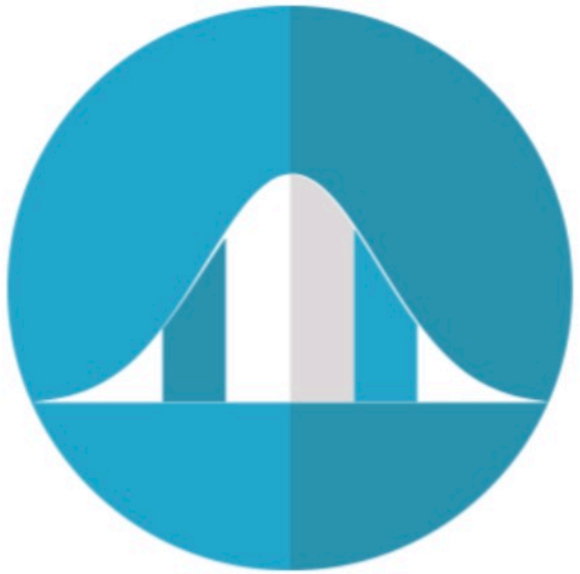

# Statistics Session-5

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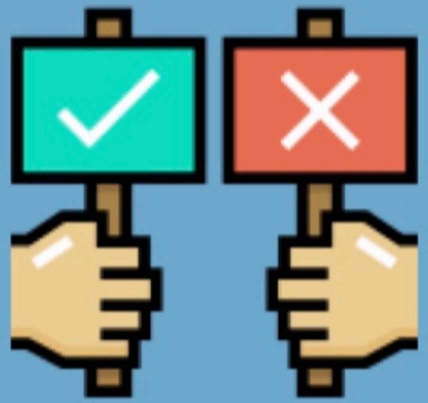
# Did you finish Statistics (Basic Concepts of Hypothesis Testing) pre-class activity?



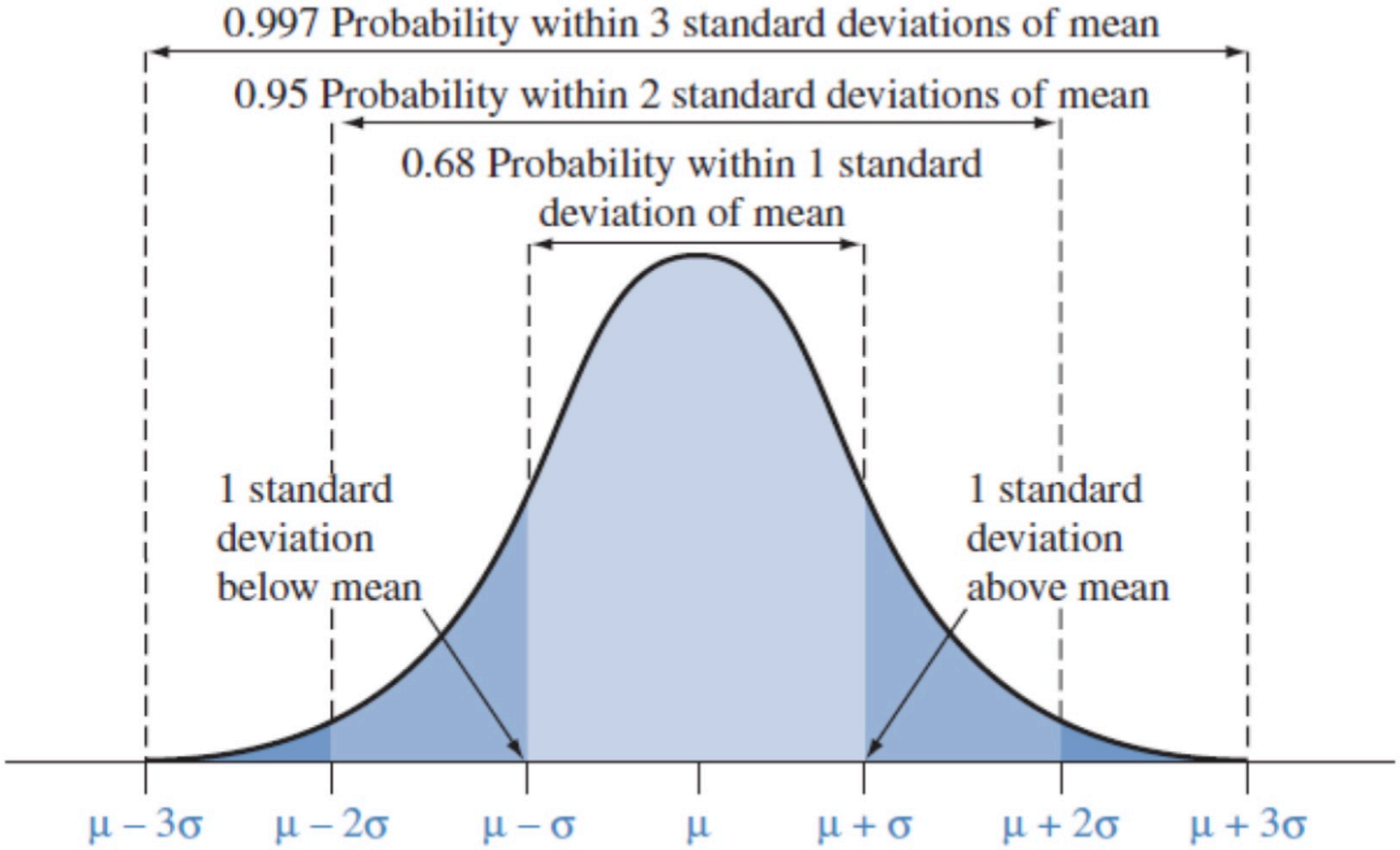
Students choose an option

Pear Deck Interactive Slide
Do not remove this bar

**No Multiple Choice Response**
You didn't answer this question

# Table of Contents

# The Empirical Rule



1

# Steps for Performing a Significance Test

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# The Steps of a Significance Test

A significance test has five steps.

- **Step 1** : Assumptions
- **Step 2** : Hypotheses
- **Step 3** : Test Statistic
- **Step 4** : P-Value
- **Step 5** : Conclusion

# Step 1: Assumptions

★ First, specify the variable and parameter. The assumptions commonly pertain to the method of data production (randomization), the sample size, and the shape of the population distribution.

Z-Test Assumptions

1. Sample is selected random
2. Observations are independent
3. Population σ is known or 30-observations sample

# Step 2 : Hypotheses

★ Each significance test has two hypotheses about a population parameter: the **null hypothesis** and an **alternative hypothesis.**

**The null hypothesis** is a statement that the parameter takes a particular value.

**The alternative hypothesis** states that the parameter falls in some alternative range of values.

# Step 2 : Hypotheses

★ The null hypothesis, denoted $H_0$, is the claim that is initially assumed to be true.

