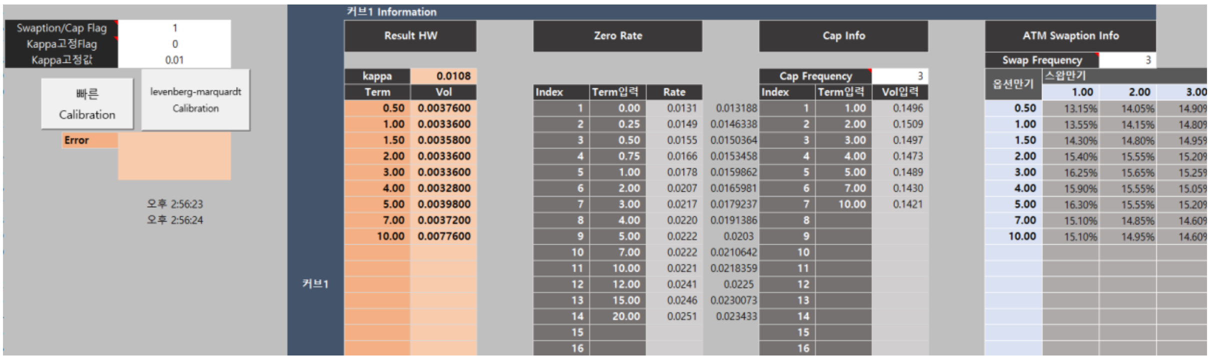
HW_Calibration 로직 설명서

빠른 Calibration, Levenberg-marquardt 알고리즘 두 가지 사용 가능



Swaption/Cap Flag	1
Kappa고정Flag	0
Kappa고정값	0.01

Result HW

kappa	0.0108
Term	Vol
0.50	0.0037600
1.00	0.0033600
1.50	0.0035800
2.00	0.0033600
3.00	0.0033600
4.00	0.0032800
5.00	0.0039800
7.00	0.0037200
10.00	0.0077600

Zero Rate

Index	Term입력	Rate	
1	0.00	0.0131	0.013188
2	0.25	0.0149	0.0146338
3	0.50	0.0155	0.0150364
4	0.75	0.0166	0.0153458
5	1.00	0.0178	0.0159862
6	2.00	0.0207	0.0165981
7	3.00	0.0217	0.0179237
8	4.00	0.0220	0.0191386
9	5.00	0.0222	0.0203
10	7.00	0.0222	0.0210642
11	10.00	0.0221	0.0218359
12	12.00	0.0241	0.0225
13	15.00	0.0246	0.0230073
14	20.00	0.0251	0.023433
15			
16			

Cap Info

Cap Frequency		3
Index	Term입력	Vol입력
1	1.00	0.1496
2	2.00	0.1509
3	3.00	0.1497
4	4.00	0.1473
5	5.00	0.1489
6	7.00	0.1430
7	10.00	0.1421
8		
9		
10		
11		
12		
13		
14		
15		
16		

ATM Swaption Info

Swap Frequency		3	
옵션만기	스왑만기		
	1.00	2.00	3.00
0.50	13.15%	14.05%	14.90
1.00	13.55%	14.15%	14.80
1.50	14.30%	14.80%	14.95
2.00	15.40%	15.55%	15.20
3.00	16.25%	15.65%	15.25
4.00	15.90%	15.55%	15.05
5.00	16.30%	15.55%	15.20
7.00	15.10%	14.85%	14.60
10.00	15.10%	14.95%	14.60

1. 빠른 Calibration

1-1 빠른 Calibration 방법론

κ는 0.002부터 0.1까지 0.002간격으로,

σ_t는 0.001부터 0.04까지 0.001간격으로 넣고 Swaption 및 Cap Pricing을 한다.

$$\text{for}(\kappa = 0.002 \ \text{ to } \ 0.1; \ \ d\kappa = 0.002)$$

$$\quad \text{for}(\sigma = 0.001 \ \text{ to } \ 0.04; \ \ d\sigma = 0.001)$$

$$\quad\quad \text{Error}(\kappa, \sigma) = \text{P}(\kappa, \sigma) - \text{P(black)}$$

$$\text{Find Min Error Point}(\kappa^*, \sigma^*)$$

찾아낸 κ, σ근방에서 위의 로직을 한 번 더 실행함

$$\text{for}(\kappa = \hat{\kappa} - 0.001 \ \text{ to } \ \hat{\kappa} + 0.001; \ \ d\hat{\kappa} = 0.0002)$$

$$\quad \text{for}(\sigma = \hat{\sigma} - 0.001 \ \text{ to } \ \hat{\sigma} + 0.001; \ \ d\hat{\sigma} = 0.0001)$$

$$\quad\quad \text{Error}(\kappa, \sigma) = \text{P}(\kappa, \sigma) - \text{P(black)}$$

$$\text{Find Min Error Point}(\kappa, \sigma)$$

1-2 Calibration 예시

Example) 다음과 같이 Swaption Vol이 주어진다고 가정하자.

	Swapmat= 1	Swapmat= 2	Swapmat= 3
Optmat= 0.5	10%	12%	14%
Optmat= 1	11%	13%	15%
Optmat= 1.5	12%	14%	16%

Calibration은 다음과 같이 실행된다.

$\mathrm{for}(\mathrm{optmat} = 0.5 \ \ \mathrm{to} \ \ 1.5)$

$\mathrm{for}(\kappa = 0.002 \ \ \mathrm{to} \ \ 0.1; \ \ d\kappa = 0.002)$

$\mathrm{for}(\sigma = 0.001 \ \ \mathrm{to} \ \ 0.04; \ \ d\sigma = 0.001)$

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$\mathrm{Error}_1(\kappa,\sigma) = \mathrm{P}(\kappa,\sigma) - \mathrm{P}(\mathrm{black}, \mathrm{Vol}_1)$

$\mathrm{Error}_2(\kappa,\sigma) = \mathrm{P}(\kappa,\sigma) - \mathrm{P}(\mathrm{black}, \mathrm{Vol}_2)$

$\mathrm{Error}_3(\kappa,\sigma) = \mathrm{P}(\kappa,\sigma) - \mathrm{P}(\mathrm{black}, \mathrm{Vol}_3)$

$Error = Error_1 + Error_2 + Error_3$

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$\mathrm{Find \ \ Min \ \ Error \ \ Point}(\kappa, \sigma_{\mathrm{optmat}})$

찾아낸 κ, σ근방에서 위의 로직을 한 번 더 실행함

2. Levenberg-marquardt

$$(\kappa^*, \sigma_t^*) = argmin_{\kappa,\sigma} \sum_{\mathrm{i}} \left\{BLACKPRICE_i^{mkt} - HWPRICE(\kappa, \sigma_t)\right\}^2$$

$$P_{\mathrm{k+1}} = P_k - (J^T J + \ \mu_k I)^{-1} J^T R(p_k)$$

(간혹 $\mu_k I$ *대신에* $\mu_k(J^T J)$*를 사용하기도 함*)

여기서 $\mathrm{J} = \begin{bmatrix} \frac{\delta r_1(p)}{\delta p_1} & \cdots & \frac{\delta r_1(p)}{\delta p_m} \\ \vdots & \ddots & \vdots \\ \frac{\delta r_n(p)}{\delta p_1} & \cdots & \frac{\delta r_n(p)}{\delta p_m} \end{bmatrix}$, $\mathrm{R(p)} = \begin{bmatrix} r_1(p) \\ \vdots \\ r_n(p) \end{bmatrix}$