Optimal Hedge Ratio and Hedging Effectiveness of Stock Index Futures

Evidence from India

Executive Summary

In a free capital mobile world with increased volatility, the need for an optimal hedge ratio and its effectiveness is warranted to design better hedging strategy with future contracts. The conventional wisdom suggest a naïve strategy of 1:1 position; to effectively hedge one unit of spot position is to hold one unit of future contract. This strategy failed to deliver as the spot and future prices behave differently. Recent advances in time series analysis comes in hand to resolve this problem with alternative model specification and methods. This study analyses four competing models names, simple ordinary least squares (OLS), vector autoregression model (VAR), vector error correction model (VECM) and a class of multivariate generalized autoregressive conditional heteroscedastic model (GARCH). With multivariate GARCH model we can estimate the time varying hedge ratio whereas the other models give a single point estimate.

Two sets of data are used in this study. For developing the model, daily data on NSE Stock Index Futures and S&P CNX Nifty Index for the time period from 4th September 2000 to 4th August 2005 and for out of sample validation daily data from 5th August 2005 to 19th September 2005 is considered. The effectiveness of the optimal hedge ratios derived from these competing models are examined in two ways. First, the mean returns of the hedged and the unhedged position and second, the average variance reduction between the hedged and the unhedged position with the hedge ratios for 1, 5, 10 and 20 days horizon.

The results clearly vote for the time varying hedge ratio derived from the multivariate GARCH model with higher mean return and higher average variance reduction across hedged and unhedged position. Even though not outperforming the GARCH model, the simple OLS based strategy performs well at shorter time horizons in terms of average variance reduction. The potential use of this multivariate GARCH model cannot be sublined because of its estimation complexities. This method bears some additional benefits over the other simple techniques in terms of mean returns and variance reduction. Sophisticated models are warranted to cut into the complexities of the dynamics of a volatile world.

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Saumitra N Bhaduri
S. Raja Sethu Durai

Madras School of Economics, Gandhi Mandapam Road, Chennai 600 025, India

1. Introduction

The effective use of futures contract in hedging decisions has become focus and center of debate on finding out an optimal hedge ratio and hedging effectiveness in empirical financial research. The recent advances in the time series econometrics has also helped to rethink on the conventional methods adopted so far and revamped entire gamut of empirical research to effectively determine the hedge ratio.

The conventional wisdom suggests about the optimal hedge ratio is to have 1:1 position; to effectively hedge one unit of spot position is to hold one unit of future contract. This strategy often called as naïve-hedging strategy failed to deliver as the movement between the spot and futures prices are not synchronized. This has brought a renewed interest at the theoretical level by the works of Johnson (1960) and Stein (1961). They adopted a portfolio approach to determine the optimal hedging strategy based on the expected-utility maximization that boils down to minimum variance analysis as a special case. Following this Ederington (1979) developed a measure of hedging effectiveness as a percent reduction in the variance between the hedged and the unhedged returns. Until then the optimal hedge ratio has been estimated from a simple regression between the historical data on realized returns of spot and futures prices and the R-squared of that regression has been considered as the measure of hedging effectiveness. Kroner and Sultan (1993) criticized the hedge ratio obtained from the regression method, as it becomes a biased one if there exists a cointegrating relationship between the spot and the futures return. They proposed a vector error correction model to estimate the hedge ratio.

Two criticisms have been suggested against these empirical methods. First, the simple regression hedge ratio has been derived from the unconditional second moments while the actual minimum variance hedge ratio is based on the conditional second moments. Second, a constant hedge ratio duly not considers the fact that the joint distribution of spot and futures prices varies over time (Cecchetti *et al*, 1988). Recent advances in time series econometric techniques have tried to address this problem. A multivariate GARCH method developed by Bollerslev *et al* (1988) has used to estimate the time varying hedge ratio by considering the conditional variance and covariance of the spot and futures returns. Following this many empirical studies have compared the constant hedge ratio with the time varying hedge ratio in perpetuating the return and the variance reduction (Holmes, 1995, Park and Switzer, 1995, Chou *et al*, 1996, Yang and Allen, 2005)

This study focuses on estimating optimal hedge ratio for stock index futures in India and comparing its hedging effectiveness. Daily data on NSE Stock Index Futures and S&P CNX Nifty Index for the time period from 4^{th} September 2000 to 4^{th} August 2005 has been considered for this study. Two important aspects contribute the significance of this study. First, compared to other countries the futures market, particularly that of stock index futures in India is fairly a new market in its earlier stage of development. Second, as an emerging market India attracts more foreign investments that induces volatility in the market. Effective hedging strategy would be highly imperative towards efficient risk management in a more volatile environment. This paper organizes as follows: Section 2 gives a brief overview of the methodology used in estimating the hedge ratio. Section 3 provides the strategy for calculating hedging effectiveness. Section 4 presents a description of the data used in this study. Section 5 discusses the empirical results and the final section concludes with a summary.