★ The alternative hypothesis, denoted by $H_a$, is an assertion that is contrary to $H_0$.

★ These are mutually exclusive (and usually exhaustive)

★ Possible conclusions from hypothesis- testing analysis are reject $H_0$ or fail to reject $H_0$.

# Step 2 : Hypotheses

The courtroom provides an analogy for a significance test.

$H_0$ — $H_0$ is that the defendant is innocent.

$H_a$ — $H_a$ is that the defendant is guilty.

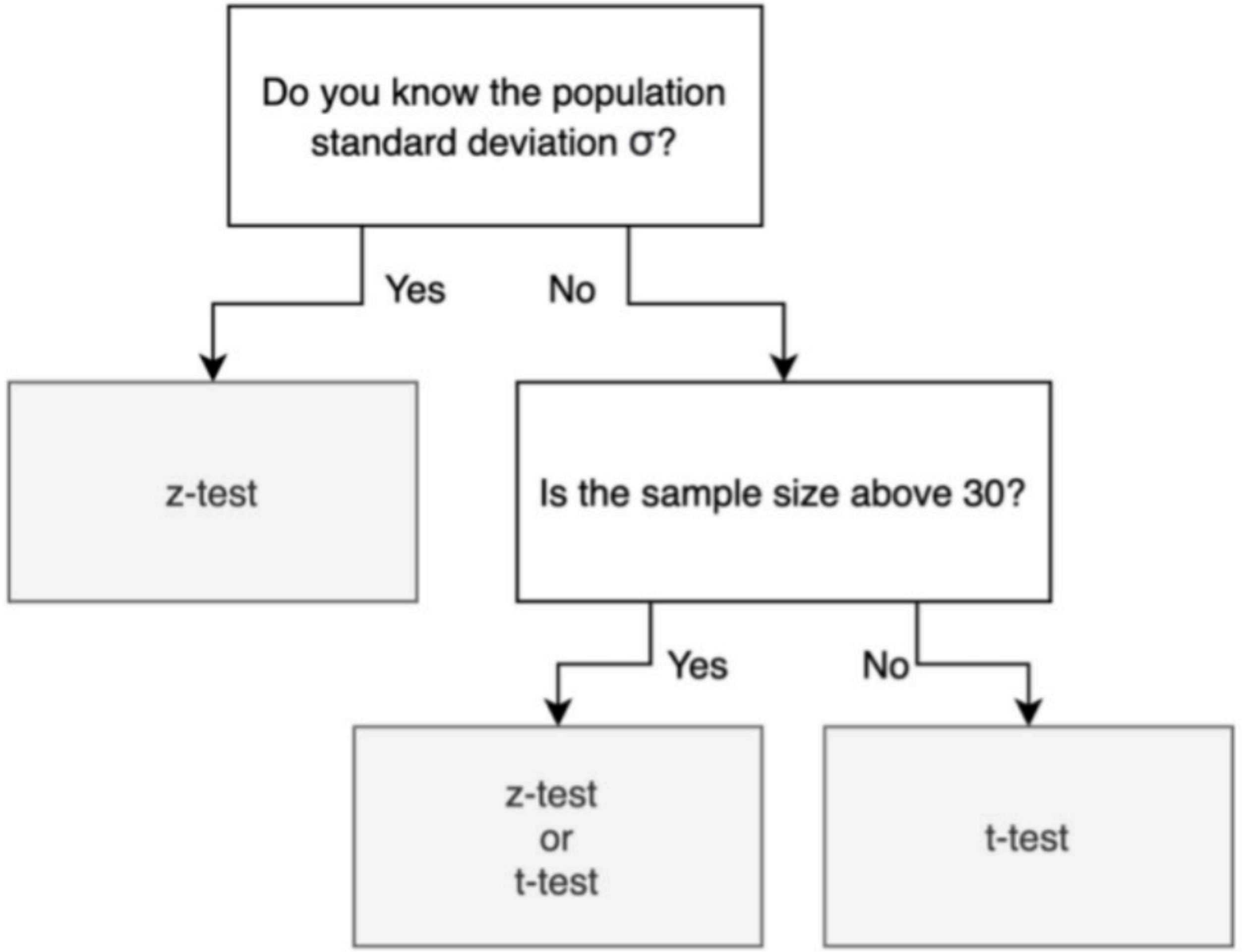
The jury presumes $H_0$ to be true unless the evidence (data) suggests otherwise.

# Step 2 : Hypotheses

- In the language of statistics convicting the defendant is called **rejecting the null hypothesis** in favor of the alternative hypothesis.
  - That is, the jury is saying that there is enough evidence to conclude that the defendant is guilty (i.e., there is enough evidence to support the alternative hypothesis).
- If the jury acquits it is stating that there is **not enough evidence to support the alternative hypothesis**.
  - Notice that the jury is not saying that the defendant is innocent, only that there is not enough evidence to support the alternative hypothesis.
  - That is why statisticians don't like to say that we "accept" the null hypothesis (prefer retain or do not reject).

