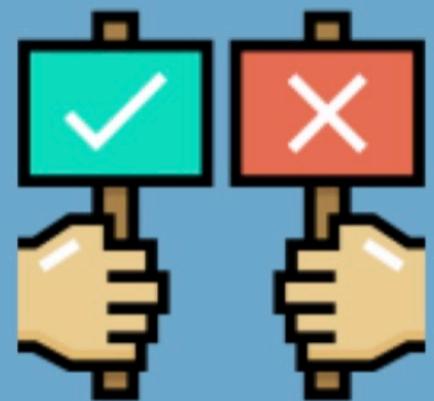


# Statistics

## Session-6

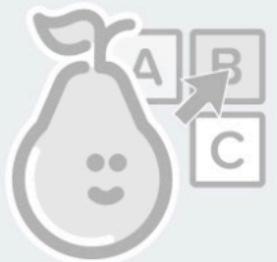


# Did you finish Statistics (Hypothesis Tests) pre-class activity?



Students choose an option

Pear Deck Interactive Slide  
Do not remove this bar



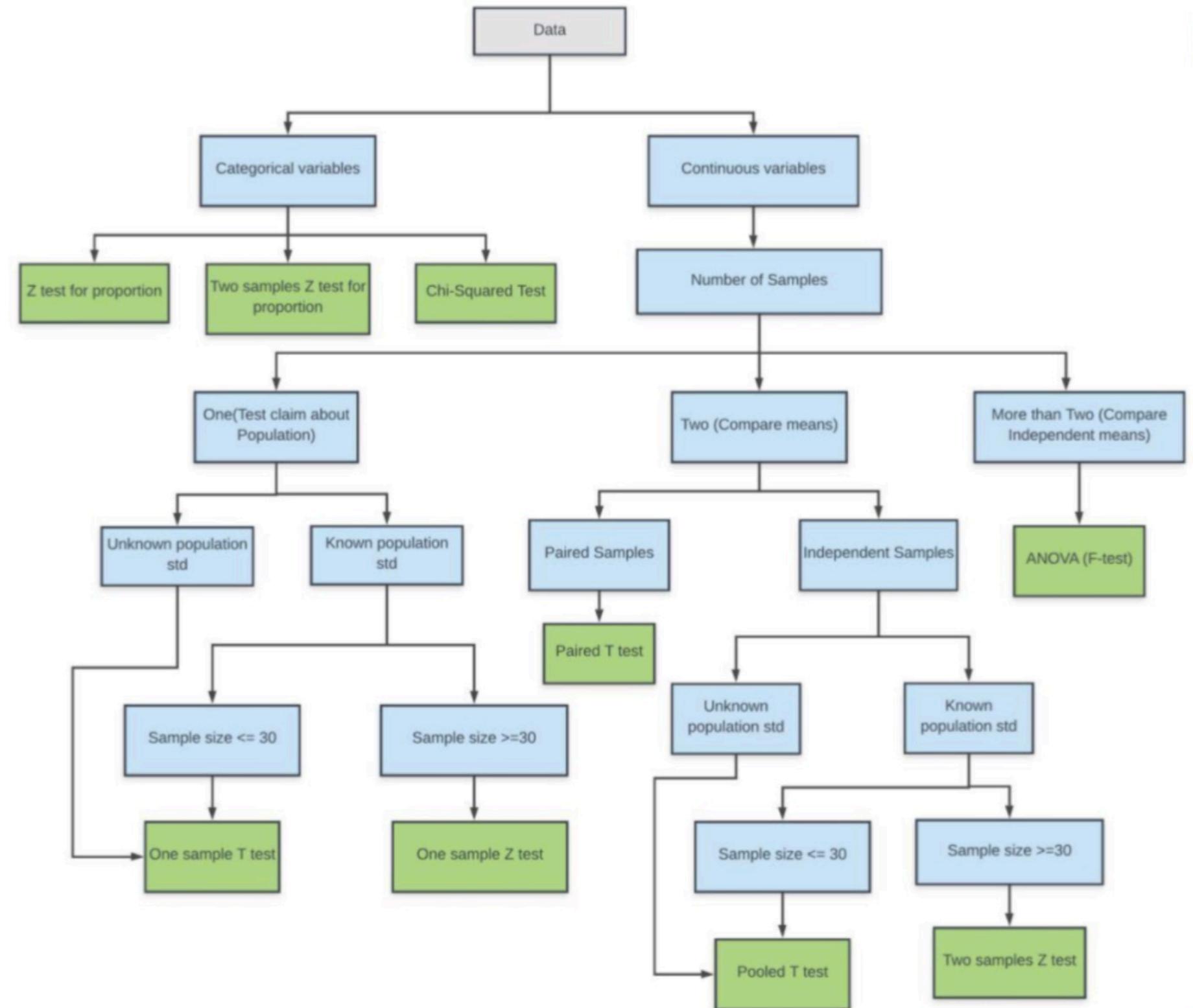
No Multiple Choice Response  
You didn't answer this question



# Table of Contents

- ▶ Basic Concepts Review  
(One Sample T Test)
- ▶ Independent Samples T Test
- ▶ Dependent T Test (Paired)
- ▶ One-way ANOVA

# ► Test Types





1

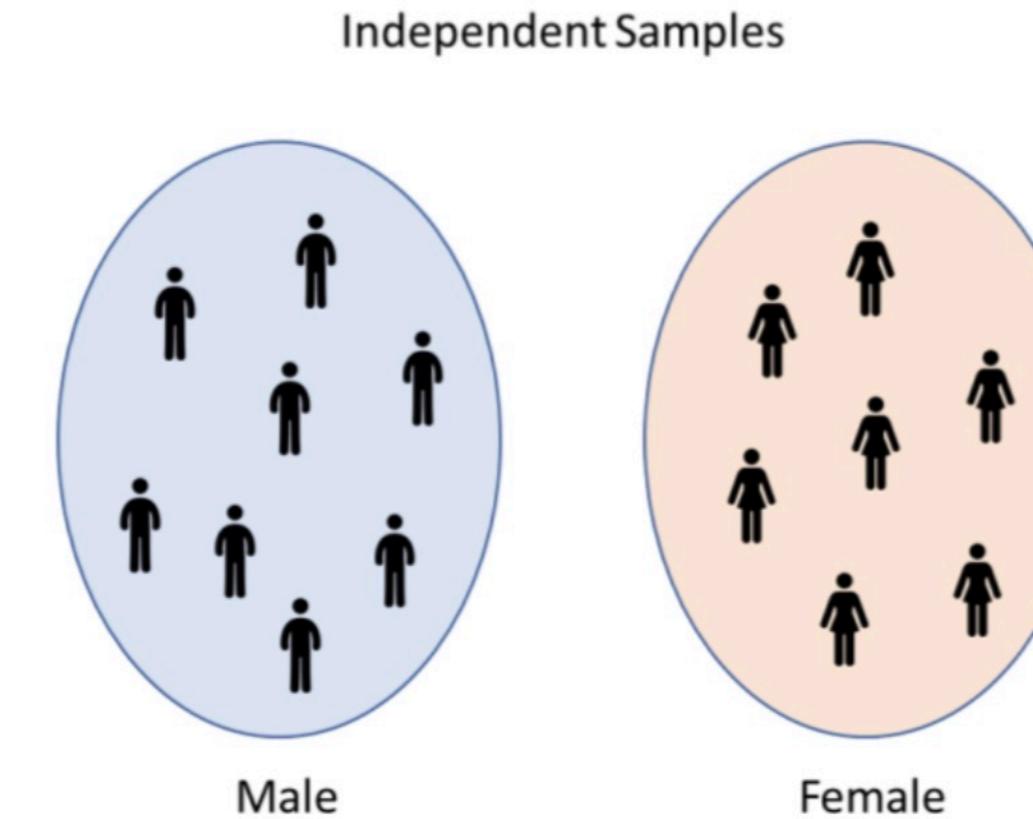
# Basic Concepts Review

# ► Independent Samples T Test

The independent-samples t-test (or independent t-test) compares the means between two independent groups on the same continuous, dependent variable.

- Does first year graduate salaries differ based on gender?

Dependent Variable	Two Independent Groups
Salary	Gender
\$35,000	Male
\$45,000	Male
\$38,000	Female
\$50,000	Female
\$28,000	Male



# ► The Steps of a Significance Test ➤

A significance test has five steps.

- **Step 1** : Assumptions
- **Step 2** : Hypotheses
- **Step 3** : Test Statistic
- **Step 4** : P-Value
- **Step 5** : Conclusion





2

# Hypothesis Test: Independent Samples Test

# ► Independent Samples T Test



## Assumptions

- A quantitative response variable for two groups
- Independent random samples
- Normal population distribution for each group.

## Hypotheses

**The null hypothesis**

$$H_0: \mu_1 = \mu_2$$

**The alternative hypothesis**  $H_a: \mu_1 \neq \mu_2$



# ► Test Statistic



Equal Variances NOT Assumed

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$df = \frac{\left( \frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right)^2}{\frac{1}{n_1-1} \left( \frac{s_1^2}{n_1} \right)^2 + \frac{1}{n_2-1} \left( \frac{s_2^2}{n_2} \right)^2}$$

Equal Variances Assumed

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} \quad df = n_1 + n_2 - 2$$

# ► Independent Samples T Test - Example

- Under normal conditions, is the average body temperature the same for men and women?
- Medical researchers interested in this question collected data from a large number of men and women, and random samples from that data are presented in the accompanying table.
- Is there sufficient evidence to indicate that mean body temperatures differ for men and women?