2. Methodology for calculating Hedge Ratio

This study focuses on four different methods for estimating the hedge ratio and test its effectiveness for both in-sample and out-sample data with 1, 5, 10 & 20 days horizon.

2.1 The Regression Method

A conventional method of finding an optimal hedge ratio is using simple ordinary least square (OLS) estimation of the following linear regression model:

$$r_{st} = \alpha + \beta r_{ft} + \varepsilon_t \tag{1}$$

where r_{st} and r_{ft} are the spot and futures returns for period t. β provides an estimate of the optimal hedge ratio.

2.2 The Bivariate VAR Method

A major disadvantage of the simple regression method described above is that there exists a possibility for the residuals being autocorrelated. To overcome this the bivariate vector autoregressive (VAR) model has be used. The optimal lag length for spot and futures returns *m, n* are decided by iterating for each lag until the autocorrelation in the residuals are fully eliminated from the system.

$$r_{st} = \alpha_s + \sum_{i=1}^{m} \beta_{si}\, r_{st-i} + \sum_{j=1}^{n} \gamma_{si}\, r_{ft-i} + \varepsilon_{st} \tag{2}$$

$$r_{ft} = \alpha_f + \sum_{i=1}^{m} \beta_{fi}\, r_{st-i} + \sum_{j=1}^{n} \gamma_{fi}\, r_{ft-i} + \varepsilon_{ft} \tag{3}$$

After estimating the system of equation, the residual series are generated to calculate the hedge ratio. Let var (ε_{st}) = σ_s, var (ε_{ft}) = σ_f and cov (ε_{st}, ε_{ft}) = σ_{sf}, then the minimum variance hedge ratio is $h^* = \sigma_{sf} / \sigma_f$

2.3 The Error Correction Method

If the level series of spot and future index are non-stationary and integrated of order one then the following vector error correction model has been used estimate the hedge ratio.

$$r_{st} = \alpha_s + \sum_{i=1}^{m} \beta_{si}\, r_{st-i} + \sum_{j=1}^{n} \gamma_{si}\, r_{ft-i} + \lambda_s Z_{t-1} + \varepsilon_{st} \tag{4}$$

$$r_{ft} = \alpha_f + \sum_{i=1}^{m} \beta_{fi}\, r_{st-i} + \sum_{j=1}^{n} \gamma_{fi}\, r_{ft-i} + \lambda_f Z_{t-1} + \varepsilon_{ft} \tag{5}$$

where $Z_{t-1} = S_{t-1} - \delta F_{t-1}$ is error correction term with (1-δ) as cointegrating vector and λ_s, λ_f as adjustment parameters. Same procedure of generating the residual series and calculate the variance, covariance of the series to estimate the minimum variance hedge ratio depicted in the bivariate VAR model has been followed.

2.4 The Multivariate GARCH Method

As most of the financial time series data posses ARCH effects, the hedge ratio from the VAR models has turned out to be extraneous. To take care of ARCH effects in the residuals of error correction model, a VEC multivariate GARCH model of Bollerslev *et al* (1988) has been deployed. The main advantage of this model is that it simultaneously model the conditional variance and covariance of two interacted series. So we can able retrieve the time varying hedge ratios based on the conditional variance and covariance of the spot and the futures prices. A standard MGARCH (1,1) model is expressed as follows

$$\begin{bmatrix} h_{ss} \\ h_{sf} \\ h_{ff} \end{bmatrix}_t = \begin{bmatrix} c_{ss} \\ c_{sf} \\ c_{ff} \end{bmatrix}_t + \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_s^2 \\ \varepsilon_s \varepsilon_f \\ \varepsilon_f^2 \end{bmatrix}_{t-1} + \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix} \begin{bmatrix} h_{ss} \\ h_{sf} \\ h_{ff} \end{bmatrix}_{t-1} \quad (6)$$

where h_{ss}, h_{ff} are the conditional variance of the errors (ε_{st} , ε_{ft}) from the mean equations. In this paper the mean equation is the bivariate vector error correction model. As the model has 21 parameters to be estimated, Engle and Wooldridge (1988) proposed a restricted version of the above model with $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ matrix have only diagonal elements. This Diagonal Vec (DVEC) model is expressed as

$$h_{sst} = c_{ss} + \alpha_{11}\varepsilon_{st-1}^2 + \beta_{11}h_{sst-1} \quad (7)$$

$$h_{sft} = c_{sf} + \alpha_{22}\varepsilon_{st-1}\varepsilon_{ft-1} + \beta_{22}h_{sft-1} \quad (8)$$

$$h_{fft} = c_{ff} + \alpha_{33}\varepsilon_{ft-1}^2 + \beta_{33}h_{fft-1} \quad (9)$$