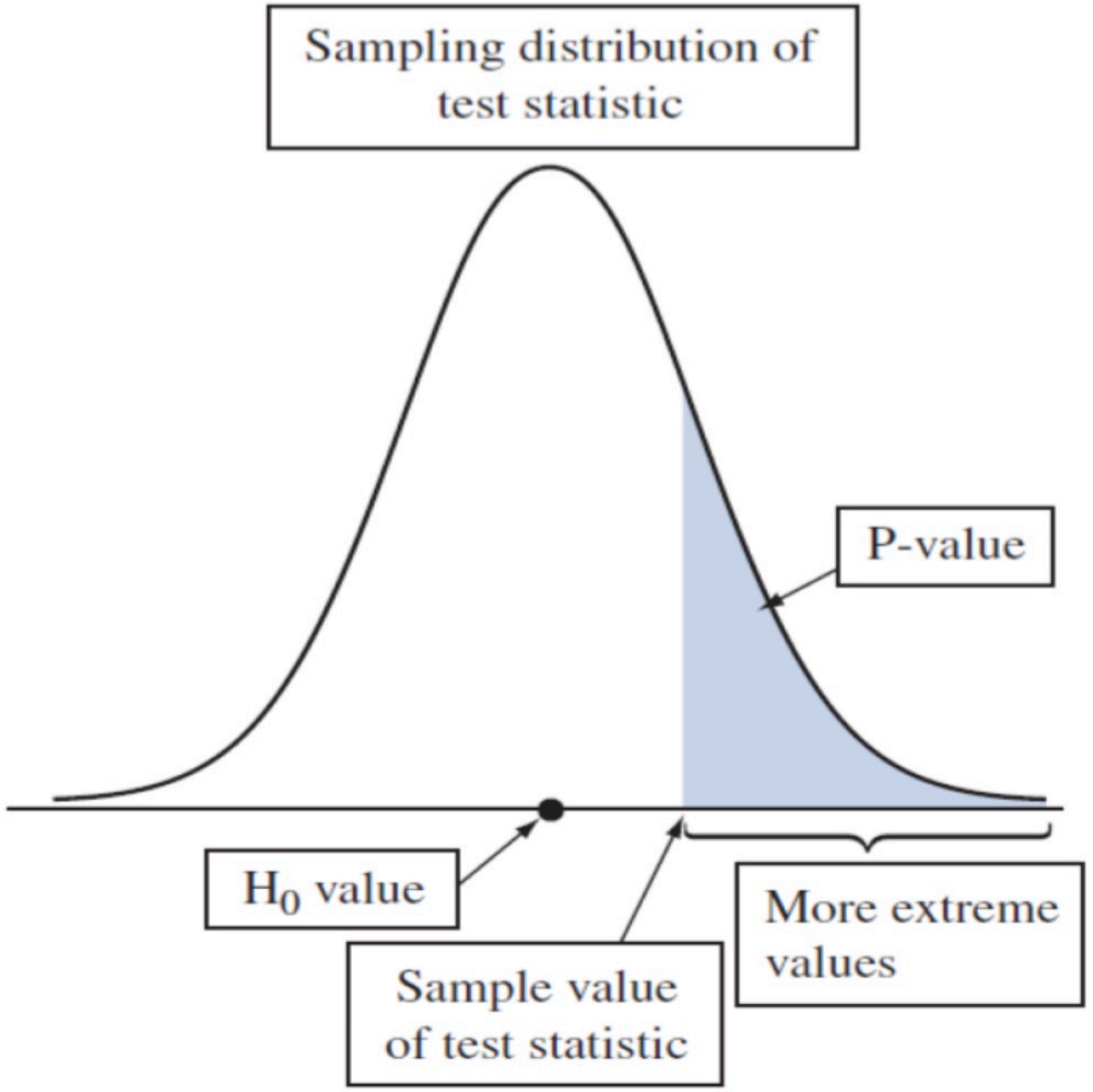
# Step 3 : Test Statistic

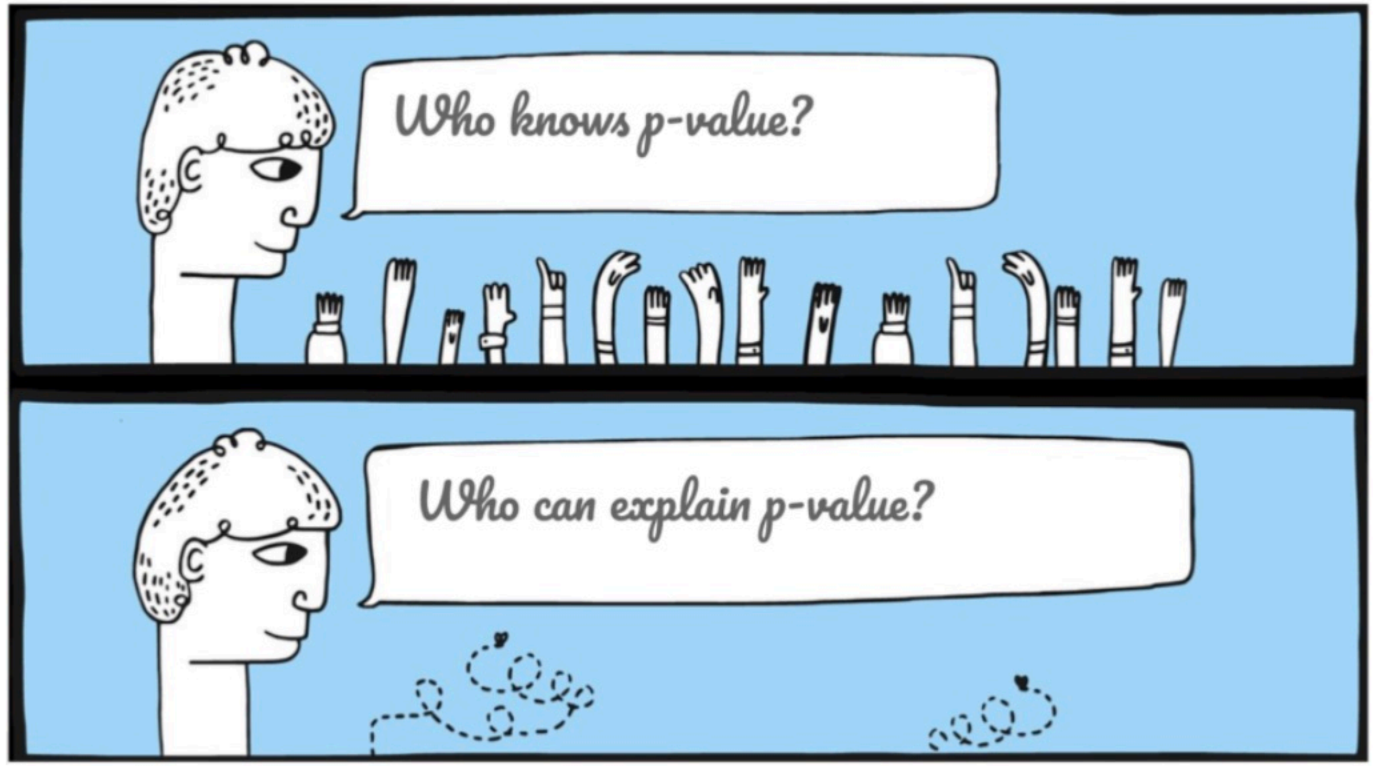
★ The test statistic measures distance between the point estimate of the parameter and its null hypothesis value, usually by the number of standard errors between them.



# Step 4 : P-Value

★ The P-value is the probability that the test statistic takes the observed value or a value more extreme if we presume $H_0$ is true. Smaller P-values represent stronger evidence against $H_0$.






Which gives stronger evidence against the null hypothesis, a P-value of 0.20 or of 0.01? Why?