Body Temperatures (°F)	
Men	Women
96.9	97.8
97.4	98.0
97.5	98.2
97.8	98.2
97.8	98.2
97.9	98.6
98.0	98.8
98.6	99.2
98.8	99.4

# ► Step 1: Assumptions

- ★ The three basic assumptions of a test about difference between means are as follows:

Independent  
Samples T-Test  
Assumptions

- 1 A quantitative response variable for two groups 
- 2 Independent random samples 
- 3 Approximately normal population distribution for each group. 

# ► Step 2 : Hypotheses

- ★ Each significance test has two hypotheses about a population parameter: the null hypothesis and an alternative hypothesis.



**The null hypothesis**  $H_0: \mu_1 = \mu_2$

**The alternative hypothesis**  $H_a: \mu_1 \neq \mu_2$

# ► The pooled estimate of $\sigma$

- Note: both  $s_1$  and  $s_2$  are estimators of  $\sigma$ .
- These can be combined to form a single estimator of  $\sigma$ ,  $S_{\text{Pooled}}$ .

- Sample Sizes
  - $n_1 = 9$
  - $n_2 = 9$
- Sample Std.Dev.
  - $s_1 = 0.5833$
  - $s_2 = 0.5487$

$$S_{\text{Pooled}} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$
$$s_p = \sqrt{\frac{(9 - 1)0.5833^2 + (9 - 1)0.5487^2}{9 + 9 - 2}} = 0.5663$$

# ► Step 3 : Test Statistic



## Calculation of test statistic

t

- $\bar{x}_1 = 97.856, \bar{x}_2 = 98.489, s_p = 0.5663$

- Test statistic:

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2 - 0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

- We find that:  $t = \frac{\bar{x} - \bar{y} - \Delta_0}{s_{Pooled} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{97.856 - 98.489 - 0}{0.5663 \times \sqrt{\frac{1}{9} + \frac{1}{9}}} = -2.371$

- $df = n_1 + n_2 - 2 = 9 + 9 - 2 = 16$

# ► Step 4 : P-Value



```
In [1]: import scipy.stats as stats
```



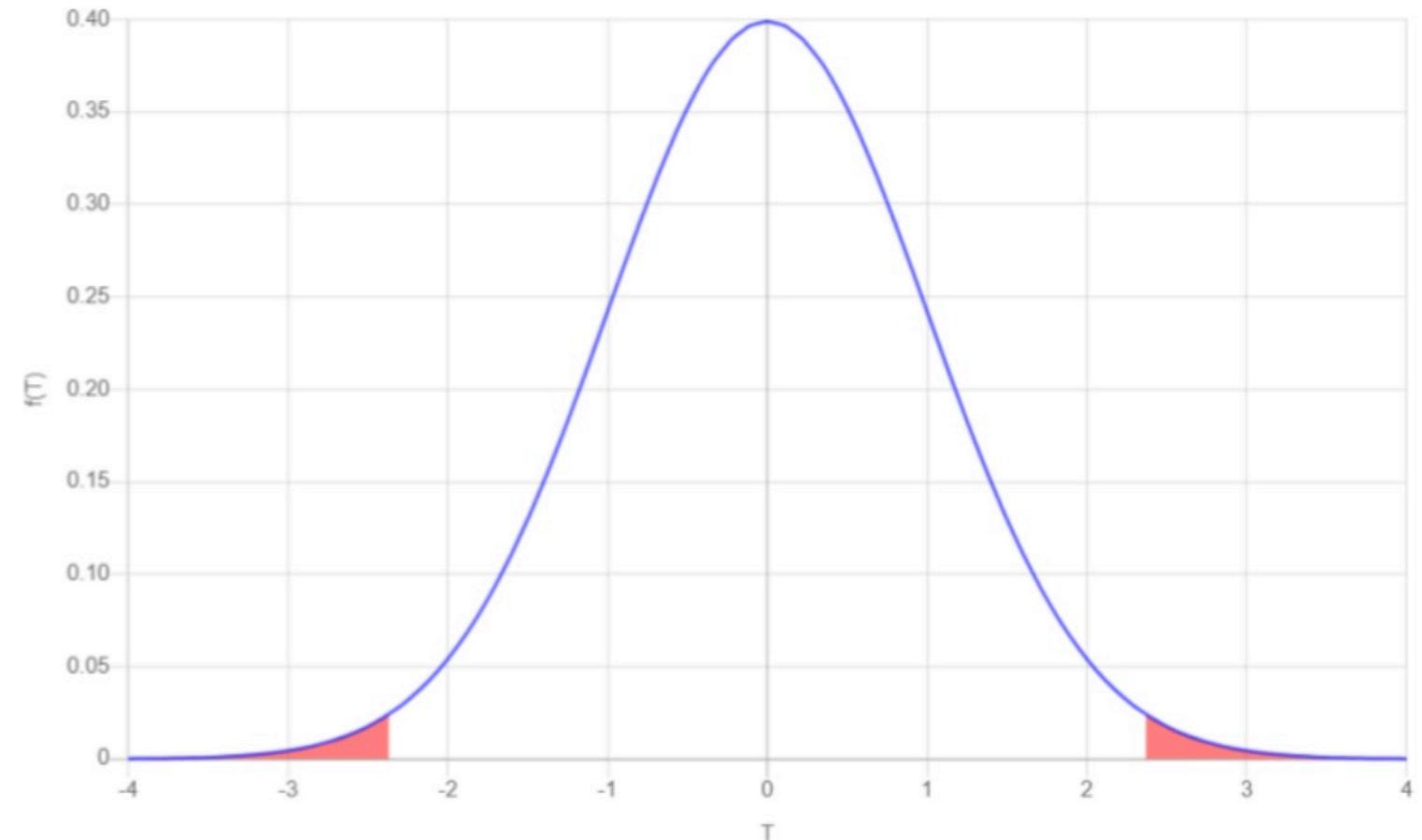
```
In [2]: 2*stats.t.cdf(-2.371, 16)
```

```
> 2*pt(-2.371,16)  
[1] 0.0306345
```

```
Out[2]: 0.030634485990952903
```

T-Test: t = -2.371, p = 0.0306

- P-value =
- $2*t_{-2.371, 16} = .0306$

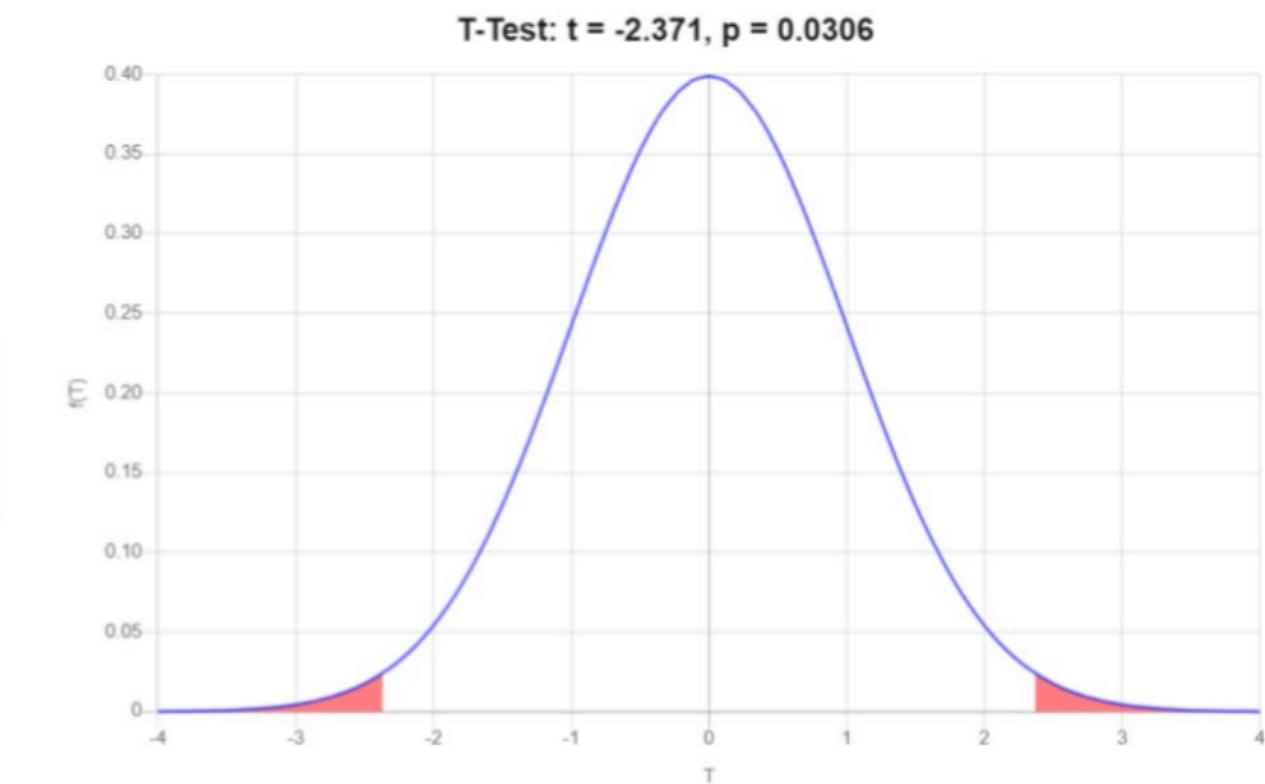


# ► Step 4 : P-Value



## One line code

```
men=[96.9, 97.4, 97.5, 97.8, 97.8, 97.9, 98, 98.6, 98.8]  
women = [97.8, 98, 98.2, 98.2, 98.2, 98.6, 98.8, 99.2, 99.4]
```



```
[1] # Calculate test statistics using stats.ttest_ind()  
indTest = stats.ttest_ind(men, women, equal_var = True)  
indTest
```

```
Ttest_indResult(statistic=-2.3724271468993643, pvalue=0.03054788637798765)
```

# ► Step 5 : Conclusion



- P-value <  $\alpha$
- We reject the null hypothesis.