The time varying hedge ratio has been calculated as the ratio between covariance of spot and futures price with variance of futures price. So h_{sft} / h_{fft} will be the time varying hedge ratio.

3. Estimating Hedging Effectiveness

The performance of the hedging strategies developed in the previous section has been examined by finding the hedging effectiveness of each strategy. To compare, the un-hedged portfolio is constructed as the composition of shares with same proportion held in the spot price index. The hedged portfolio is constructed with the combination of both the spot and the futures contract held. The hedge ratios estimated from each strategy determines the number of futures contract. The hedging effectiveness is calculated by the variance reduction in the hedged portfolio compared to that of un-hedged portfolio. The return of un-hedged and hedged portfolios are simply expressed as follows:

$$R_{unhedged} = S_{t+1} - S_t \tag{10}$$

$$R_{hedged} = (S_{t+1} - S_t) - h^*(F_{t+1} - F_t) \tag{11}$$

where $R_{unhedged}$ and R_{hedged} are return on un-hedged and hedged portfolio. S_t and F_t are logged spot and futures prices at time t with h^* is optimal hedge ratio. Similarly the variance of the un-hedged and hedged portfolio is expressed as

$$Var_U = \sigma_s^2 \tag{12}$$

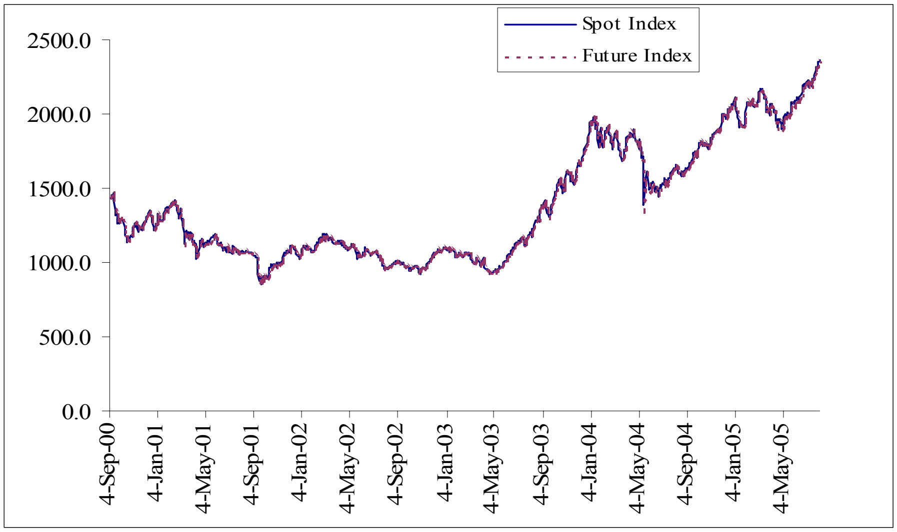
$$Var_H = \sigma_s^2 + h^{*2}\sigma_f^2 - 2h^*\sigma_{sf} \tag{13}$$

where Var_U and Var_H are variance of un-hedged and hedged portfolios with σ_s, σ_f and σ_{sf} are standard deviations of spot and futures price and covariance between them respectively. Ederington (1979) proposed a measure of hedging effectiveness as the percentage reduction in variance of the hedged and the un-hedged portfolios. The hedging effectiveness is calculated as $\frac{Var_U - Var_H}{Var_U}$. This measure is calculated for both in-sample and out-sample data with 1, 5, 10 & 20 days horizon for evaluation.

4. Data

This study uses Daily data on NSE Stock Index Futures and S&P CNX Nifty Index for the time period from 4th September 2000 to 4th August 2005. The data are collected from the NSE website (www.nseindia.com). Figure 1 graphs the data on spot index and the future index.

