Algo. Trading and Quant. Trading (2021 Fall)

Homework #3

Due date: 12/09 18:59

1. (Simulation of Targeting POV-VWAP)

The agent aims to maximize the performance criterion

$$E_0\left[X_T^v + Q_T^v(S_T^v - \alpha Q_T^v) - \varphi\int_0^T (v_u - \rho\mu_u)^2 du\right]$$

Here ρ is the target ratio of the market speed, μ_t is the market trading speed, v_t is the agent's execution speed, Q_t^v denotes the agent's inventory, X_t^v is the cash process, and S_t^v is the midprice; each is assumed to satisfy the following dynamics:

$$dQ_t^v = -v_t dt$$

$$dX_t^v = (S_t^v - kv_t)v_t dt$$

$$dS_t^v = -bv_t dt + \sigma dW_t$$

and W_t is a Brownian motion.

In the double limit of first ensuring full liquidation, and then forcing the tracking penalty to infinity, the optimal strategy is given by

$$\lim_{\varphi\to\infty}\lim_{\alpha\to\infty} v_t^* = \rho\left[\mu_t - \frac{1}{T-t}\int_t^T E_t[\mu_u]du\right] + \frac{Q_t^{v*}}{T-t}$$

Furthermore, when the market's order-flow rate μ_t satisfies the following dynamics:

$$d\mu_t = -\kappa\mu_t dt + \eta_{1+N_{t-}} dN_t$$

where N_t is a Poisson process with intensity λ and $\eta_1, \eta_2, \dots$ are iid exponential random variables with mean η, then we may express the optimal trading rate as

$$\lim_{\varphi\to\infty}\lim_{\alpha\to\infty} v_t^* = \rho\left[\mu_t - \frac{1}{T-t}\left((\mu_t - \psi)\frac{1-e^{-\kappa(T-t)}}{\kappa} + \psi(T-t)\right)\right] + \frac{Q_t^{v*}}{T-t}$$

where $\psi = \frac{\lambda\eta}{\kappa}$