Students choose an option



No Multiple Choice Response

You didn't answer this question

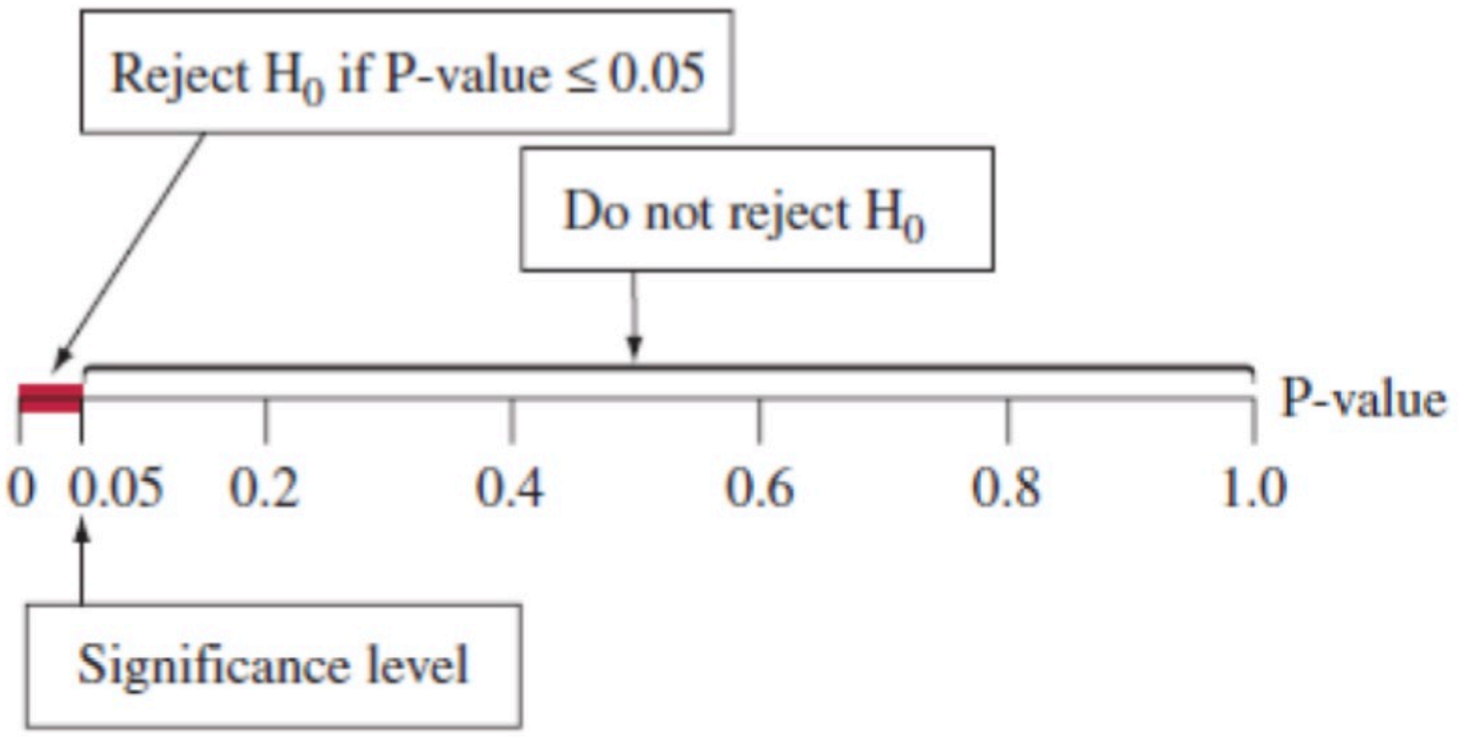
# Step 5 : Conclusion

★ Report and interpret the P-value in the context of the study. Based on the P-value, make a decision about $H_0$ (either reject or do not reject $H_0$) if a decision is needed.

NULL HYPOTHESIS → Fail to Reject the Null / Reject the Null

# Significance Level

The significance level is a number such that we reject $H_0$ if the P-value is less than or equal to that number. In practice, the most common significance level is 0.05.



I. The P-value is greater than the significance level.
II. The P-value is computed from the significance level.
III. The P-value is the parameter in the null hypothesis.
IV. The P-value is a test statistic.
V. The P-value is a probability.

## In hypothesis testing, which of the following statements is always true?

Students choose an option



**No Multiple Choice Response**
You didn't answer this question

# 2 Type I and Type II Errors

# What happens when we mess up?


# Type I and Type II Errors

- ★ A *type I error* consists of rejecting the null hypothesis $H_0$ when it was true.
  - E.g., we convict an innocent person
  - E.g., we say a fair coin is biased.
- ★ A *type II error* involves not rejecting $H_0$ when $H_0$ is false.
  - E.g., we fail to convict a guilty person.
  - E.g., we say a biased coin is fair.

**Note:** ***Type I*** errors are usually considered to be more serious

# Decision Table

## Decision

conclusion from statistical analysis

| Reality: the true state of nature | Accept the Null | Reject the Null |
| --- | --- | --- |
| Null hypothesis is **true** | Correct Conclusion | Type I Error<br>**reject a true null hypothesis** |
| Null hypothesis is **false** | Type II Error<br>**accept a false null hypothesis** | Correct Conclusion |

# Classification Error Metrics

| | Predicted class POSITIVE (spam) | Predicted class NEGATIVE (normal) | |
|---|---|---|---|
| Actual class POSITIVE (spam) | TRUE POSITIVE (TP) 320 | FALSE NEGATIVE (FN) 43 | *Recall* $= \frac{TP}{TP+FN} = \frac{320}{320+43} = 0.882$ |
| Actual class NEGATIVE (normal) | FALSE POSITIVE (FP) 20 | TRUE NEGATIVE (TN) 538 | |
| | *Precision* $= \frac{TP}{TP+FP} = \frac{320}{320+20} = 0.941$ | | |