- P-value = .0306
- Significance Level ( $\alpha$ ) = 0.05

- P-value <  $\alpha$

Reject the Null

- We have sufficient evidence to indicate that mean body temperatures differ for men and women. (at the  $\alpha=0.05$ )



3

# Hypothesis Test: Dependent T Test

# ► Dependent T Test (Paired)

The dependent t-test (or the paired-samples t-test) compares the means between two related groups on the same continuous, dependent variable.

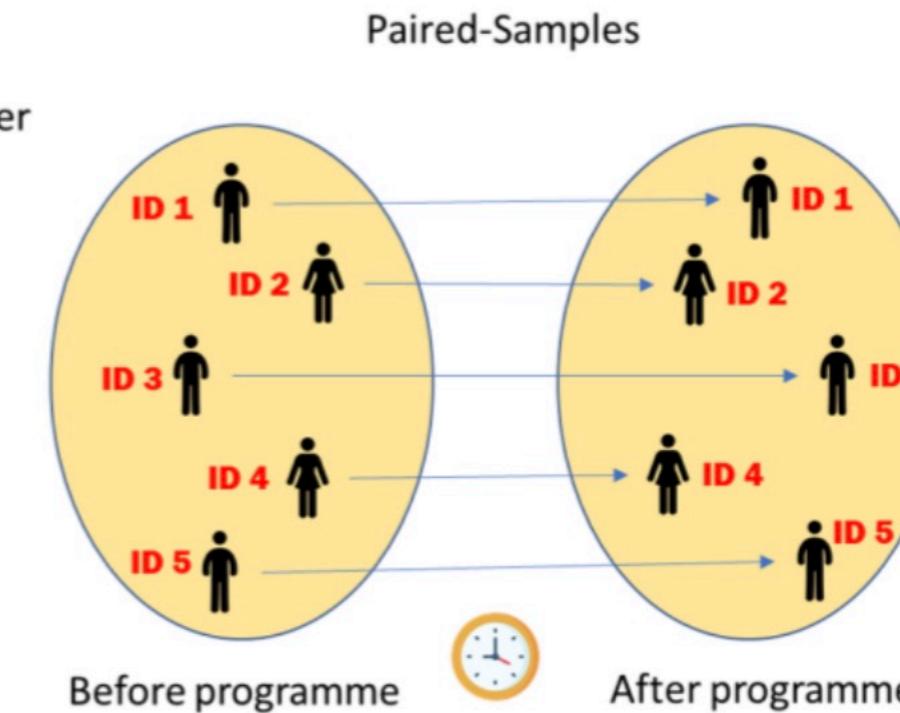
- Is there statistical difference in smokers' daily cigarette consumption before and after a 6 week hypnotherapy programme?

daily cigarette consumption before programme      daily cigarette consumption after programme

Participant ID

ID	Before	After	Difference
1	12	10	2
2	18	7	11
3	23	22	1
4	10	12	-2
5	8	4	4

Before-After



# ► Dependent T Test Assumptions

- The dependent variable is continuous, and it is measured twice on the same sample of subjects.
- The independent variable should consist of two categorical, "related groups" or "matched pairs".
- The difference between the scores of the variables are normally distributed.



## Hypotheses

**The null hypothesis**

$$H_0: \mu_D = 0 \quad (\mu_D = \mu_1 - \mu_2)$$

**The alternative hypothesis**  $H_a: \mu_D \neq 0$



# ► Test Statistic



$$t = \frac{\bar{x}_{diff} - 0}{s_{\bar{x}}}$$

$$s_{\bar{x}} = \frac{s_{diff}}{\sqrt{n}}$$

$\bar{x}_{diff}$  = Sample mean of the differences

$n$  = Sample size (i.e., number of observations)

$s_{diff}$  = Sample standard deviation of the differences

$s_{\bar{x}}$  = Estimated standard error of the mean ( $s/\sqrt{n}$ )

# ► Dependent Samples T Test - Example

- An article\* reports a comparison of several methods for predicting the shear strength for steel plate girders.
- Data for two of these methods, the Karlsruhe and Lehigh procedures, when applied to nine specific girders, are shown in the table below.
- Determine whether there is any difference (on the average) for the two methods. ( $\alpha = 0.05$ )

\*Journal of Strain Analysis  
[1983, Vol. 18(2)]

Girder	Karlsruhe Method	Lehigh Method	Difference $d_j$
S1/1	1.186	1.061	0.125
S2/1	1.151	0.992	0.159
S3/1	1.322	1.063	0.259
S4/1	1.339	1.062	0.277
S5/1	1.2	1.065	0.135
S2/1	1.402	1.178	0.224
S2/2	1.365	1.037	0.328
S2/3	1.537	1.086	0.451
S2/4	1.559	1.052	0.507

# ► Step 1: Assumptions



- ★ The three basic assumptions of a test about difference between means are as follows:

Paired  
Samples  
T-Test

- |                  |  |  |
|------------------|--|--|
| Assumptions<br>1 | The dependent variable is continuous.                        |  |
| 2                | Observations are independent of one another.                 |  |
| 3                | Approximately normal population distribution for each group. |  |

# ► Step 2 : Hypotheses

- ★ Each significance test has two hypotheses about a population parameter: the null hypothesis and an alternative hypothesis.



**The null hypothesis**  $H_0: \mu_D = 0$

**The alternative hypothesis**  $H_a: \mu_D \neq 0$

# ► Step 3 : Test Statistic



## Calculation of test statistic

t

- $\bar{d} = 0.2769$ ,  $s_d = 0.1350$ ,  $n=9$
- Test statistic:  $t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$
- We find that:  $t_0 = \frac{\bar{d}}{s_d / \sqrt{n}} = \frac{0.2769}{0.1350 / \sqrt{9}} = 6.15$

# ► Step 4 : P-Value



```
In [1]: import scipy.stats as stats
```



```
In [2]: 2*(1-stats.t.cdf(6.15, 8))
```

```
Out[2]: 0.00027399606897193785
```

- P-value = 0.0003

# ► Step 4 : P-Value



One line code

```
karlsruhe=[1.186, 1.151, 1.322, 1.339, 1.2, 1.402, 1.365, 1.537, 1.559]
lehight=[1.061, 0.992, 1.063, 1.062, 1.065, 1.178, 1.037, 1.086, 1.052]
```

```
# Calculate test statistics using stats.ttest_rel()
pairedtest = stats.ttest_rel(karlsruhe, lehight)
pairedtest
```

```
Ttest_relResult(statistic=6.0819394375848255, pvalue=0.00029529546278604066)
```

# ► Step 5 : Conclusion



- P-value <  $\alpha$
- We reject the null hypothesis.

- P-value = .0003
- Significance Level ( $\alpha$ ) = 0.05

- P-value <  $\alpha$

Reject the Null

- The data indicate that the Karlsruhe method produces, on the average, higher strength predictions than does the Lehigh method.



