Local Volatility Calibration 로직설명서

1. Dupire Local Volatility 계산로직

$$\sigma_{\mathrm{L}}^2(K,T) = \frac{\sigma^2 + \sigma T\left(\frac{\Delta\sigma}{\Delta T} + (r-q)K\frac{\Delta\sigma}{\Delta K}\right)}{\left(1 + K\cdot d_1\cdot\frac{\Delta\sigma}{\Delta K}\sqrt{T}\right)^2 + \sigma K^2 T\left(\left(\frac{\Delta^2\sigma}{\Delta K^2}\right) - d_1\left(\frac{\Delta\sigma}{\Delta K}\right)^2\sqrt{T}\right)}$$

2. SABR Implied Volatility

$$\sigma_{\mathrm{SABRIV}} = \frac{\alpha\left\{1 + \left[\frac{(1-\beta)^2}{24}\frac{\alpha^2}{(FK)^{\frac{1-\beta}{2}}} + \frac{1}{4}\cdot\frac{\rho\beta v\alpha}{(FK)^{\frac{1-\beta}{2}}} + \frac{2-3\rho^2}{24}v^2\right]T\right\}}{(FK)^{\frac{(1-\beta)}{2}}\left[1 + \frac{(1-\beta)^2}{24}\ln^2\left(\frac{F}{K}\right) + \frac{(1-\beta)^4}{1920}\ln^4\left(\frac{F}{K}\right)\right]} \times \frac{z}{\chi(z)}$$

$$\mathrm{z} = \frac{\mathrm{v}}{\alpha}(FK)^{\frac{1-\beta}{2}}\ln\left(\frac{F}{K}\right)$$

$$\chi(\mathrm{z}) = \ln\left[\frac{\sqrt{1-2\rho z+z^2}+z-\rho}{1-\rho}\right]$$

3. Estimating α, v and ρ.

$$(\alpha^*, v^*, \rho^*) = argmin_{\alpha,\rho,v}\sum_{\mathrm{i}}\left\{\sigma_i^{mkt} - \sigma_{SABRIV}(F_i,\alpha,K_i,\rho,v)\right\}^2$$

$$s.t. \quad [-1<\rho<1, \qquad \& \ v>0]$$

4. SABR Calibration을 위한 Levenberg-Marquardt 로직

$$P_{\mathrm{k+1}} = P_k - (J^TJ + \ \mu_k I)^{-1}J^TR(p_k)$$

(간혹 $\mu_k I$ 대신에 $\mu_k(J^TJ)$를 사용하기도 함)

여기서 $\mathrm{J} = \begin{bmatrix} \frac{\delta r_1(p)}{\delta p_1} & \cdots & \frac{\delta r_1(p)}{\delta p_m} \\ \vdots & \ddots & \vdots \\ \frac{\delta r_n(p)}{\delta p_1} & \cdots & \frac{\delta r_n(p)}{\delta p_m} \end{bmatrix}$, $\mathrm{R(p)} = \begin{bmatrix} r_1(p) \\ \vdots \\ r_n(p) \end{bmatrix}$

5. SABR LocalVol

➔ 2~4를 통해 SABR Implied Vol을 추정하고 1 Dupire 식을 통해 산출