**Accuracy =**

$$\frac{\sum TP + \sum TN}{\sum \text{total population}}$$

**F1 Score =**

$$\frac{2}{\frac{1}{\text{Recall}} + \frac{1}{\text{Precision}}}$$

## Confusion Matrix


3
One and Two Tailed Tests
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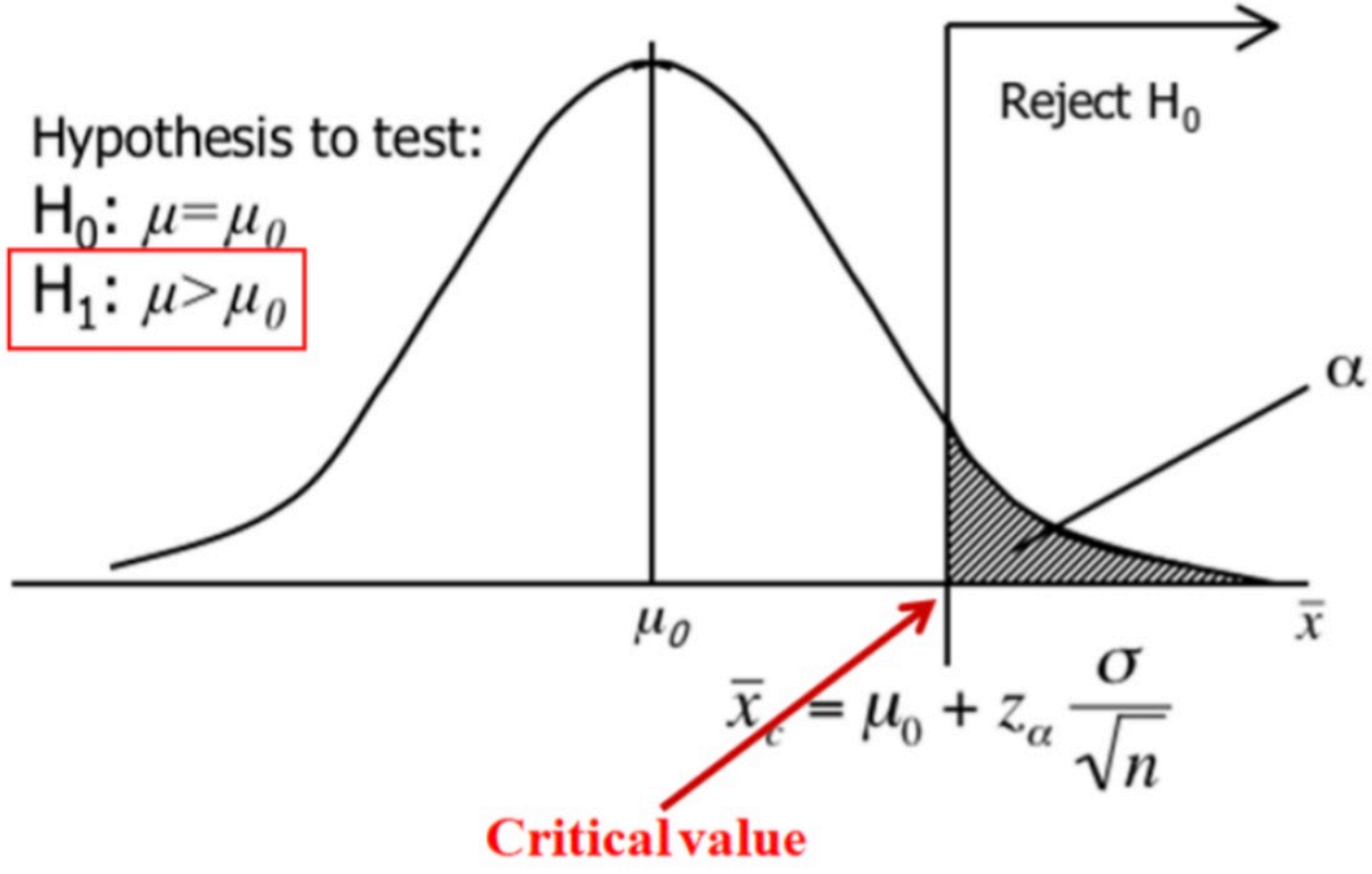
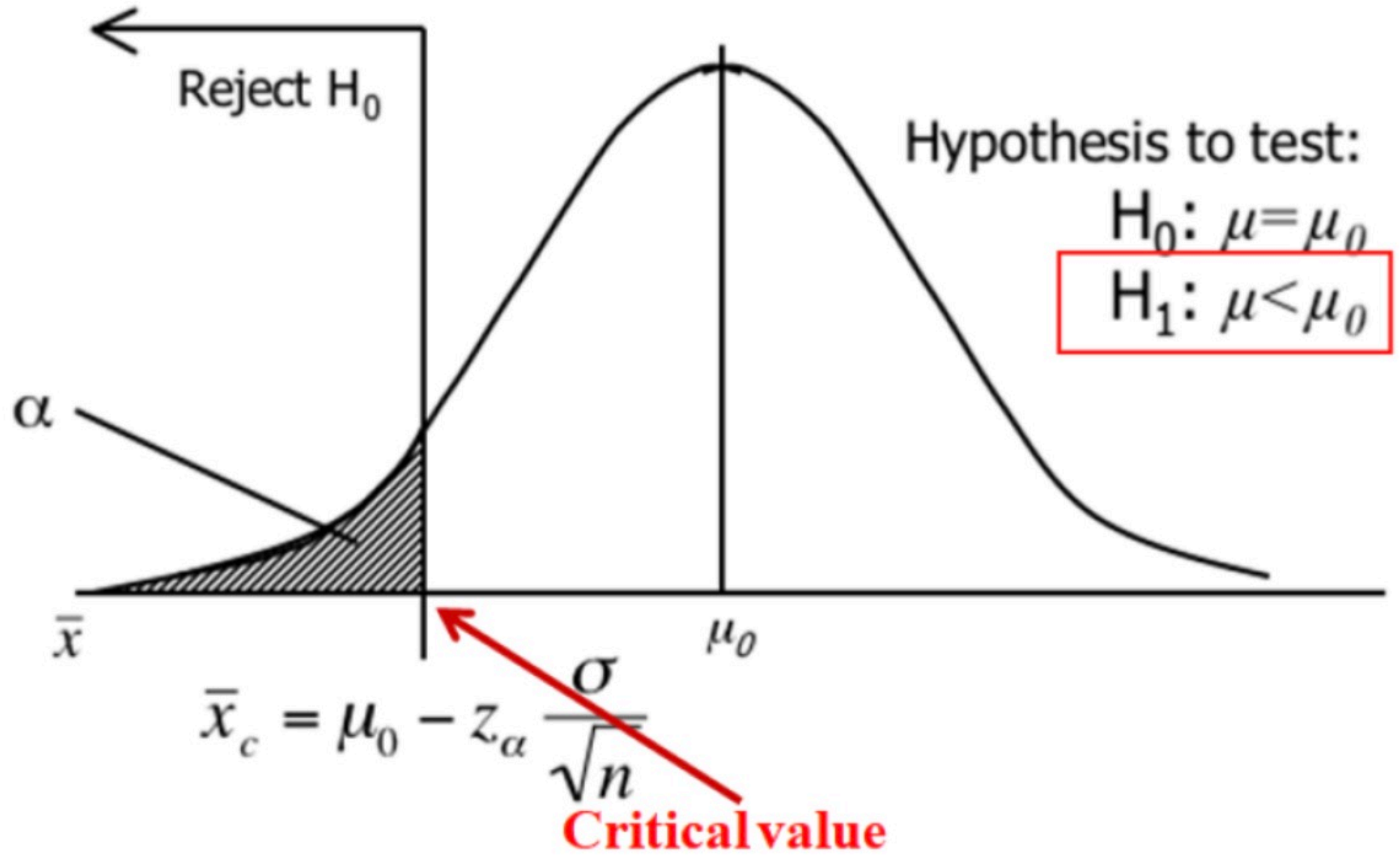
# Right-Tail Testing

