지는 informed trader만 볼 수 있다.

• MM은 buy/sell 주문흐름을 관찰하면서 bid/ask price를 제출해야 한다.

# The Model

An event occurs with probability  $\alpha$

- Good news with probability  $1 - \delta$
- Bad news with probability  $\delta$

$$S_i = \begin{cases} \bar{S}_i & \text{with } 1 - \delta \\ \underline{S}_i & \text{with } \delta \end{cases}$$

Assume you are a **market maker**

At what price will you post your bid and ask volume?

Period  $i$

(1) **Informed trader**  $\sim \text{Poisson}(\mu)$   
: Buy if the event is a good news,  
sell if the event is a bad news

(2) **Uninformed buyer**  $\sim \text{Poisson}(\varepsilon)$   
: Buy regardless of the event

(3) **Uninformed seller**  $\sim \text{Poisson}(\varepsilon)$   
: Sell regardless of the event

# The Model (cont'd)

$(\alpha, \delta, (\bar{S}_i, \underline{S}_i), \mu, \varepsilon)$  값은 다 알고 있음.

A market maker uses his knowledge of these parameters to determine the price at which he is willing to go long, the bid, and the price at which he is willing to go short, the ask

These prices differ, and so there is a bid-ask spread, because the market maker does not know whether the counterparty to his trade is informed or not

(B) MM이 다른 거래자의 매수주문을 받았다고 하자.  
 • 거래상대방이 Informed 일까?  $S = \bar{S}$  라는 뜻이긴 하나서  $S = \bar{S}$ 에 팔아야 손해는 안본다.  
 • " " Uninformed 일까?  $E[S_i] < \bar{S}$ 에 팔면 된다.

Let  $P(t) = (P_n(t), P_b(t), P_g(t))$  be the MM's belief about the events "no news", "bad news", and "good news"

The time  $t$  expected value of the asset, conditional on the history prior to time  $t$  :

$$E[S_i | t] = P_n(t)S_i^* + P_b(t)\underline{S}_i + P_g(t)\bar{S}_i \quad \text{where} \quad S_i^* = \delta\underline{S}_i + (1 - \delta)\bar{S}_i$$

The bid and ask :

$$A(t) = \underbrace{\frac{\mu P_g(t)}{\varepsilon + \mu P_g(t)}}_{\text{거래상대방이 informed 일 확률}} \times \bar{S} + \underbrace{\frac{\varepsilon}{\varepsilon + \mu P_g(t)}}_{\text{거래상대방이 uninformed 일 확률}} \times E[S_i | t] \Rightarrow A(t) = E[S_i | t] + \underbrace{\frac{\mu P_g(t)}{\varepsilon + \mu P_g(t)} (\bar{S}_i - E[S_i | t])}_{\text{거래상대방이 informed 일 때 expected loss}}$$

$$B(t) = E[S_i | t] - \frac{\mu P_b(t)}{\varepsilon + \mu P_b(t)} (E[S_i | t] - \underline{S}_i)$$

# The Model (cont'd)

The bid-ask spread :

$$\Sigma(t) = \frac{\mu P_g(t)}{\varepsilon + \mu P_g(t)} (\bar{S}_i - E[S_i | t]) + \frac{\mu P_b(t)}{\varepsilon + \mu P_b(t)} (E[S_i | t] - \underline{S}_i)$$

- The first term is the probability that a buy is an information-based trade times expected loss to an informed buyer
- The second term is the probability that a sell is an information-based trade times expected loss to an informed seller

Assume the natural case in which good and bad news are equally likely, that is,  $\delta = 1 - \delta \Rightarrow \delta = \frac{1}{2}$

Then,

$$\Sigma = \frac{\alpha\mu}{\alpha\mu + 2\varepsilon} (\bar{S}_i - \underline{S}_i)$$

degree of informed trading
~ volatility

The probability that an order is from an informed trader, which is called PIN, is given by :

$$PIN = \frac{\alpha\mu}{\alpha\mu + 2\varepsilon}$$

# VPIN

The standard approach to computing the PIN model uses maximum likelihood to estimate the unobservable parameters and then derives PIN from these parameter estimates

In a HFT world, however, this intermediate numerical estimation (i.e. MLE) of unobservable parameters is computationally heavy and time-consuming