Figure 1: Data on Spot Index and Future Index



Let S be the log of S&P CNX Nifty Index and F is the log of NSE Stock Index Futures. The standard unit root test establishes that the series S and F are non-stationary at levels and the return series (r_s and r_f) are stationary. Table 1 provides the unit root test results.

Table 1: Unit Root Test

Variable	ADF Statistics	Variable	ADF Statistics
S	0.290	r_s	-26.57**
F	0.246	r_f	-26.47**

** denote significance at 1% level.

The cointegration test is conducted using the Johansen's (1991) maximum likelihood method. We have used four lags in short-run specification of the model as suggested by Akaike information, Schwartz, Hannen-Quin criteria and likelihood ratio test. The results of cointegration tests are presented in Table 2. The trace and max-eigen value statistics suggest existence of one cointegrating vector at 1 % level of significance. The cointegrating vector normalized with respect to S show that the long run cointegrating coefficients with respect to F is statistically significant.

Table 2: Cointegration Analysis

Hypothesis	Eigenvalue	λ_{TRACE}	95 % Critical Value	λ_{MAX}	95 % Critical Value
$r = 0$	0.0475	60.96^{**}	12.53	60.21^{**}	11.44
$r \leq 1$	0.0006	0.75	3.84	0.75	3.84

r is valid cointegration vectors. ** denote significance at 1% level.

The corresponding unrestricted cointegrating vector normalized on S is given as

S	F
1	-1.000185 (0.00008)

Standard errors are in the parentheses.

5. Empirical Results

In this section, we calculate the optimal hedge ratio from four different models described in section 2 and compare the measures of hedging effectiveness of these hedging models.

5.1. Estimates of Optimal Hedge Ratio

First, the optimal hedge ratio is calculated from a simple OLS regression (1). Table 3 reports the results from the regression model. The optimal hedge ratio is 0.928642.

Table 3: OLS Regression Model

	Coefficient
α	0.00003 (0.000104)
β	0.928642** (0.0070)
$\boldsymbol{R^2}$	**0.933977**

Standard errors are in the parentheses.
** denote significance at 1% level.

To calculate the optimal hedge ratio from a Bivariate VAR model, we estimated the equations (2) and (3) with four lags and the results are presented in Table 4.

Table 4: Estimates of Bivariate VAR Model

Equation (2)	Coefficient	Equation (3)	Coefficient
α_s	0.0003 (0.0003)	α_f	0.0003 (0.0004)
β_{s1}	0.3849** (0.117)	β_{f1}	0.6840** (0.121)
β_{s2}	0.0380 (0.121)	β_{f2}	0.2779* (0.126)
β_{s3}	0.0044 (0.120)	β_{f3}	0.1045 (0.125)
β_{s4}	0.1540 (0.114)	β_{f4}	0.2359* (0.119)
γ_{s1}	-0.2437* (0.112)	γ_{f1}	-0.5765** (0.116)
γ_{s2}	-0.1731 (0.117)	γ_{f2}	-0.3885** (0.122)
γ_{s3}	0.0524 (0.116)	γ_{f3}	-0.0267 (0.121)
γ_{s4}	-0.0614 (0.110)	γ_{f4}	-0.1367 (0.115)
$\boldsymbol{R^2}$	**0.04737**	$\boldsymbol{R^2}$	**0.04922**

In the parentheses are standard errors. * (**) denote significance at 5 % and 1% level respectively.

The optimal hedge ratio is derived as $h^* = \sigma_{sf} / \sigma_f$. Where σ_{sf} is covariance ($\varepsilon_s\ \varepsilon_f$) and σ_f is variance (ε_f) with ε_s and ε_f are the residuals from the equations (2) and (3). Table 5 presents the estimates of optimal hedge ratio from the Bivariate VAR Model.

Table 5: Optimal Hedge Ratio from the Bivariate VAR Model

	Values
Covariance ($\varepsilon_s\ \varepsilon_f$)	0.000195
Variance (ε_f)	0.000209
h*	**0.932921**

To calculate the optimal hedge ratio from a Vector Error Correction (VEC) model, we estimated the equations (4) and (5) with four lags and the results are presented in Table 6. From the results we see that the speed of adjustment parameter λ_f is significant only in the futures equation with a positive value, which signifies that the future index is converging to movements in spot index and not the vice versa.

