Swaption on Forward-Starting Swap "Replication"?

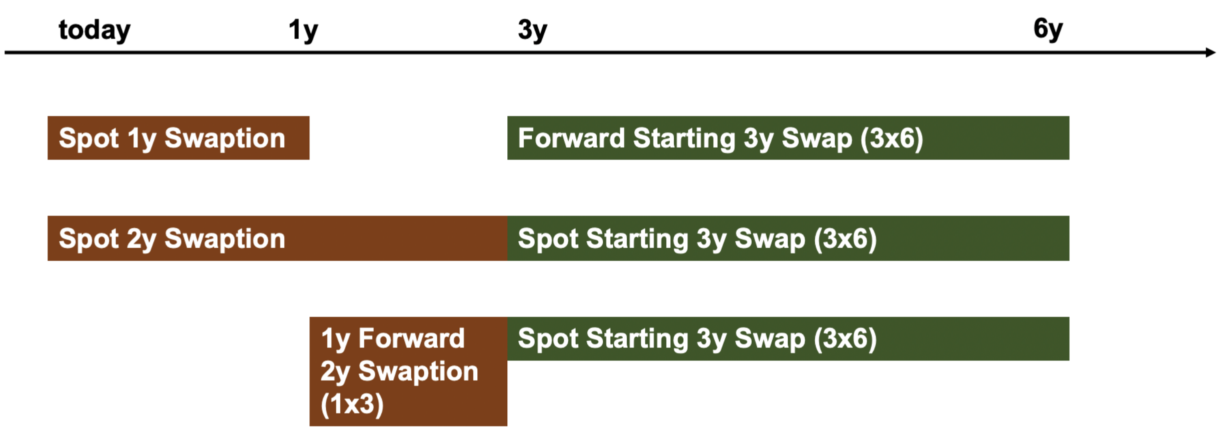
Lately I was thinking about forward-starting swaptions vs. options on forward-starting swaps a bit, and I started wondering about the following:

Suppose we are at time T_0 (today) and we want to price a swaption that expires in T_1 and entitles us to enter into a swap which lives from T_2 to T_3. Clearly, I work in the setting $T_0 < T_1 < T_2 < T_3$.

I was asking myself whether it is reasonable (*possible*?) to approximate (*replicate*?) the price of above mentioned option by looking at a combination of the prices of:

- a spot (T_0) starting swaption with expiry T_2 that delivers the (then, i.e., at T_2) spot-starting swap and
- a forward-starting swaption that lives from T_1 to T_2 and delivers the (then, i.e., at T_2) spot-starting swap

I have drawn a little picture to illustrate what I mean ($T_0 = 0$ (today), T_1 is 1 year from today, T_2 is 3 years from today, and T_3 is 6 years from today):



I intuitively have the feeling that it's not working out, and my first line of thought is that it's because the swap underlying the three options is not 100% the same (although it's always the 3x6 swap, the forward starting swap seems more uncertain to me compared to the then-spot starting swap, as the optionality ends after 1y and not after 3y). Maybe someone can provide a little more information and/or some formulae that would confirm my conjecture?

1 Answer

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The way to think about this is an option to enter a basket of two swaps. The basket contains these positions:

P_1: a long position in a swap that starts at T_1 and finishes at T_3

P_2: a short position in a swap that starts at T_1 and finishes at T_2.

This basket replicates the payoff of the forward starting swap. Denoting $S(\tau_1, \tau_2)$ as the swap rate for the swap starting at τ_1 and ending at τ_2, and $A(\tau_1, \tau_2)$ as the corresponding Annuity (PVBP), then the payoff (for a payer) can be written as:

$$\max\left\{\underbrace{A(T_1, T_3)(S(T_1, T_3) - K)}_{P_1} - \underbrace{A(T_1, T_2)(S(T_1, T_2) - K)}_{P_2}, 0\right\}$$

This is effectively a spread option between two swap rates (obviously with some weights). The present value of the spread option therefore depends on the joint distribution between the two swap rates, $S(T_1, T_2)$ and $S(T_1, T_3)$. So you will not be able to perfectly replicate this payoff with vanilla swaptions, though some (upper / lower bound) approximations may be possible.

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answered Dec 18, 2020 at 9:30

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