Easley et al. (2012) propose a direct analytic estimation of toxicity that does not require the above complicated estimation

They update their measure in volume time, which is the reason why they term VPIN

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- 차점: 트레이더가 체결 데이터와 호가정보에 access가 가능하므로 하자.
- Tick data (Time / Price / Volume)
  - Quote data (Bid prices & volumes, ask prices & volumes)

# VPIN (cont'd)

## Construction of VPIN

- 1) Volume bucketing  
: Group sequential trades into equal volume buckets of an exogenously defined size  $V$

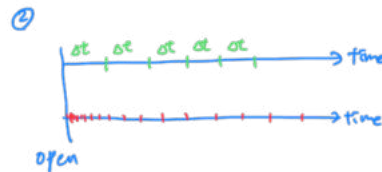
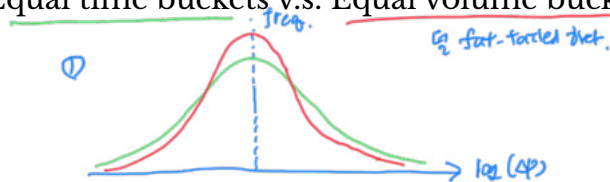
ex) 샘플기간 동안 하루 평균 총 거래량의  $1/50$

A volume bucket is a collection of trades with total volume  $V$

If the last trade needed to complete a bucket is for a size greater than required, the excess size is given to the next bucket

Sampling by volume buckets allows us to divide the trading session into periods of **comparable information content** over which trade imbalances have a meaningful economic impact on the MMs

Q) Equal time buckets v.s. Equal volume buckets?



# VPIN (cont'd)

## Construction of VPIN

- 2) Buy volume and sell volume classification

: Aggregate trades over short intervals and then use the standardized price changes between the beginning and end of the interval to determine the percentage of buy and sell volume (**bulk classification**)

Calculate buy and sell volume using one-minute time bars (or the analysis can also be down using volume bars)

$\Delta p > 0$  :  $\Delta p > 0$  :  $\Delta p < 0$  :  
 buy volume  $\uparrow$  sell volume  $\downarrow$   
 sell volume  $\uparrow$  buy volume  $\downarrow$

$$V_{\tau}^B = \sum_{i=t(\tau-1)+1}^{t(\tau)} V_i \cdot Z\left(\frac{P_i - P_{i-1}}{\sigma_{\Delta P}}\right)$$

$$V_{\tau}^S = \sum_{i=t(\tau-1)+1}^{t(\tau)} V_i \cdot \left(1 - Z\left(\frac{P_i - P_{i-1}}{\sigma_{\Delta P}}\right)\right) = V - V_{\tau}^B$$

Q1) Bulk classification v.s. Tick-based classification?

Q2) Time bars v.s. Volume bars?

ex) Lee-Ready algorithm

Tick rule + Quote rule

Best ask  
 x midpoint  
 Best bid  
 buyer-initiated  
 $\Delta p > 0$  : buyer-initiated  
 $\Delta p < 0$  : seller-initiated  
 seller-initiated

# VPIN (cont'd)

## Construction of VPIN

- 3) VPIN calculation  
: Calculate the moving average of order imbalances over  $n$  buckets

$$E[|V_{\tau}^S - V_{\tau}^B|] \cong \alpha\mu$$

$$E[|V_{\tau}^S + V_{\tau}^B|] \cong \alpha\mu + 2\varepsilon$$

$$VPIN = \frac{\alpha\mu}{\alpha\mu + 2\varepsilon} \cong \frac{1}{n} \sum_{\tau=1}^n \frac{|V_{\tau}^S - V_{\tau}^B|}{V}$$