4

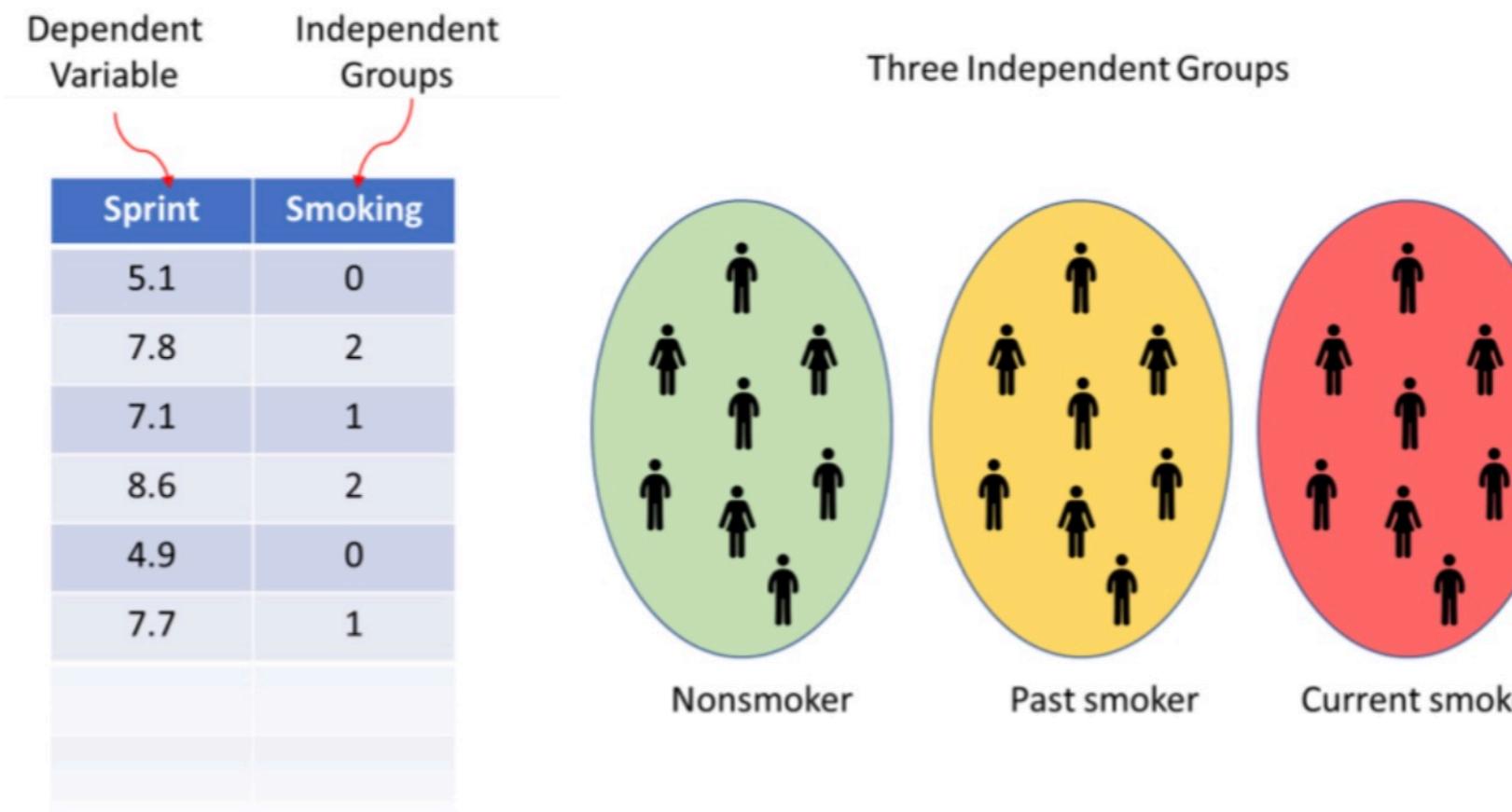
# Hypothesis Test: One-way ANOVA

# ► One-way ANOVA



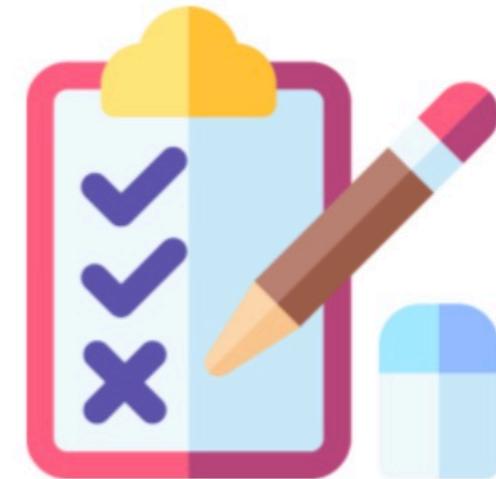
**Both the One-Way ANOVA and the Independent Samples t Test can compare the means for two groups. However, only the One-Way ANOVA can compare the means across three or more groups.**

- Is there a statistically significant difference in sprint time with respect to smoking status?  
(0 = Nonsmoker, 1 = Past smoker, 2 = Current smoker)



# ► One-way ANOVA Assumptions

- Dependent variable that is continuous
- Independent variable that is categorical (two or more groups)
- Independent samples/groups
- Normal distribution of the DV for each group
- Homogeneity of variances



## Hypotheses

**The null hypothesis**

$H_0: \mu_1 = \mu_2 = \mu_3 = \dots = \mu_k$

(all k population means are equal)

**The alternative hypothesis**  $H_a:$  At least one  $\mu_i$  different



# ► Test Statistic ANOVA Table

$$SSR = \sum(\hat{y}_i - \bar{y})^2$$

regression  
sum of squares

model degrees of  
freedom  
k: group number

regression  
mean square

**F statistic**

	Sum of Squares	df	Mean Square	F
Group (Between)	SSR	k-1	MSR = SSR/(k-1)	MSR/MSE
Error (Within)	SSE	n-k	MSE = SSE/(n-k)	
Total	$SST = SSR+SSE$	n-1		

error sum  
of squares

$$SSE = \sum(\hat{y}_i - y_i)^2$$

total sum  
of squares

error degrees  
of freedom

total degrees  
of freedom

mean  
square  
error

# ► One-way ANOVA - Scenarios



★ Variation in Time to Relief of Symptoms  
Between and Within 3 Treatment



★ Music compressed by four MP3  
compressors are with the same quality

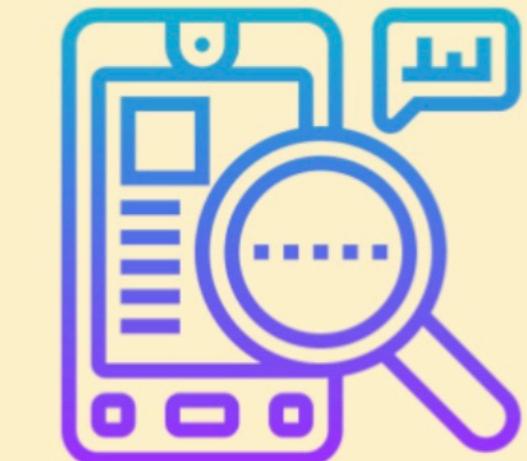
★ Three new drugs are all as effective as a placebo



★ Lectures, group studying, and computer assisted instruction are equally effective

# ► One-way ANOVA - Example

- As an example, we analyze the Cushings data set, which is available from the MASS package.
- The Type variable in the data set shows the underlying type of syndrome, which can be one of four categories:
  - adenoma (a),
  - bilateral hyperplasia (b),
  - carcinoma (c),
  - unknown (u).
- Our objective is to find whether the four groups are different with respect to urinary excretion rate of Tetrahydrocortisone.



## ► One-way ANOVA - Example

- The total number of observations is  $n=27$ ,
- The number of observations in each group is
  - $n_1 = 6$ ,  $n_2 = 10$ ,  $n_3 = 5$ , and  $n_4 = 6$ .
- We also find the group specific means, which are
  - 3.0, 8.2, 19.7, and 14.0, respectively.
- The degrees of freedom parameters are
  - $df_1 = 4-1=3$  ( $k-1$ ) and  $df_2 = 27-4=23$  ( $n-k$ ).
  - ( $k$ : number of groups)
- $SS_B = 893.5$  and  $SS_W = 2123.6$ .

# ► Step 1: Assumptions

- ★ The basic assumptions of one-way ANOVA test are as follows:

## One-way ANOVA Assumptions

**1**

The dependent variable is continuous.



**2**

Independent variable that is categorical.



**3**

Approximately normal population distribution for each group.



**4**

Homogeneity of variances



# ► Step 2: Hypotheses

- ★ The null and alternative hypotheses of one-way ANOVA can be expressed as:



**The null hypothesis  $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$**

**The alternative hypothesis  $H_a: \text{At least one } \mu_i \text{ different}$**

# ► Step 3: Test Statistic



## ANOVA Table

F

	Sum of Squares	df	Mean Square	F
Group (Between)	$SSR = 893.5$	$k-1 = 3$	$MSR = 893.5 / 3$ $= 297.8$	$MSR/MSE = 297.8 / 92.3$ $= 3.226$
Error (Within)	$SSE = 2123.6$	$n-k = 23$	$MSE = 2123.6 / 23$ $= 92.3$	
Total	$SST = 3017.1$	$n-1 = 27$		

