Algo. Trading and Quant. Trading (2021 Fall)

Homework #4

Due date: 12/16 18:59

1. (Market making at-the-touch with filled probability)

In the setting of market making at-the-touch covered in the lecture note, we have assumed that when a matching MO arrives, the agent's LO is filled with probability 1. Now we loose this assumption.

(a) Consider the framework developed in page 21-25 in Lecture 4, where the agent ("market maker") posts only at-the-touch, but assume that when an MO arrives, and the agent is posted on the matching side of the LOB, his order is filled with probability $\rho < 1$. Derive the DPE and compute the optimal strategy in feedback form.

(b) Consider the framework developed in page 26-33 in Lecture 4, where the MM is subject to adverse selection from active traders, but assume that when an MO arrives, and the agent is posted on the matching side of the LOB, his order is filled with probability $\rho < 1$. Derive the DPE and compute the optimal strategy in feedback form.

(a)

$$H(t, X, S, Q) = \max_{\substack{l_u^+, l_u^- \\ t \le u \le T}} \mathbb{E}\left[X_T + Q_T\left(S_T - \left(\tfrac{1}{2}\Delta + \varphi Q_T\right)\right) - \phi \int_t^T Q_u^2 \, du \right]$$

HJB EQ)

$$0 = \max_{l_t^+, l_t^-} \Big[\partial_t H + \frac{1}{2} \sigma^2 \partial_{SS} H - \phi Q_t^2$$

$$+ \underline{\mathbb{1}} \; l \lambda^+ \left(H(t, X + (S + \tfrac{1}{2}\Delta) l^+, S, Q - l^+) - H() \right)$$

$$+ \underline{\mathbb{1}} \; l \lambda^- \left(H(t, X - (S - \tfrac{1}{2}\Delta) l^-, S, Q + l^-) - H() \right)$$

$$H(t, X, S, Q) = X + QS + h(t, Q)$$

$$0 = \partial_t h - \phi Q^2 + \lambda^+ \cdot e \cdot \max_{\ell^+} \left[\ell^+ \frac{\Delta}{2} + \left(h(t, Q - \ell^+) - h \right) \right] \cdot \mathbb{1}$$

$$+ \lambda^- e \max_{\ell^-} \left[\ell^- \frac{\Delta}{2} + \left(h(t, Q + \ell^-) - h() \right) \right] \cdot \mathbb{1}$$

$$h(T, Q) = -Q\left(\frac{1}{2}\Delta + \phi Q \right)$$

$$\vdots$$

$$\ell^{+,*} = \mathbb{1}\left\{ \frac{\Delta}{2} + \left[h(t, Q-1) - h(t, Q) \right] > 0 \right\} \cap \left\{ Q > \underline{Q} \right\}$$

$$\ell^{-,*} = \mathbb{1}\left\{ \frac{\Delta}{2} + \left[h(t, Q+1) - h(t, Q) \right] > 0 \right\} \cap \left\{ Q \leq \overline{Q} \right\}$$

(b).

$$H(t, X, S, \alpha, \theta) = \max_{\substack{l_u^+, l_u^- \\ t \le u \le T}} E_t \Big[$$

$$X_T + Q_T\left(S_T - \left(\tfrac{1}{2}\Delta + \phi Q_T\right)\right) - \phi \int_t^T Q_u^2 \, du \Big]$$

$\vdots$

HJB EQ)

$$0 = \partial_t H + \alpha \cdot \partial_S H + \frac{1}{2}\sigma^2 \partial_{SS} H - \zeta \alpha \partial_\alpha H$$

$$+ \frac{1}{2}\eta^2 \partial_{\alpha\alpha} H - \phi Q^2$$

$$+ \rho \cdot \lambda^+ \cdot \max_{l^+} \Big[l^+ \mathbb{1}(Q > \underline{Q}) \, E\Big[H(t, X + (S + \tfrac{1}{2}\Delta l^+) l^+, S, \alpha + \epsilon^+, Q - l^+) - H \Big]$$

$$+ \rho \lambda^+ \max_{l^+} \Big[(1 - l^+ \mathbb{1} \quad) \cdot E\Big[H(t, X, S, \alpha + \epsilon^+, Q) - H(\,) \Big] \Big]$$

$$+ \rho \lambda^{-} \max_{\ell^{-}} \Big[\ell^{-} \cdot \mathbb{1}(Q < \bar{Q}) \, \mathbb{E}\Big[H(t, X + (S - \tfrac{1}{2}\Delta \ell^{-})\ell^{-}, S, \alpha - \epsilon^{-}, Q + \ell^{-}) - H \Big] \Big]$$

$$+ \rho \lambda^{-} \max_{\ell^{-}} \Big[(1 - \ell^{-} \cdot \mathbb{1}) \, \mathbb{E}\Big[H(t, X, S, \alpha - \epsilon^{-}, Q) - H(\,) \Big] \Big]$$

$$H(T, X, S, \alpha, Q) = X + Q\left(S - \left(\tfrac{1}{2}\Delta + \phi Q\right)\right).$$

$$\vdots$$

$$H(t, X, S, \alpha, Q) = X + QS + h(t, \alpha, Q)$$

$$0 = \partial_t H - \zeta \alpha \cdot \partial_\alpha H + \tfrac{1}{2}\eta^2 \partial_{\alpha\alpha} H + \alpha Q - \phi Q^2$$

$$+ \rho \lambda^{+} \max_{\ell^{+}} \Big[\mathbb{1}(Q > \underline{Q}) \, \mathbb{E}\Big[\qquad \Big] \Big]$$

$$+ \rho \lambda^{-} \max_{\ell^{-}} \Big[\qquad \Big]$$

$+ \, p\lambda^{+} E[\qquad] + p\lambda^{-} E[\qquad]$

$\Leftrightarrow$

$$\ell^{+,*} = 1\left\{ \frac{1}{2}\Delta + E[h(t, \alpha+\epsilon^{+}, \theta+1) - h(t, \alpha+\epsilon^{+}, \theta)] > 0 \right\}$$

$$\cap \{ \alpha > \underline{\theta} \}$$

$$\ell^{-,*} = 1\left\{ \frac{1}{2}\Delta + E(h(t, \alpha-\epsilon^{-}, \theta+1) - h(t, \alpha-\epsilon^{-}, \theta)) > 0 \right\}$$

$$\cap \{ \alpha < \overline{\theta} \}$$