Financial Markets

Exercise Set 1

1. [Exercises 1.B.1 and 1.B.2 in Mas-Colell et al. (copied below)]

Prove the following:

Proposition If $\succsim$ is rational then:

i. $\succ$ is both *irreflexive* ($x \succ x$ never holds) and transitive.
ii. $\sim$ is reflexive ($x \sim x \; \forall x \in X$), transitive, and symmetric (if $x \sim y$ then $y \sim x$).
iii. if $x \succ y \succsim z$, then $x \succ z$.

2. Lexicographic preference

Consider the lexicographic preference relation $\succsim$ over $\mathbb{R}^2$ defined by $(x_1, x_2) \succsim (y_1, y_2)$ if $x_1 > y_1$ or $x_1 = y_1$ and $x_2 \geq y_2$.

i. Show that $\succsim$ is transitive.
ii. Show that $\succsim$ is not continuous.

Hint: Consider the sequence $(x_1^n, x_2^n) = (1 - 1/n, 1)$ and $(y_1, y_2) = (1, 0)$.

3. [Exercise 3.C.6 in Mas-Colell et al. (copied below) + Plotting]

Suppose that in a two-commodity world, the consumer's utility function takes the form

$$U(x) = \left[\alpha_1 x_1^\rho + \alpha_2 x_2^\rho\right]^{1/\rho}.$$

This utility function is known as the *constant elasticity of substitution* (or *CES*) utility function.

i. Show that when $\rho = 1$, indifference curves become linear.
ii. Show that as $\rho \to 0$, this utility function comes to represent the same preferences as the (generalized) Cobb-Douglas utility function $U(x) = x_1^{\alpha_1} x_2^{\alpha_2}$.
iii. Plot the lower contour set for $U(x) = 1$ for $\rho = 1/2$, when $\alpha_1 = \alpha_2 = 1$.
iv. Plot indifference curves for $\rho = 1/2$ and $\rho = 2$, when $\alpha_1 = \alpha_2 = 1$.
v. What happens to the shape of the indifference curves as $\rho \to -\infty$? You can use a plot for this.

Hint: For ii., use L'Hôpital's rule and the fact that monotonic transformations of a utility function represents the same preference relation by considering $log(U(x))$.