The VPIN metric is updated after each volume bucket

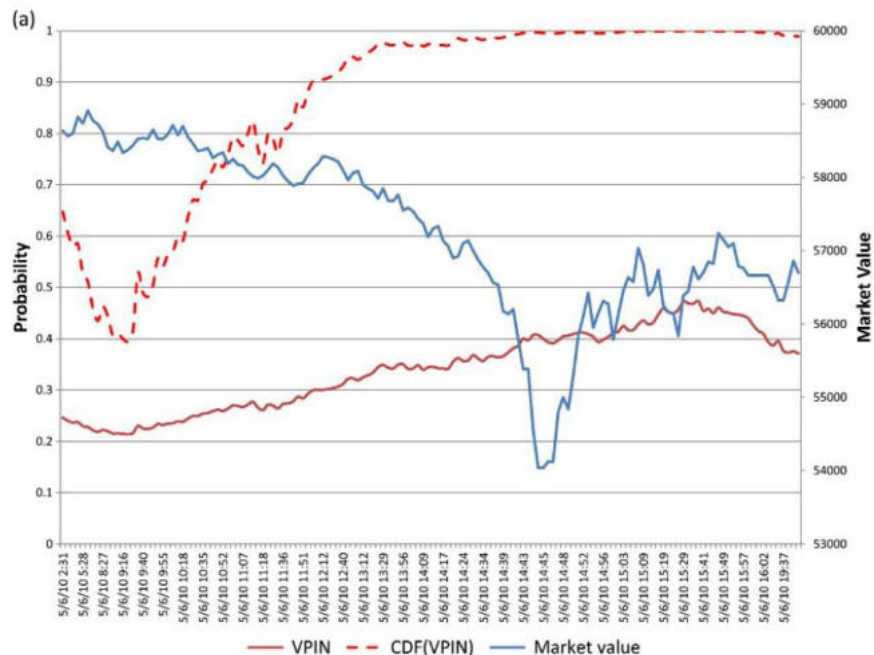
: Want the speed at which we update VPIN to mimic the speed at which information arrives at the marketplace

: Would like each update to be based on a comparable amount of information

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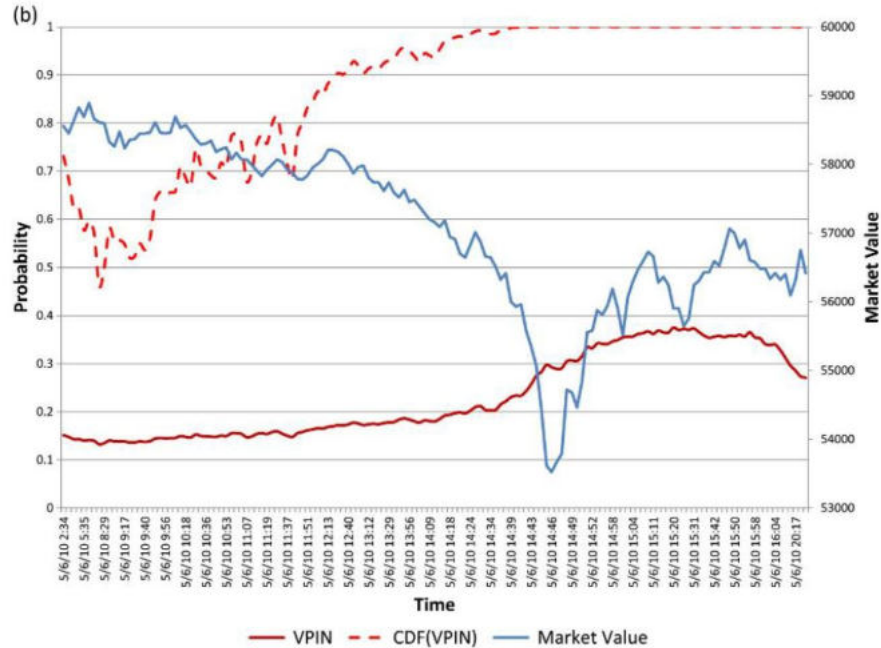
# Flash Crash and VPIN



**Figure 5-3a.** The VPIN toxicity metric during the Flash Crash. VPIN estimated on one-minute bars bulk classification

Signals an extreme level for the VPIN flow toxicity metric at least two hours before the crash (see the CDF(VPIN) dashed line crossing the 0.9 threshold)