Table 6: Estimates of Vector Error Correction Model

Equation (4)	Coefficient	Equation (5)	Coefficient
α_s	0.0003 (0.0003)	α_f	0.0003 (0.0004)
β_{s1}	0.2898* (0.130)	β_{f1}	0.4780** (0.135)
β_{s2}	-0.0408 (0.130)	β_{f2}	0.1070 (0.135)
β_{s3}	-0.0555 (0.1258)	β_{f3}	-0.0253 (0.130)
β_{s4}	0.1100 (0.117)	β_{f4}	0.1408 (0.121)
γ_{s1}	-0.1508 (0.126)	γ_{f1}	-0.3750** (0.130)
γ_{s2}	-0.0974 (0.122)	γ_{f2}	-0.2244* (0.135)
γ_{s3}	0.1098 (0.125)	γ_{f3}	0.0976 (0.126)
γ_{s4}	-0.0195 (0.113)	γ_{f4}	-0.0459 (0.117)
λ_s	0.1352 (0.082)	λ_f	0.2931** (0.085)
R^2	**0.04943**	R^2	**0.05817**

In the parentheses are standard errors. * (**) denote significance at 5 % and 1% level respectively.

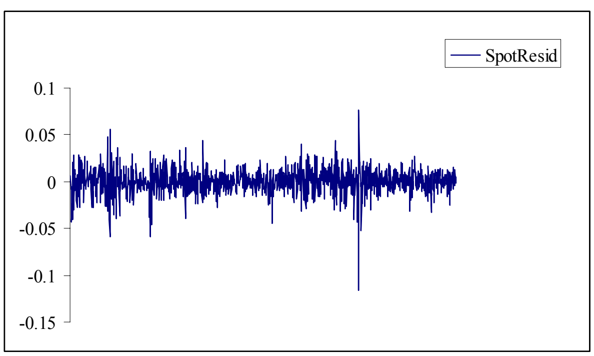
The optimal hedge ratio is derived as $h^* = \sigma_{sf} / \sigma_f$. Where σ_{sf} is covariance ($\varepsilon_s\ \varepsilon_f$) and σ_f is variance (ε_f) with ε_s and ε_f are the residuals from the equations (4) and (5). Table 7 presents the estimates of optimal hedge ratio from the VEC Model.

Table 7: Optimal Hedge Ratio from the VEC Model

	Values
Covariance (ε_s ε_f)	0.000194
Variance (ε_f)	0.000207
h*	**0.937400**

To examine the efficiency of both the Bivariate VAR model and the VEC Model, the features of the residuals are examined. Figure 2 plots the residuals from equation (2) and (3) and Figure 3 plots the residuals from equation (4) and (5). It clearly shows the presence of ARCH effects. This is also confirmed by the analysis proposed by McLeod and Li (1983), which examine the sample autocorrelation functions of the mean equation. In that the squared residuals from the estimated mean equation is checked for a significant Q-statistic at a given lag. The results, which show a high significance for the Q-statistic for each given lag, are reported in Table 8 for Bivariate VAR model and Table 9 for VEC model.

Figure 2: Residual series from Spot and Future equation in Bivariate VAR model.



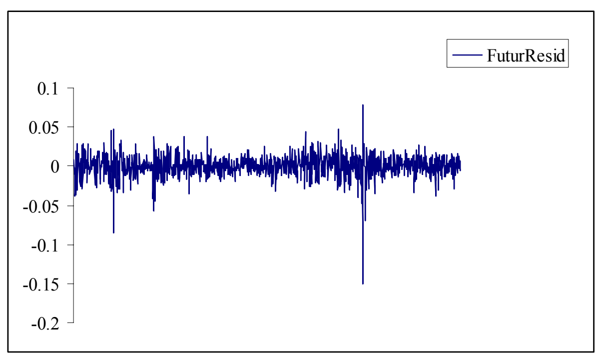
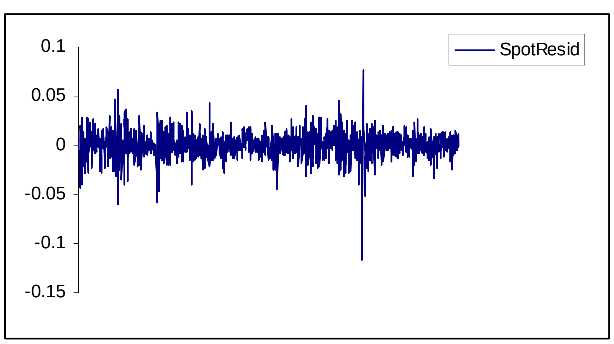


Figure 3: Residual series from Spot and Future equation in VEC model.



