Consider a dividend paying stock. On an ex-dividend date, the stock prices before and after the dividend payment are related by

$$S(t+) = S(t-)(1-y) - c, \tag{1}$$

where y and c are the relative and absolute dividends, respectively. Assuming constant risk free rate r and dividend yield q, the forward stock price in the presense of multiple dividends is

$$F(t,T) = S(t)e^{(r-q)(T-t)} \prod_{t\le t_i \le T} (1-y_i) - \sum_{t\le t_j\le T} c_j e^{(r-q)(T-t_j)} \prod_{t_j<t_i\le T}(1-y_i), \tag{2}$$

where t_i's and t_j's are the relative and discrete dividend payment times.

Define the multiplicative forward factor between two times as

$$M(t,T) = e^{(r-q)(T-t)} \prod_{t\le t_i\le T}(1-y_i), \tag{3}$$

and denote the price of a European call option written on the stock as a function of discrete dividends,

$$\mathrm{Call}(C_i) \equiv \mathrm{Call}\left(S(0), K; C_i\right), \tag{4}$$

the sensitivity of the European call price with respect to the discrete dividends in the vanishing limit is [1]

$$\begin{aligned} & \frac{\partial^k \mathrm{Call}}{\partial C_{i_1}\cdots\partial C_{i_k}}(\mathbf{0}) \\ = \quad & (-)^k \frac{\partial^k \mathrm{Call}_{BS}}{\partial S^k}\left(S(0)e^{-\sigma^2\sum_{p=1}^k T_{i_p}}, K, T\right) \prod_{p=1}^k M(0,T_{i_p})^{-1} e^{-\sigma^2\sum_{p=2}^k (p-2)T_{i_p}}, \end{aligned} \tag{5}$$

where

$$\mathrm{Call}_{BS}\left(S(0), K, T\right) = P(0,T)(S(0)M(0,T)N(d_+) - KN(d_-)), \tag{6}$$

is the vanilla call option price without any discrete dividend, with $P(0,T)$ as the discount factor between time 0 and T,

$$d_\pm = \frac{1}{\sigma\sqrt{T}}\log\left(\frac{S(0)M(0,T)}{K}\right) \pm \frac{1}{2}\sigma\sqrt{T}, \tag{7}$$

and $N(x) = (2\pi)^{-1/2}\int_{-\infty}^x e^{-u^2/2}du$ is the standard normal cumulative distribution function.

Following [1], define a proxy European option price as

$$\mathrm{Proxy}\left(C_1,\cdots,C_k\right) = \mathrm{Call}_{BS}\left(S^*\left(C_1,\cdots,C_k\right), K^*\left(C_1,\cdots,C_k\right), T\right), \tag{8}$$

where, up to second order,

$$S^*\left(C_1,\cdots,C_k\right) = S(0) + \sum_{i=1}^k\left(a_iC_i + \frac{1}{2}a_{ii}C_i^2\right) + \sum_{i=1}^k\sum_{j>i} a_{ij}C_iC_j, \tag{9}$$

and

$$K^*\left(C_1,\cdots,C_k\right)=K+\sum_{i=1}^{k}\left(b_iC_i+\frac{1}{2}b_{ii}C_i^2\right)+\sum_{i=1}^{k}\sum_{j>i}^{k}b_{ij}C_iC_j. \tag{10}$$

We can determine the coefficients by two relations. First, the sensitivity of the proxy in the vanishing dividend limit should coincide with that of the real option price, *i.e.*,

$$\frac{\partial^k \mathrm{Proxy}}{\partial C_{i_1}\cdots\partial C_{i_k}}\left(\mathbf{0}\right)=\frac{\partial^k \mathrm{Call}}{\partial C_{i_1}\cdots\partial C_{i_k}}\left(\mathbf{0}\right). \tag{11}$$

For both sides of the above equation, we have closed form expressions. The other relation is the call-put parity,

$$P(0,T)\left(S^*M(0,T)-K^*\right)=S(0)-P(0,T)K-\sum_{i=1}^{k}P(0,T_i)C_i. \tag{12}$$

References

[1] Arnaud Gocsei and Fouad Sahel, *Analysis of the sensitivity to discrete dividends: A new approach for pricing vanillas*, arXiv:1008.3880.