- Calculate the critical value of the mean ( ) and compare against the observed value of the sample mean ( $\bar{x}$ )…



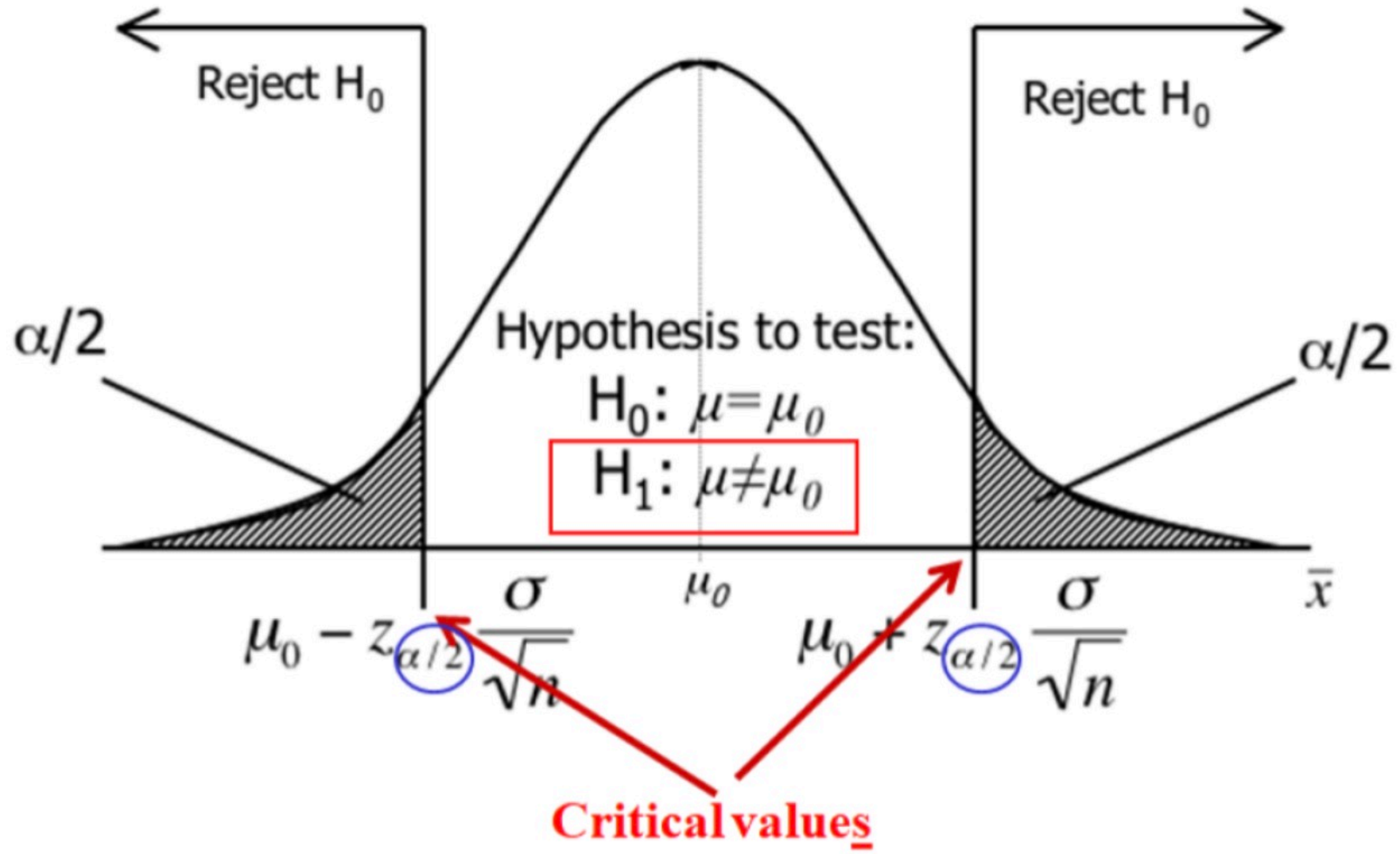
# Left-Tail Testing

- Calculate the critical value of the mean ( $\bar{x}_c$ ) and compare against the observed value of the sample mean ( $\bar{x}$ )…



# Two-Tail Testing

- Two tail testing is used when we want to test a research hypothesis that a parameter is not equal (≠) to some value (Note the use of $\alpha/2$ in determining the critical value)



# 4 Significance Test About a Population Mean

# Large Sample - *Example*

- Suppose that a beach is safe to swim if the mean level of lead in the water is 10.0 ($\mu_0$) parts/million.
- We assume $X_i \sim N(\mu, \sigma = 1.5)$
- Water safety is going to be determined by taking 40 water samples and using the test statistic.
- Sample mean = 10.5
- $\alpha = 0.05$

# Step 1: Assumptions

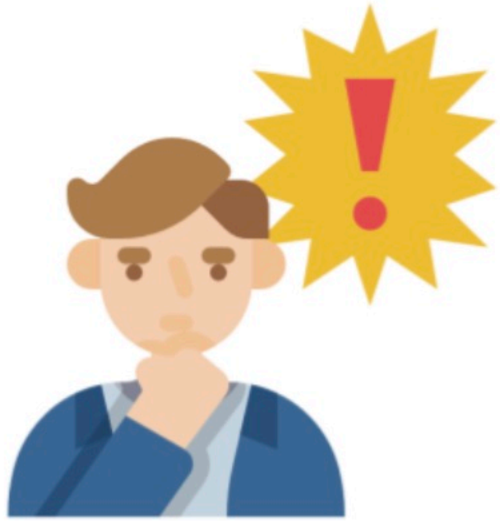
★ The three basic assumptions of a test about a mean are as follows:

Z-Test Assumptions

| | |
|---|---|
| 1 | Sample is selected random |
| 2 | Observations are independent |
| 3 | Population σ is known or 30-observations sample |

# Step 2 : Hypotheses

★ Each significance test has two hypotheses about a population parameter: the null hypothesis and an alternative hypothesis.



