# Hull White Simulation 로직

1. 이론적 기초

   HW 1Factor Process를 다음과 같이 가정한다.

$$\mathrm{dr_t} = [\theta_t - \kappa r_t]dt + \sigma_t dW$$

2. $\mathrm{e}^{\kappa t} \cdot r_t$에 대하여 $\mathrm{d}(\mathrm{e}^{\kappa t} \cdot r_t) = \kappa e^{\kappa t} r_t dt + e^{\kappa t} dr_t = \kappa e^{\kappa t} r_t dt + e^{\kappa t}[(\theta_t - \kappa r_t)dt + \sigma_t dW]$

$$= \kappa e^{\kappa t} r_t dt + e^{\kappa t}(\theta_t - \kappa r_t) + \sigma_t e^{\kappa t} dW = e^{\kappa t} \theta_t dt + \sigma_t e^{\kappa t} dW$$

   양 변을 T1에서 T2까지 적분하면

$$\mathrm{e}^{\kappa T_2} \times r_{T_2} - \mathrm{e}^{\kappa T_1} \times r_{T_1} = \int_{T1}^{T2} \theta_t e^{\kappa t} dt + \int_{T1}^{T2} \sigma_t e^{\kappa t} dW$$

$$\mathrm{r_{T2}} = \frac{e^{\kappa T_1}}{e^{\kappa T_2}} r_{T_1} + \int_{T1}^{T2} \theta_t e^{\kappa(t-T_2)} dt + \int_{T1}^{T2} \sigma_t e^{\kappa(t-T_2)}\, dW$$

   해당 모듈은 $\theta_t = 0$을 가정합니다.

   $\mathrm{r_{T_2}} = e^{-\kappa(T_2 - T_1)} \times r_{T_1} + \int_{T1}^{T2} \sigma_t e^{\kappa(t-T_2)}\, dW$

3. 따라서,

$$\mathrm{r(t)|r(s)} \sim N\left(e^{\kappa(t-s)} r(s), \int_s^t e^{-2\kappa(t-u)} du\right)$$

$$\mathrm{r(t_{i+1}) = XA(t_i)} \cdot r(t_i) + XV(t_i) \cdot \epsilon_i$$

$$\mathrm{XA(t_i)} = e^{-\kappa(t_{i+1} - t_i)}, \qquad \mathrm{XV(t_i)} = \left(\int_{t_i}^{t_{i+1}} e^{-2\kappa(t_{i+1} - t_i)} \sigma^2(\tau) d\tau\right)^{0.5}$$

4. XA, XV는 미리 Generate 해놓고 epsilon을 시뮬레이션을 통해 산출하여 shortrate path 시뮬레이션한다.