# ► Step 4 : P-Value



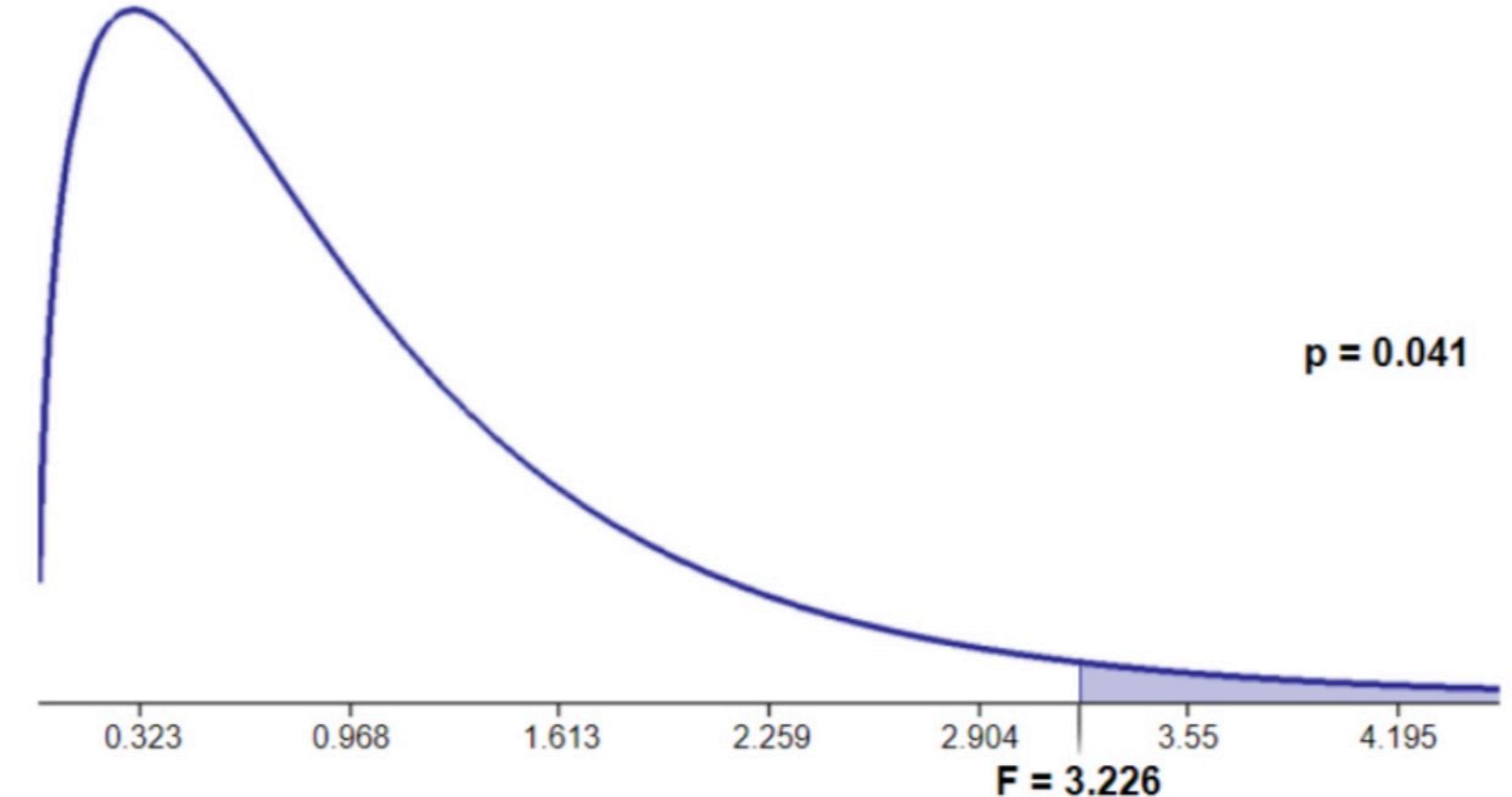
```
In [1]: import scipy.stats as stats
```



```
In [2]: 1 - stats.f.cdf(3.226, dfn=3, dfd=23)
```

```
Out[2]: 0.041207862659964456
```

- F statistic = 3.226
- P-value = 0.041



# ► Step 4 : P-Value

## One line code



	Tetrahydrocortisone	Pregnanetriol	Type
a1	3.1	11.70	a
a2	3.0	1.30	a
a3	1.9	0.10	a
a4	3.8	0.04	a
a5	4.1	1.10	a

```
df.Type.unique()
```

```
array(['a', 'b', 'c', 'u'], dtype=object)
```

```
f_oneway((df.Tetrahydrocortisone[df.Type=="a"]),
          (df.Tetrahydrocortisone[df.Type=="b"]),
          (df.Tetrahydrocortisone[df.Type=="c"]),
          (df.Tetrahydrocortisone[df.Type=="u"]))
```

```
F_onewayResult(statistic=3.2257394791378426,
                pvalue=0.0412182793672776)
```

# ► Step 5 : Conclusion



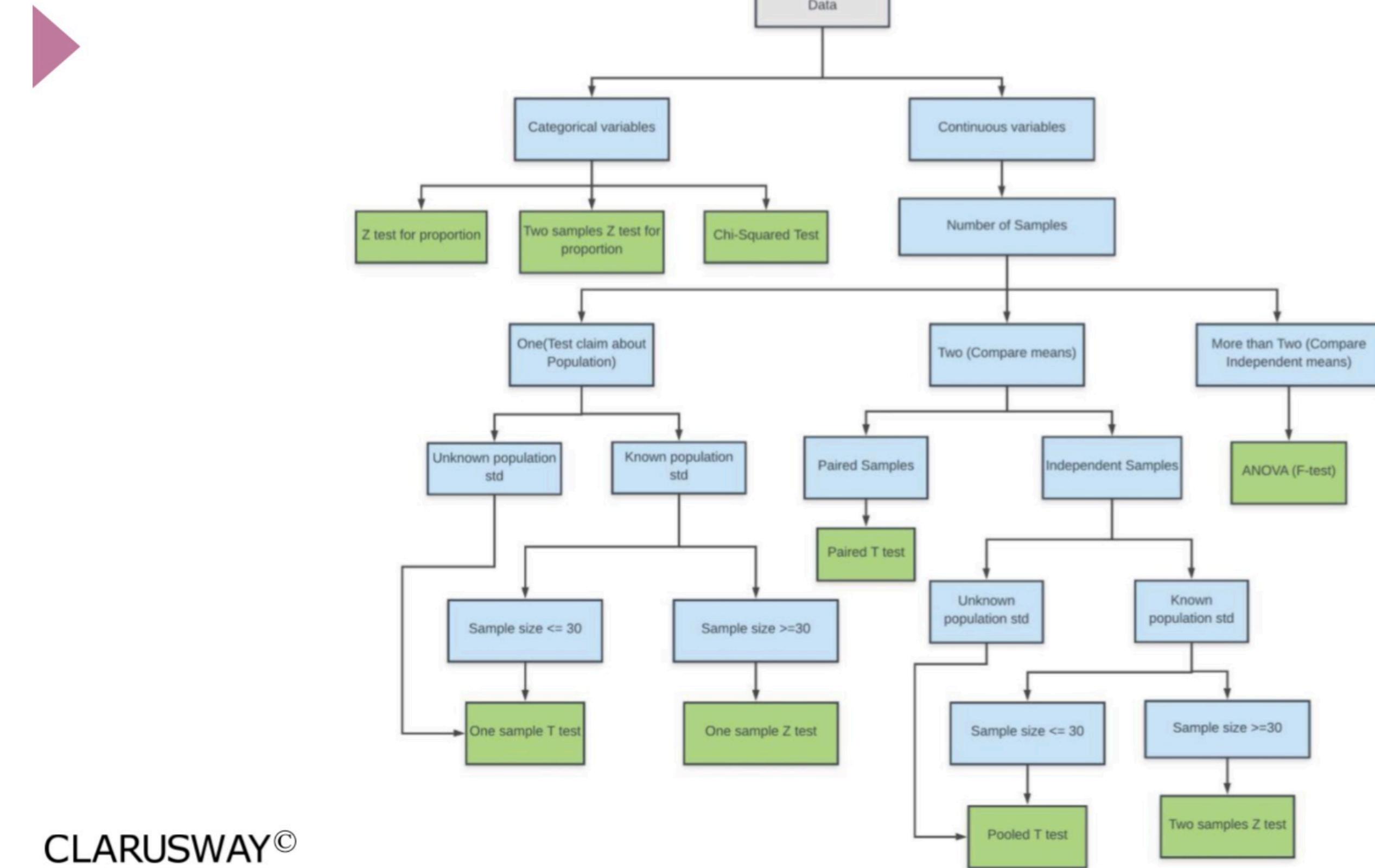
- $P\text{-value} < \alpha$
- We reject the null hypothesis.