**The null hypothesis** $H_0$: $\mu = 10.0$

**The alternative hypothesis** $H_a$: $\mu > 10.0$

# Step 3 : Test Statistic

**Calculation of test statistic**

- From the data, we calculate $\bar{x} = 10.5$
- Using our standardized test statistic: $z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$
- We find that: $z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \dfrac{10.5 - 10}{1.5 / \sqrt{40}} = 2.1$
- Since z = 2.1 is greater than 1.645, so we can reject the null hypothesis in favor of $H_a$.

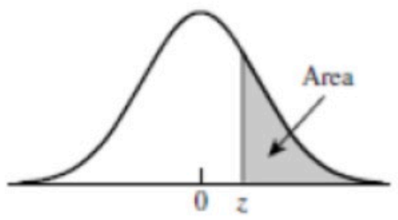
# Step 4 : P-Value

```
In [1]: import scipy.stats as stats
```

```
In [2]: 1-stats.norm.cdf(2.1)
Out[2]: 0.017864420562816563
```

- P-value =
- $P(z > 2.1 \mid H_0 \text{ true}) = .0179$

**Table 4 Normal Curve Areas**
**Standard normal probability in right-hand tail**
**(for negative values of *z*, areas are found by symmetry)**




| | Second decimal place of $z$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $z$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
| 0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |
| 0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| 0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| 0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| 0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| 0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| 0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| 0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| 0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| 0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| 1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| 1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| 1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| 1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| 1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0722 | .0708 | .0694 | .0681 |
| 1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| 1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| 1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| 1.8 | .0359 | .0352 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| 1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| 2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| 2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| 2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| 2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |

# Step 5 : Conclusion

★ We reject the null hypothesis when the P-value is less than or equal to the preselected significance level.

- P-value = .0179
- Significance Level ($\alpha$) = 0.05
- P-value < $\alpha$

# Step 5 : Conclusion

- P-value < α
- We can reject the null hypothesis in favor of $H_a$.

NULL HYPOTHESIS → Fail to Reject the Null

Reject the Null

- That is “there is sufficient evidence to close the beach.”

A department store manager determines that a new billing system will be cost-effective only if the mean monthly account is more than $170.

A random sample of 400 monthly accounts is drawn, for which the sample mean is $178. The accounts are approximately normally distributed with a standard deviation of $65.

**Can we conclude that the new system will be cost-effective?**

Students, write your response!



No Text Response
You didn't answer this question

# Step 1: Assumptions

★ The three basic assumptions of a test about a mean are as follows:

Z-Test Assumptions

1. Sample is selected random
2. Observations are independent
3. Population σ is known or 30-observations sample

# Step 2 : Hypotheses

★ Each significance test has two hypotheses about a population parameter:
*the null hypothesis* and *an alternative hypothesis*

**The null hypothesis** $H_0$: $\mu = 170$

**The alternative hypothesis** $H_a$: $\mu > 170$

# Step 3 : Test Statistic

**Calculation of test statistic**

- From the data, we calculate $\bar{x} = 178$
- Using our standardized test statistic: $z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$
- We find that: $z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \dfrac{178 - 170}{65 / \sqrt{400}} = 2.46$
- Since z = 2.46 is greater than 1.645, so we can reject the null hypothesis in favor of $H_a$.

# Step 4 : P-Value

```
In [1]: import scipy.stats as stats
```

```
In [2]: 1-stats.norm.cdf(2.46)
Out[2]: 0.006946850788624337
```

- P-value =
- $P(z > 2.46 \mid H_0 \text{ true}) = .0069$

| | Second decimal place of $z$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $z$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
| 0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |
| 0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| 0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| 0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| 0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| 0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| 0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| 0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| 0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| 0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| 1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| 1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| 1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| 1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| 1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0722 | .0708 | .0694 | .0681 |
| 1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| 1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| 1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| 1.8 | .0359 | .0352 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| 1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| 2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| 2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| 2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| 2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| 2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |

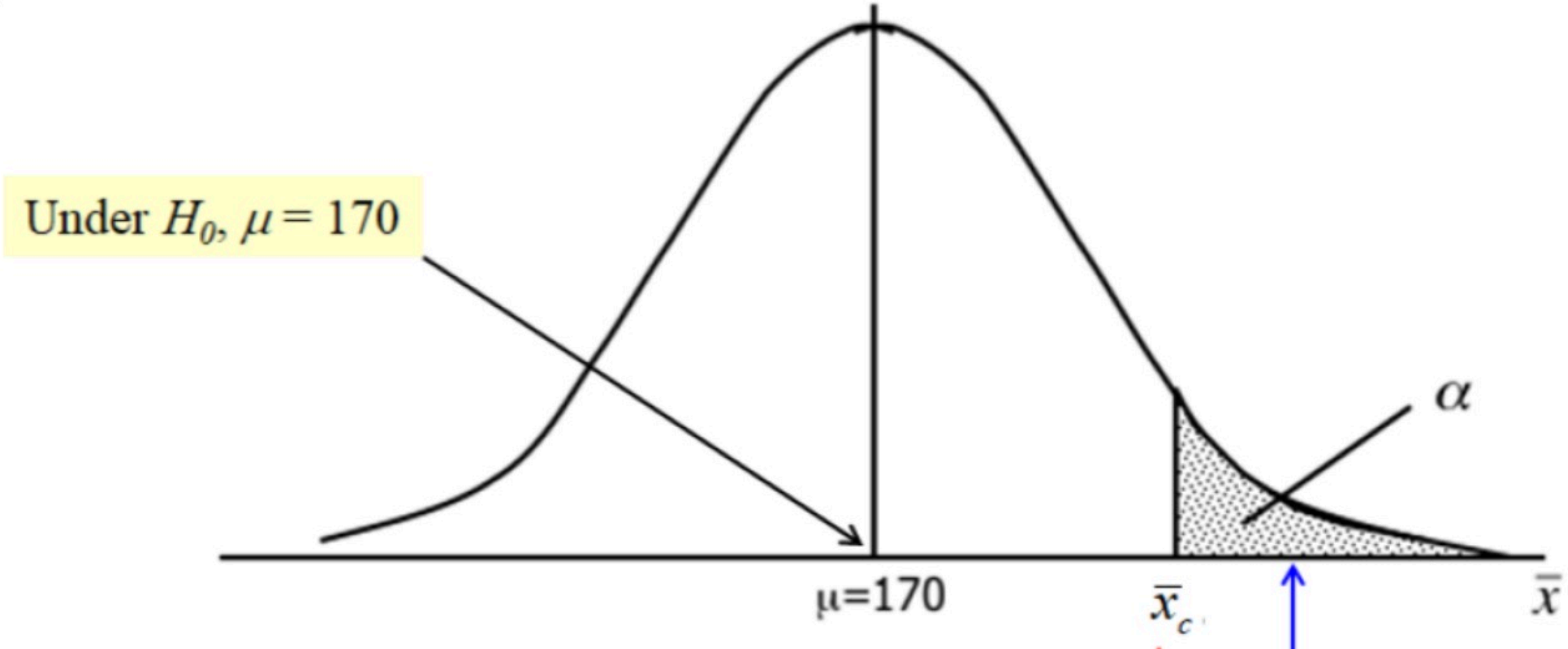
# Step 5 : Conclusion

- P-value < α
- We can reject the null hypothesis in favor of $H_a$.

