Indhold

1 | Opgave 1

Prove the following:

Proposition: If $\succeq$ is rational then:

1. **$\succ$ is both irreflexive and transitive**
   For the irreflexive part completeness is used, which is given by

   $$\forall x, y \in X \Rightarrow x \succeq y \quad \text{or} \quad x \preceq y$$

   In this given case we have two x which gives

   $$\forall x \in X \Rightarrow x \succeq x$$

   which is the reflexive property, since it applies $\forall x$ and it can not be both $\succ$ and $\succeq$ it must be a contradiction and therefore it is seen that $x \succ x$ never holds.
   Next, we prove transitivity
   If it is rational we know that by transitivity we have: $\forall x, y, z \in X$: if $x \succsim y$ and $y \succsim z$ then $x \succsim z$. It is possible to only operate with $\succ$, and then we have $x \succ y$ and $y \succ z$ so that $x \succ z$, which means that $\succ$ is transitive.

2. **$\sim$ is reflexive ($x \sim x$ $\forall x \in X$), transitive and symmetric (if $x \sim y$ then $y \sim x$).**

   Reflective: By completeness we have, from the exercise above, that $x \succsim x$ $\forall x \in X$ this implies by definition of indifference that $x \sim x$

   Transitive: Suppose $x \sim y$ and $y \sim z$ we know that by transitivity we have: $\forall x, y, z \in X$: if $x \succsim y$ and $y \succsim z$ then $x \succsim z$. we also, by definition of indifference, have the inequalities $x \precsim y$ and $y \precsim z$ then $x \precsim z$. Since $x \succsim z$ and $x \precsim z$, it is therefore known that $x \sim z$, which means that $\sim$ is transitive.

   Symmetric: By completeness we again have, that $x \succsim y$ or $y \succsim x$ $\forall x, y \in X$, so when only operating with $\sim$ this implies by definition of indifference that $x \sim y$ and $y \sim x$

3. **If $x \succ y \succeq z$, then $x \succ z$**
   Since $\succeq$ is rational, $x \succsim z$. If we assume $z \succsim x$. Then since $y \succsim z$, then by transitivity, $y \succsim x$. But we also have that $x \succ y$, which is a contradiction. Therefore $x \succ z$.

2 | Opgave 2

Consider the lexicographic preference relation $\succsim$ over $\mathbb{R}^2$ defined by $(x_1, x_2) \succsim (y_1, y_2)$ if $x_1 > y_1$ or $x_1 = y_1$ and $x_2 \geq y_2$
Proposition: If $\succeq$ is rational then:

1. **Show that $\succsim$ is transitive.**
   To show $X \succsim Y \succsim Z \implies X \succsim Z$ let first $Y \succsim Z$ be fulfulled, then $y_1 > z_1$ or $y_1 = z_1$ and $y_2 \geq z_2$. Since $x_1 > y_1$ or $x_1 = y_1$ and $x_2 \geq y_2$ is fulfilled, then $x_1 > y_1 \geq z_1$ or $x_1 = y_1 \geq z_1$ and $x_2 \geq y_2 \geq z_2$ implying that $X \succsim Z$

2. **Show that $\succsim$ is not continuous**
   Let $(x_1^n, x_2^n) = (1 - \frac{1}{n}, 1)$ and $(y_1, y_2) = (1, 0)$ then it is clear that $X \succ Y$, but taking the limit
   $$\lim_{n\to\infty} X = (1, 0) = Y$$
   Thus $X \succ Y$ but $n \to \infty \implies X \sim Y$