- $P\text{-value} = 0.041$
- Significance Level ( $\alpha$ ) = 0.05

- $P\text{-value} < \alpha$

Reject the Null

- Therefore, we can reject  $H_0$  at 0.05 significance level and conclude that the differences among group means are statistically significant

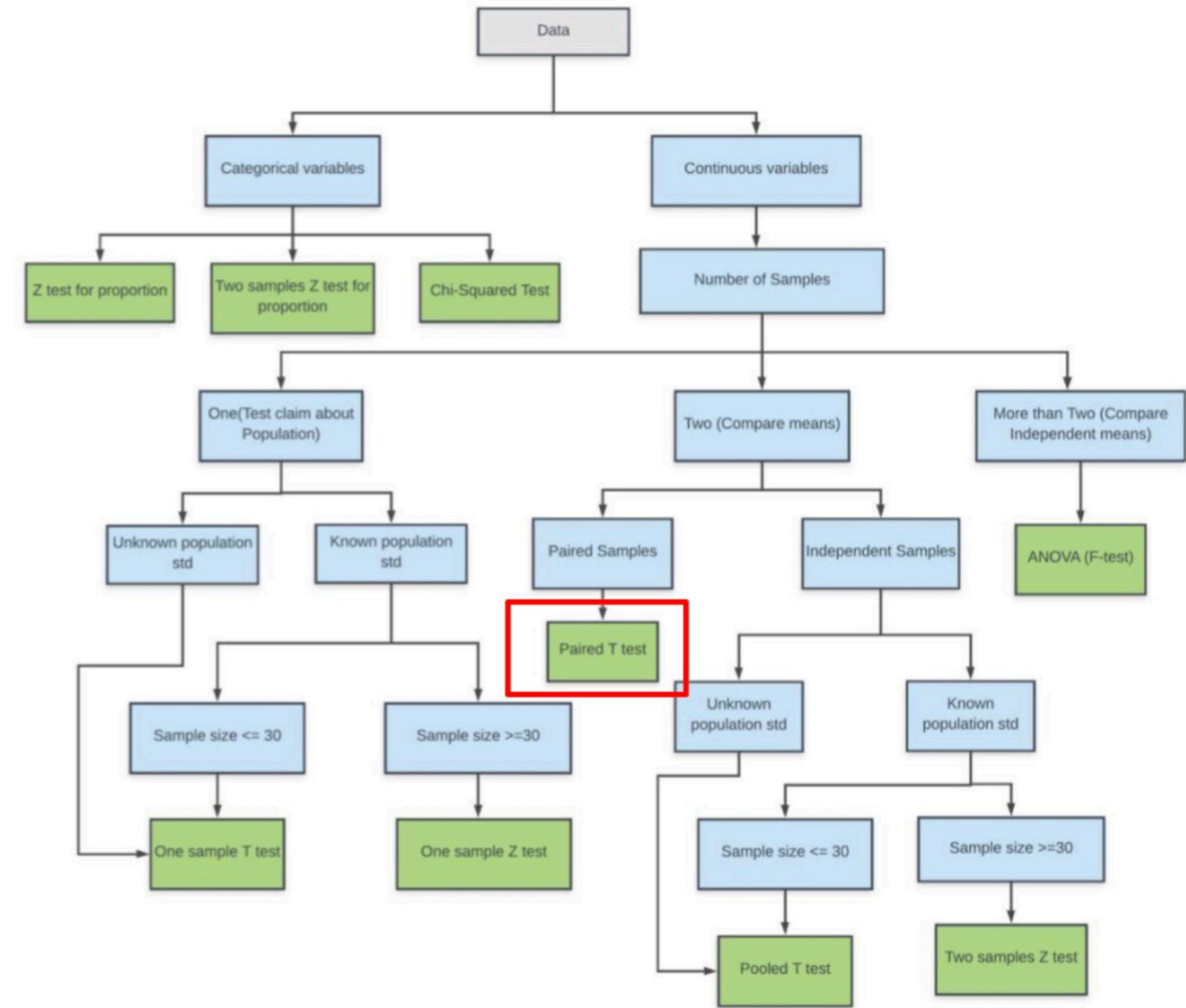




## LETS PRACTICE

- **(B)** Is there a significant mean difference between Wednesday and Saturday gas prices if we check the same 20 stations on both days?

# SOLUTION

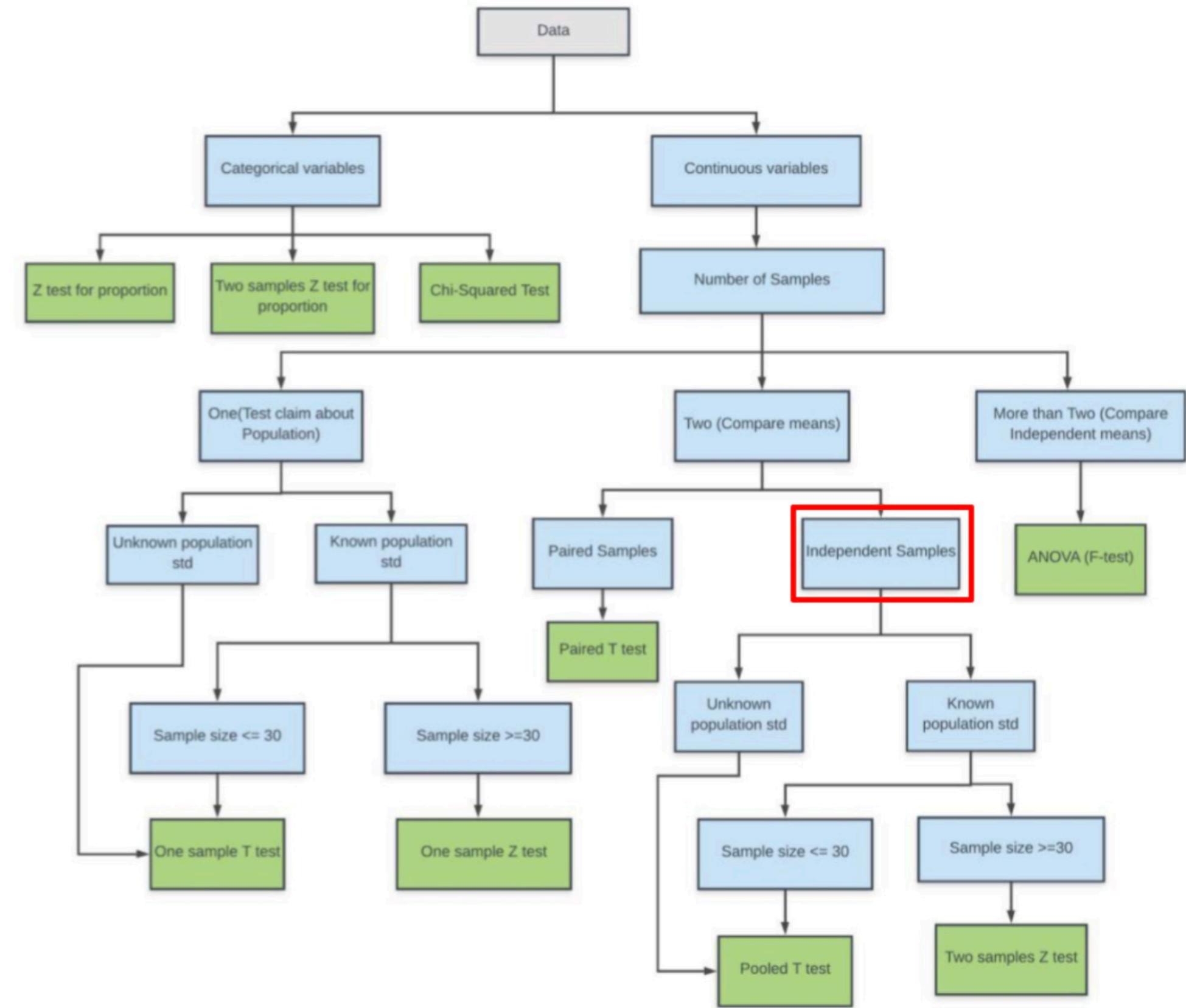




## LETS PRACTICE

- **(C)** Is there a significant difference between the average Boston gas price and the average New York gas price today?