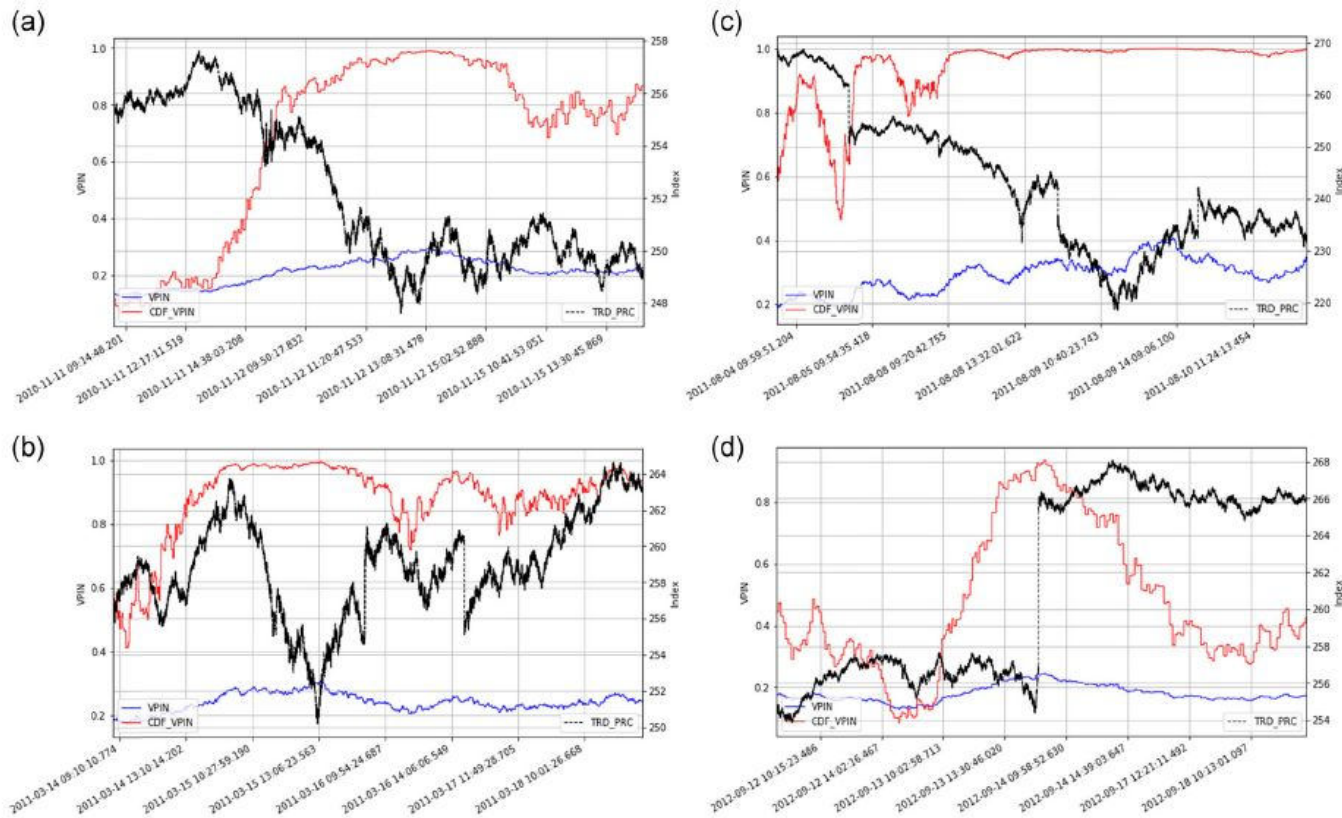
# Flash Crash and VPIN (cont'd)



**Figure 5-3b.** The VPIN toxicity metric during the Flash Crash. VPIN estimated on ten-second bars bulk classification

# **Application of VPIN : KOSPI 200 Index Futures**

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**FIGURE 1** BV-VPIN around extreme price volatilities. The left top panel (a), the left bottom panel (b), the right top panel (c), and the right bottom panel (d) describe the following episodes, respectively: (1) Expiration-day effect of KOSPI 200 options (11/11/2010), (2) Fukushima Daiichi nuclear disaster following the 2011 Tohoku earthquake and tsunami (03/11/2011), (3) downgrade of the US credit rating (08/05/2011), and (4) upgrade of the Korean credit rating (09/14/2012). BV-VPIN, Volume-Synchronized Probability of Informed Trading using bulk-volume classification; KOSPI 200, Korea Composite Stock Price Index 200 [Color figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]