3 | Opgave 3

Suppose that in a two-commodity world, the consumer's utility function takes the form:

$$U(x) = [\alpha_1 x_1^\rho + \alpha_2 x_2^\rho]^{1/\rho}$$

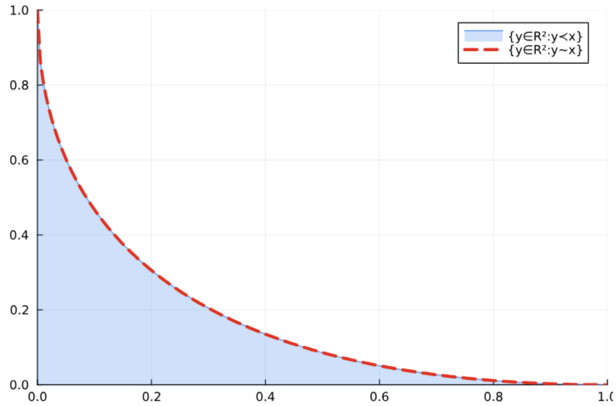
This utility function is known as the constant elasticity of substitution (or CES) utility function.

3.0.1 Show that when $\rho = 1$ indifference curves become linear.

When $\rho = 1$ then $U(x) = [\alpha_1 x_1^1 + \alpha_2 x_2^1]^{1/1} \Rightarrow \alpha_1 x_1 + \alpha_2 x_2$. Solves x_2 we get: $x_2 = -\frac{\alpha_1}{\alpha_2} x_1$ and thus $U(x)$ become linear.

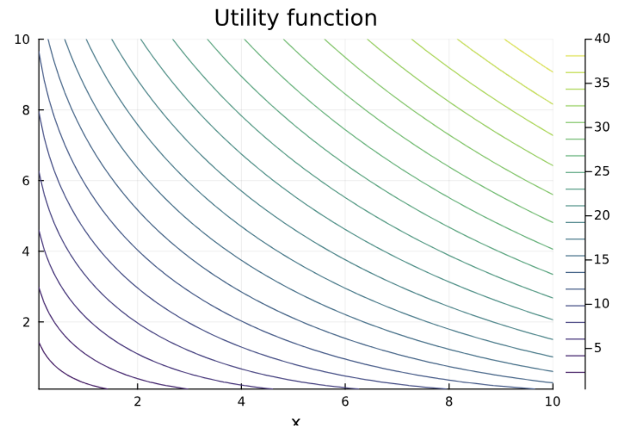
3.0.2 Show that as $\rho \to 0$ this utility function comes to represent the same preferences as the Cobb-Douglas utility function $U(x) = x_1^{\alpha_1} x_2^{\alpha_2}$

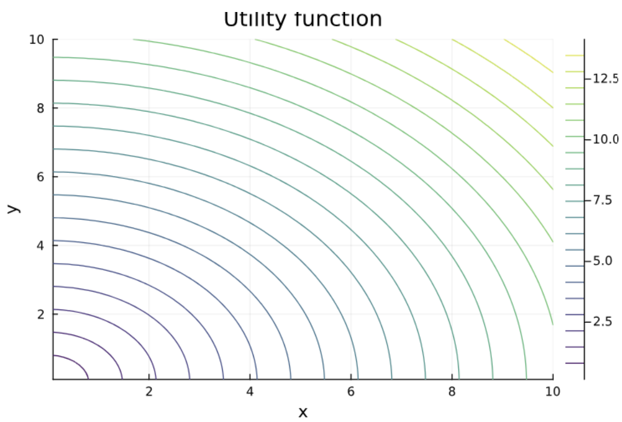
3.0.3 Plot the lower contour set for $U(x) = 1$ for $\rho = \frac{1}{2}$ when $\alpha_1 = \alpha_2 = 1$



3.0.4 Plot indifference curves for $\rho = \frac{1}{2}$ and $\rho = 2$ when $\alpha_1 = \alpha_2 = 1$

3.0.5 What happens to the shape of the indifference curves as $\rho \to -\infty$? you can use plot for this





Kode (står i kommentar i overleaf

plot!(X,y,line = :dash, color = :red, linewidth = 3, label="yR^2:y x")

5)

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Figur 3.1: opgave V