# SOLUTION

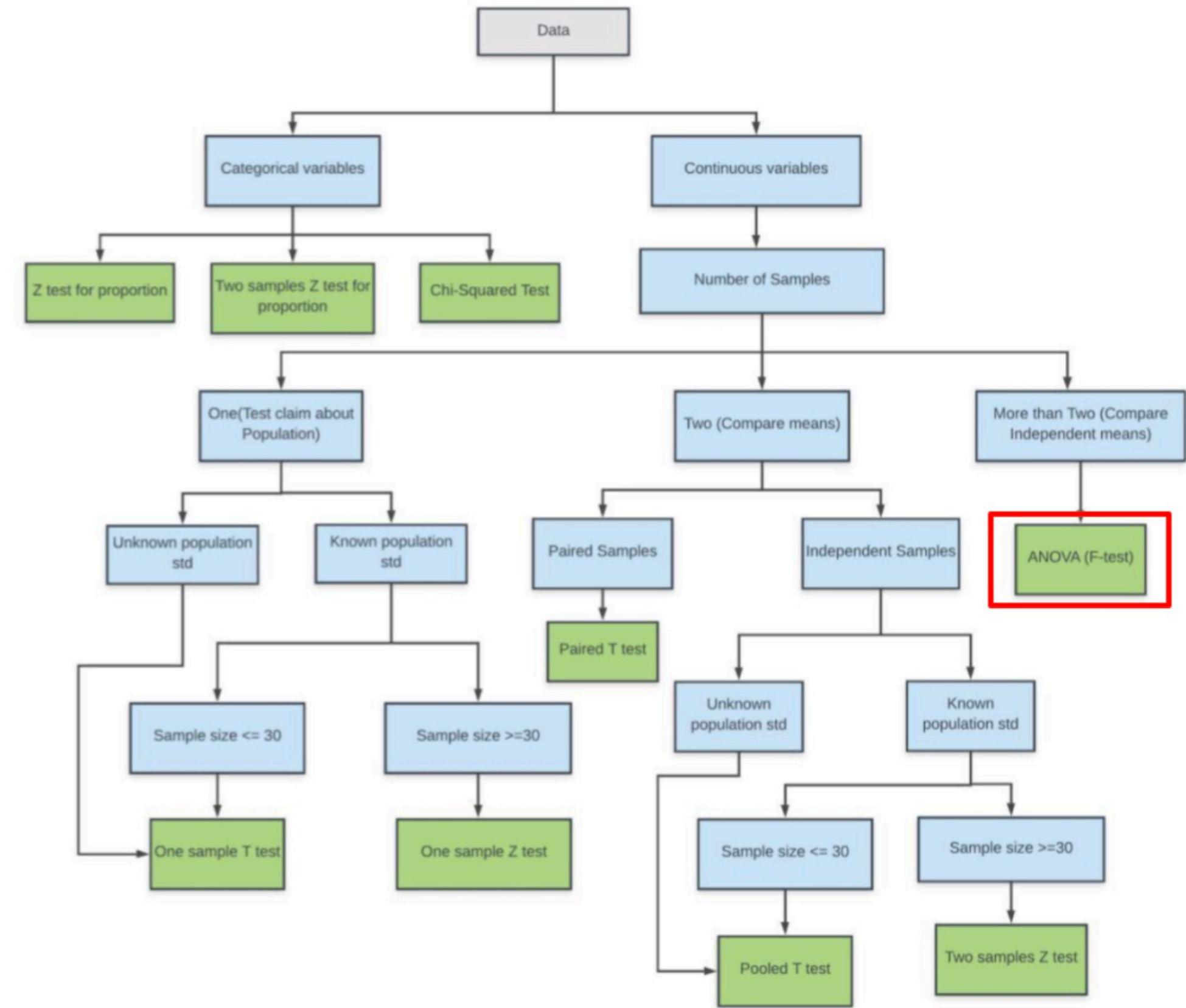




## LETS PRACTICE

- (H) Is there a difference in the mean mileage for the four populations of vehicle size (small cars, large cars, trucks, and SUVs)?

# SOLUTION

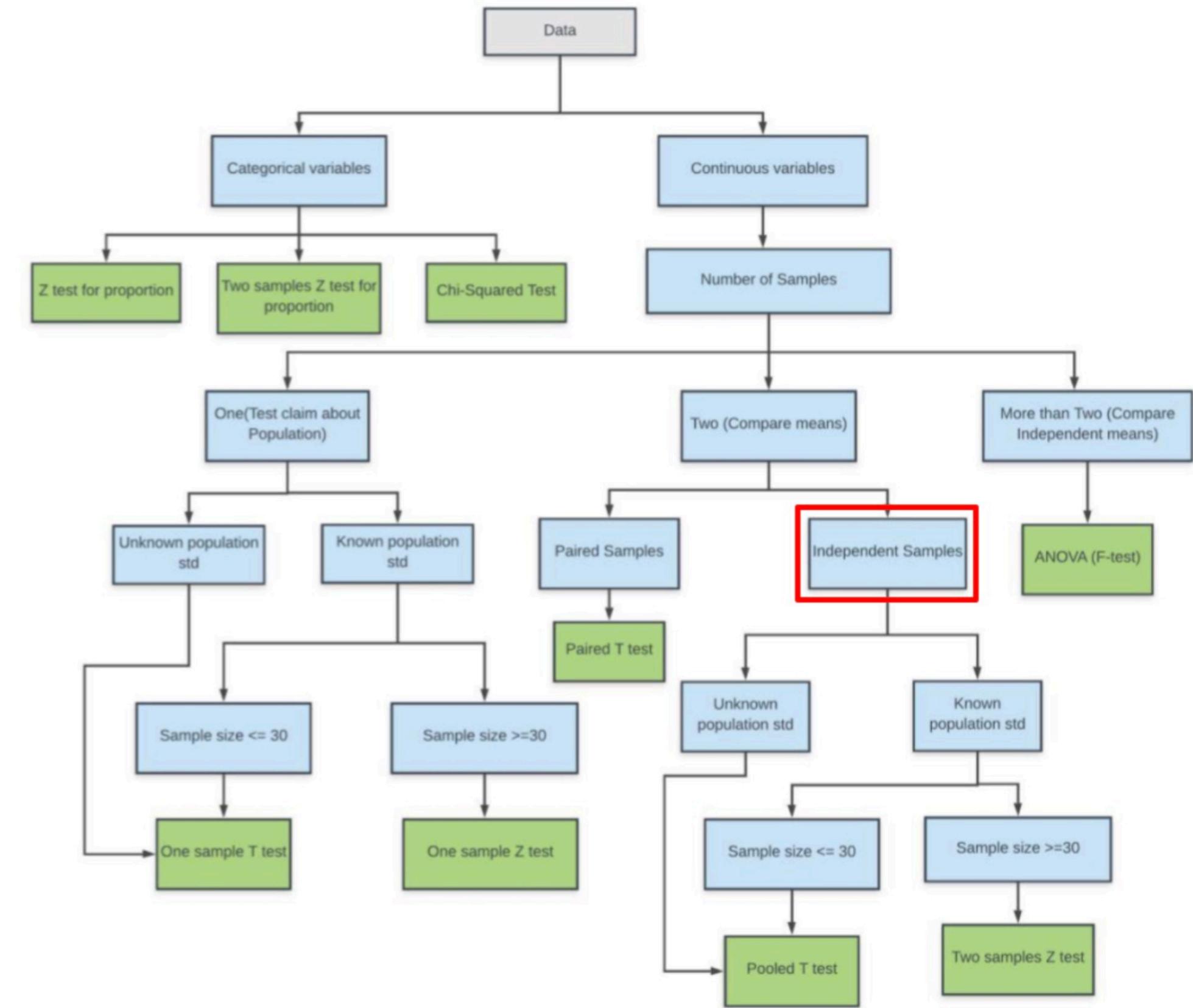




## LETS PRACTICE

- (C) Are the average delays for two airlines are *different*?

# SOLUTION

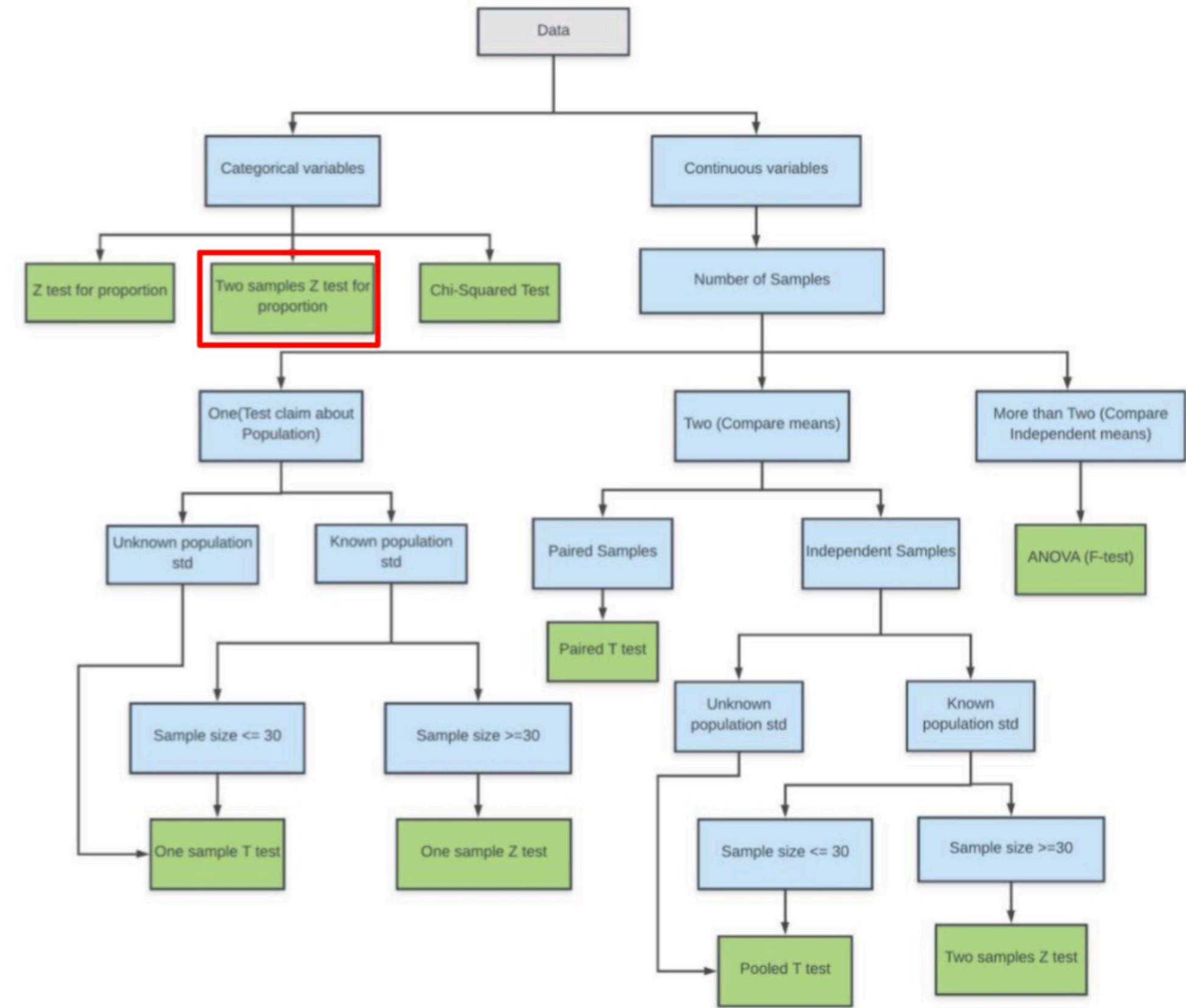




## LETS PRACTICE

- (E) Is there a difference in **proportions** of male and female students who missed at least one discussion during this semester?