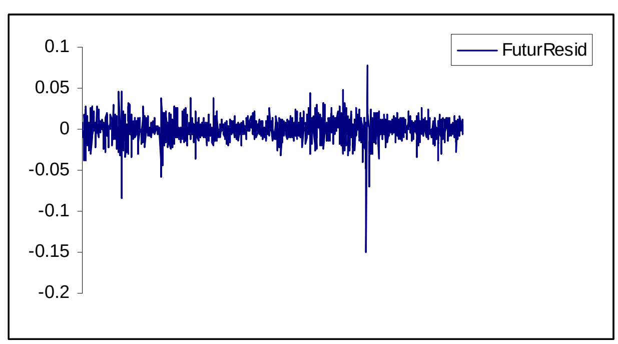


Table 8: Squared residuals from the BivariateVAR Model

Equation (2)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.521	0.521	336.53	0.000
		2	0.156	-0.159	366.76	0.000
		3	0.051	0.056	370.00	0.000
		4	0.107	0.108	384.12	0.000
		5	0.100	-0.015	396.50	0.000
		6	0.051	-0.002	399.68	0.000
		7	0.039	0.034	401.55	0.000
		8	0.071	0.046	407.88	0.000
		9	0.114	0.062	423.98	0.000
		10	0.064	-0.043	429.02	0.000
		11	0.010	-0.006	429.13	0.000
		12	0.021	0.035	429.69	0.000
		13	0.055	0.016	433.43	0.000
		14	0.078	0.040	441.07	0.000
		15	0.025	-0.046	441.86	0.000
		16	-0.004	0.003	441.88	0.000
		17	0.001	-0.001	441.88	0.000
		18	0.007	-0.019	441.94	0.000
		19	0.031	0.046	443.17	0.000
		20	0.020	-0.017	443.66	0.000

Equation (3)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.426	0.426	224.58	0.000
		2	0.092	-0.109	235.06	0.000
		3	0.040	0.054	237.07	0.000
		4	0.111	0.101	252.31	0.000
		5	0.083	-0.010	260.78	0.000
		6	0.039	0.009	262.63	0.000
		7	0.018	0.003	263.04	0.000
		8	0.061	0.054	267.67	0.000
		9	0.151	0.121	296.14	0.000
		10	0.046	-0.091	298.80	0.000
		11	0.012	0.039	298.97	0.000
		12	0.015	-0.003	299.25	0.000
		13	0.051	0.020	302.47	0.000
		14	0.069	0.045	308.47	0.000
		15	0.015	-0.045	308.74	0.000
		16	0.004	0.023	308.77	0.000
		17	0.002	-0.021	308.78	0.000
		18	-0.002	-0.032	308.78	0.000
		19	0.025	0.061	309.57	0.000
		20	0.004	-0.044	309.59	0.000

Table 9: Squared residuals from the VEC Model

Equation (4)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.515	0.515	327.84	0.000
		2	0.145	-0.163	353.77	0.000
		3	0.047	0.062	356.47	0.000
		4	0.105	0.104	370.16	0.000
		5	0.093	-0.021	380.94	0.000
		6	0.049	0.008	383.89	0.000
		7	0.038	0.028	385.65	0.000
		8	0.069	0.045	391.60	0.000
		9	0.116	0.071	408.29	0.000
		10	0.066	-0.045	413.70	0.000
		11	0.008	-0.009	413.78	0.000
		12	0.018	0.033	414.17	0.000
		13	0.057	0.022	418.18	0.000
		14	0.077	0.033	425.56	0.000
		15	0.022	-0.044	426.18	0.000
		16	-0.006	0.005	426.22	0.000
		17	-0.001	-0.004	426.22	0.000
		18	0.008	-0.014	426.30	0.000
		19	0.036	0.050	427.90	0.000
		20	0.025	-0.016	428.66	0.000

Equation (5)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.401	0.401	198.81	0.000
		2	0.073	-0.104	205.40	0.000
		3	0.030	0.047	206.50	0.000
		4	0.104	0.098	220.02	0.000
		5	0.070	-0.016	226.05	0.000
		6	0.034	0.017	227.50	0.000
		7	0.015	0.000	227.80	0.000
		8	0.054	0.048	231.37	0.000
		9	0.153	0.132	260.65	0.000
		10	0.051	-0.081	263.92	0.000
		11	0.008	0.028	264.01	0.000
		12	0.008	-0.002	264.10	0.000
		13	0.053	0.027	267.66	0.000
		14	0.065	0.036	272.88	0.000
		15	0.013	-0.038	273.07	0.000
		16	0.002	0.019	273.08	0.000
		17	0.001	-0.019	273.08	0.000
		18	0.000	-0.030	273.08	0.000
		19	0.032	0.062	274.37	0.000
		20	0.010	-0.035	274.50	0.000

