Investigations - Pairs Project

Physical model and asumptions

model the bowl as

- rigid body of mass m
- geometrically spherical with radius R
- centre of mass C displaced a from geometric centre O
- displacement lies along a ~~fixed~~ body fixed symmetry axis
- rolling without slipping on horizontal plane
- gravity $\vec{g} = -g\hat{z}$
- no air resistence
- bowl remains upright

These assumptions isolate dominate first order effect

Kinematics of rolling without slipping

let

- P be the intanteneous point of contact
- ω be the angular velocity
- v be the velocity of geometric centre

rolling without slipping gives

$$v = \omega \times \vec{R} \rightarrow \vec{R} = Rz$$ vector from P to O

since bowl rolls upright $|\omega| = \frac{v}{R}$

Forces and Torques

Forces acting:

- gravity, $m\vec{g}$ acting at C
- normal reaction N at P
- static friction f at P

Torque

$r_{pc} = R\hat{z} + a$

$\tau = r_{pc} \times mg$ since g vertical

$|\tau| = mga$

Angular Momentum of Rolling Bowl

$L = I\omega$

$I \approx \frac{2}{5} m R^2$ (for near spherical bowl)

Rotational Equation of Motion

Eulers Equation

$$\frac{dL}{dt} = \tau$$

$$\frac{d}{dt}(I\omega) = mga\,\hat{n}$$ perpendicular to direction of motion

$$I\frac{d\omega}{dt} = mga$$

since I and $|\omega|$ are approx constant, this equation describes precession of ω, not a change in its magnitude.

Torque and Precession

$$\left|\frac{d\omega}{dt}\right| = \Omega|\omega|$$

$$I\omega\Omega = mga$$

$$\Omega = \frac{mga}{I\omega}$$

$$\Omega = \frac{mgaR}{IV}$$

Direction of motion

$\theta(t)$ angle between velocity and inital x direction

$$\frac{d\theta}{dt} = \Omega = \text{constant}$$

The direction of motion rotates uniformly in time

Radius of Curvature

$$K = \frac{1}{\rho} = \frac{1}{V}\frac{d\theta}{dt}$$

$$\rho = \frac{V}{\Omega}$$

$$\Rightarrow \quad \rho = \frac{I V^2}{m g a R}$$

Parametric Equation of the Path

$$\dot{x} = V \cos(\Omega t)$$

$$\dot{y} = V \sin(\Omega t)$$

$$\Rightarrow \quad x(t) = \frac{V}{\Omega} \sin(\Omega t)$$

$$y(t) = \frac{V}{\Omega}(1 - \cos(\Omega t))$$