- P-value = .0069
- Significance Level (α) = 0.05
- P-value < α   Reject the Null

- So it is cost effective to install the new billing system.

# Step 5 : Conclusion




$$\bar{x}_c = \mu_0 + z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right) = 170 + 1.645\left(\frac{65}{\sqrt{400}}\right) = 175.34$$

$\bar{x} = 178$

- So it is cost effective to install the new billing system.

# Hypothesis testing using t-tests

- The observed value of the test statistic is obtained as follows:
  - We refer to t as the t-score.
  - Then,
    - If $H_A$: $\mu < \mu_0$, $p_{obs} = P(T \leq t)$,
    - If $H_A$: $\mu > \mu_0$, $p_{obs} = P(T \geq t)$,
    - If $H_A$: $\mu \neq \mu_0$, $p_{obs} = 2 \times P(T \geq |t|)$,
- Here, T has a t-distribution with n − 1 degrees of freedom, and t is our observed t-score.

# Small Sample - *Example*

- Bon Air ELEM has 1000 students. The principal of the school thinks that the average IQ of students at Bon Air is at least 110. To prove her point, she administers an IQ test to 20 randomly selected students.
- Among the sampled students, the average IQ is 108 with a standard deviation of 10.
- Based on these results, should the principal accept or reject her original hypothesis?
- $\alpha = 0.01$

# Step 1: Assumptions

★ The three basic assumptions of a test about a mean are as follows:

t-Test Assumptions

1. The variable is quantitative.
2. The data production employed randomization.
3. The population dist. is approximately normal.

# Step 2 : Hypotheses

★ Each significance test has two hypotheses about a population parameter:
*the null hypothesis* and *an alternative hypothesis*

**The null hypothesis** $H_0$: μ >= 110

**The alternative hypothesis** $H_a$: μ < 110

# Step 3 : Test Statistic

**Calculation of test statistic**

t

- From the data, we calculate $\bar{x} = 108$
- Using our standardized test statistic: $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$
- We find that: $t = \frac{108 - 110}{10/\sqrt{20}} = \frac{-2}{2.236} = -0.894$
- Since t = -0.894 is greater than -2.539, so we fail to reject the null hypothesis.

# Step 4 : P-Value

```
In [1]: import scipy.stats as stats

In [2]: stats.t.cdf(-0.894, 19)

Out[2]: 0.19125344283171025
```

- P-value =
- $P(t < -0.894 \mid H_0 \text{ true}) = .1913$

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# Step 5 : Conclusion

- P-value > α
- We cannot reject the null hypothesis.

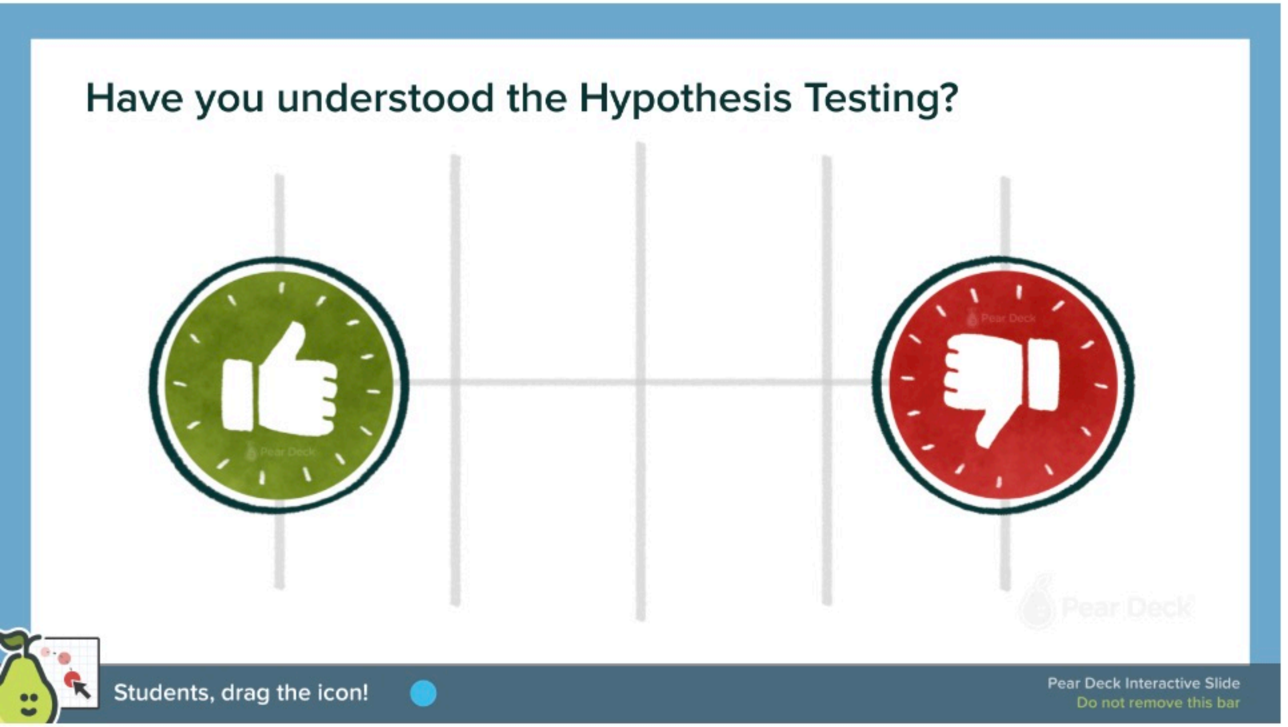
- P-value = .1913
- Significance Level (α) = 0.01
- P-value > α

Fail to Reject the Null

- The principal should accept her original hypothesis.

# More Hypothesis Tests

- ★ Diff between means
- ★ Diff between pairs
- ★ Proportions
- ★ Diff between props
- ★ Goodness of fit test
- ★ Homogeneity
- ★ Independence
- ★ Regression slope




**No Draggable™ Response**

You didn't answer this question

# THANKS!

## Any questions?

You can find me at:

- jason@clarusway.com