The residual plots and Q-Statistic from the squared residual series denotes the presence of ARCH effects. This implies that the assumption of constant variance over time and the estimation of constant hedge ratios may be inappropriate. The estimation of time-varying variances and covariances and as a consequence time-varying hedge ratios based on a GARCH model are therefore expected to give better results. We estimated the Diagonal VEC multivariate GARCH model of Engle and Wooldridge (1988). The estimated results of the DVEC model specified in equations (7)-(9) are presented in Table 10.

Table 10: Estimates of the DVEC-GARCH Model

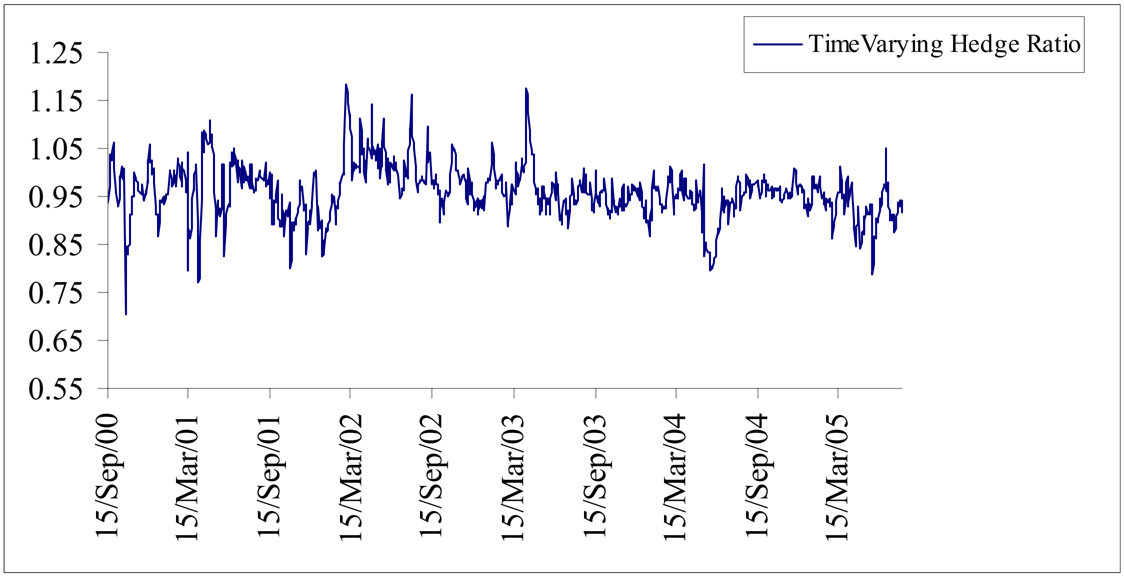
	Coefficient
C_{ss}	0.0000156** (0.0000017)
C_{sf}	0.0000146** (0.0000015)
C_{ff}	0.0000148** (0.0000015)
α_{11}	0.7746040** (0.0156636)
α_{22}	0.7883055** (0.0146405)
α_{33}	0.7917804** (0.0139856)
β_{11}	0.1293227** (0.0126181)
β_{22}	0.1183799** (0.0115152)
β_{33}	0.1182199** (0.0106835)

In the parentheses are standard errors.

** denote significance at 1% level.

Figure 4 depicts the time varying hedge ratio derived from the DVEC GARCH model. The average value of the time varying hedge ratio series is 0.95885.

Figure 4: Estimates of Time Varying Hedge Ratio from DVEC GARCH Model



The optimal hedge ratio estimated from four different models are listed in Table 11.

Table 11: Estimates of Optimal Hedge Ratio

Method	$h*$
OLS	0.92864
BVAR	0.93292
VECM	0.93740
DVEC-GARCH	0.95885

5.2. Estimates of Hedging Effectiveness

Daily data on NSE Stock Index Futures and S&P CNX Nifty Index for the time period from 4th September 2000 to 4th August 2005 has been used for constructing the optimal hedge ratio and test its effectiveness with 1, 5, 10 & 20 day horizon. For out of sample validation, data from 5th August 2005 to 19th September 2005 has been used.

Traditionally the hedging effectiveness is equal to R-squared of the OLS regression. But to compare across competing strategies, we consider a standard method explained in section 3 to test the hedging effectiveness for the optimal hedge ratios derived from all the models. A hedging strategy is effective only if the mean return from the strategy is higher than the competing strategies and it reduced a significant portion of the variance with respect to its unhedged strategy. The mean returns and average variance reduction has been calculated for non overlapping 1, 5, 10 & 20 day horizon for both within sample and out of sample validations. Table 12 gives within sample mean returns and Table 13 gives the average variance reduction for different hedging ratios.

Table 12: Mean Return for within sample

Method	***h****	***1-Day***	***5-Day***	***10-Day***	***20-Day***
OLS	0.92864	0.041%	0.040%	0.037%	0.031%
BVAR	0.93292	0.041%	0.040%	0.037%	0.032%
VECM	0.93740	0.042%	0.041%	0.038%	0.032%
DVEC-GARCH	0.95885	0.043%	0.041%	0.038%	0.033%
DVEC-GARCH	Time Varying	**0.044%**	**0.042%**	**0.038%**	**0.034%**

The table clearly establishes the fact that the time varying hedge ratio from the DVEC-GARCH specification has given a higher mean returns compared to any other derived optimal hedge ratios.

Table 13: Average Variance Reduction for within sample

Method	***h****	***1-Day***	***5-Day***	***10-Day***	***20-Day***
OLS	0.92864	**93.36%**	**83.67%**	89.69%	91.41%
BVAR	0.93292	93.36%	83.60%	89.68%	91.41%
VECM	0.93740	93.35%	83.52%	89.66%	91.41%
DVEC-GARCH	0.95885	93.26%	83.07%	89.52%	91.36%
DVEC-GARCH	Time Varying	92.96%	83.24%	**89.71%**	**91.44%**

The variance reduction depicts a slightly different picture, for smaller time horizons the optimal hedge ratio derived from OLS is performing better than the other competing strategies whereas for longer time horizons it is DVEC-GARCH time varying hedge ratio performs better. But the out of sample mean returns and average variance reduction vote for time varying hedge ratio from the DVEC-GARCH specification. Table 14 and Table 15 present the results.

Table 14: Mean Return for out of sample

Method	*h**	*1-Day*	*5-Day*	*10-Day*	*20-Day*
OLS	0.92864	0.029%	0.024%	0.024%	0.024%
BVAR	0.93292	0.028%	0.020%	0.020%	0.020%
VECM	0.93740	0.027%	0.015%	0.015%	0.015%
DVEC-GARCH	0.95885	0.025%	-0.008%	-0.008%	-0.008%
DVEC-GARCH	Time Varying	**0.037%**	**0.026%**	**0.025%**	**0.025%**

Table 15: Average Variance Reduction for out of sample

Method	*h**	*1-Day*	*5-Day*	*10-Day*	*20-Day*
OLS	0.92864	91.92%	93.20%	93.27%	93.16%
BVAR	0.93292	91.84%	93.11%	93.19%	93.07%
VECM	0.93740	91.75%	93.00%	93.09%	92.97%
DVEC-GARCH	0.95885	91.25%	92.43%	92.56%	92.44%
DVEC-GARCH	Time Varying	**92.36%**	**93.35%**	**93.37%**	**93.18%**

6. Conclusion

The conventional naïve strategy of 1:1 position for hedging has faced several criticisms as the spot and future prices behave differently. In a free capital mobile world with an increased volatility the need for an effective hedging strategy is highly imperative for the fund managers to optimize. This paper tries to give an overview of the competing models in calculating optimal hedge ratio. The effectiveness of these strategies is compared with mean returns and average variance reduction with respect to the unhedged position. Daily data on NSE Stock Index Futures and S&P CNX Nifty Index for the time period from 4th September 2000 to 4th August 2005 has been considered for developing the optimal hedge ratio and the data from 5th August 2005 to 19th September 2005 has been considered for out of sample validation. The results clearly establishes that the time varying hedge ratio derived from DVEC-GARCH model gives a higher mean returns compared to other counterparts. On the average variance reduction front the DVEC-GARCH model gives better performance only in the long time horizons compared to the simple OLS method that scores well in the short time horizons. The DVEC-GARCH model imparts a slight edge over the OLS in the out of sample validation. This DVEC-GARCH model cannot be ignored for its modeling complexities as it provides an improved outcome in terms of effective hedging against simple naïve and other strategies. However, from a cost of computation point of view, given the complexities involved in estimating the DVEC-GARCH model, one can equally consider the simple OLS strategy that performs well at the shorter time horizons.

7. References

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