# SOLUTION



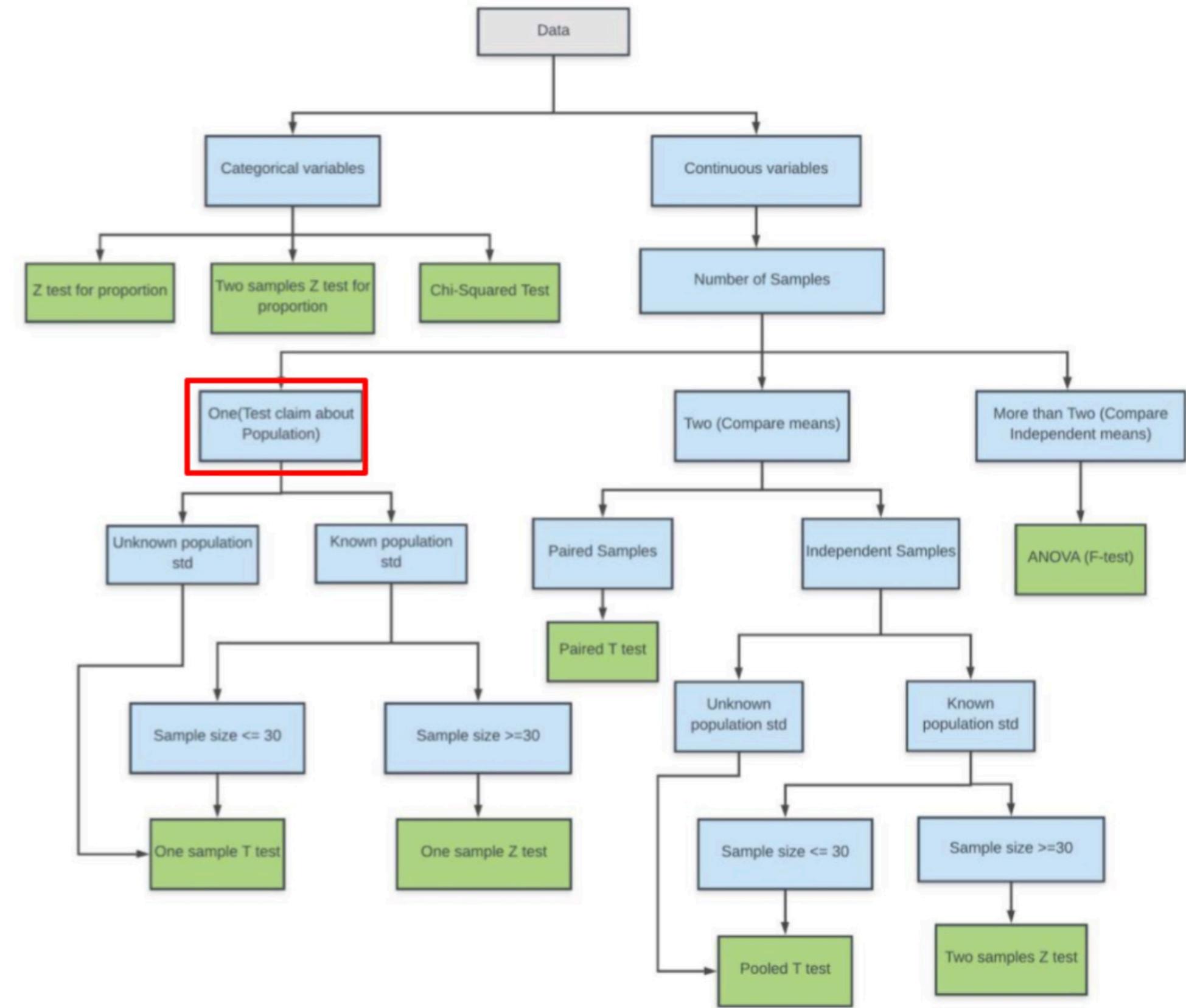


## LETS PRACTICE

**Q:** Ted Copper of the local police unit is concerned about the safety of drivers along Interstate 93. He would like your help in deciding which statistical test might be used for each of his questions. Read each of the research questions in parts (a) through (e) below and then write the appropriate statistical test:

- The police are interested in seeing how fast drivers typically go. The speed limit is 75 mph, but the officer suspects that they go faster.
- A large sample of male and female drivers is taken and each driver is asked the maximum mph over the speed limit they have ever gone. The police suspect that males will have driven faster than females, but want to test their theory.
- A sample of drivers is asked how fast they drove on Friday (max speed) and how fast they drove on Thursday. The police want to investigate if drivers head to their destinations faster on the weekend.
- A sample of 60 drivers is taken and each person is asked if they have ever used a cell phone while driving. Police are curious as to whether or not the rate has increased over a 60% rate from two years ago.
- A large sample of young, middle and older drivers is taken and each driver is asked how fast they drove. The police suspect that young drivers are faster than the others, but want to test their theory.

# SOLUTION



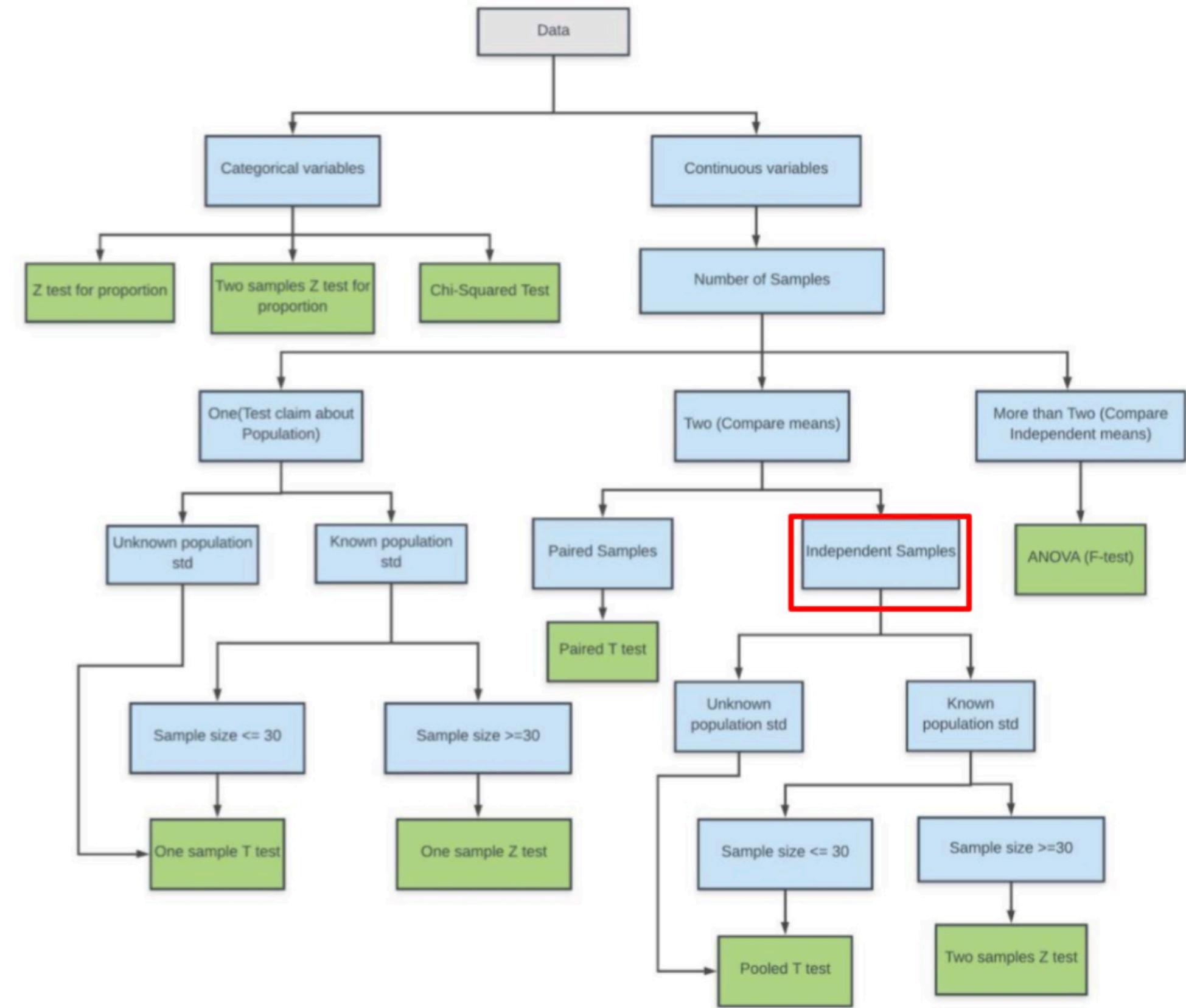


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# SOLUTION



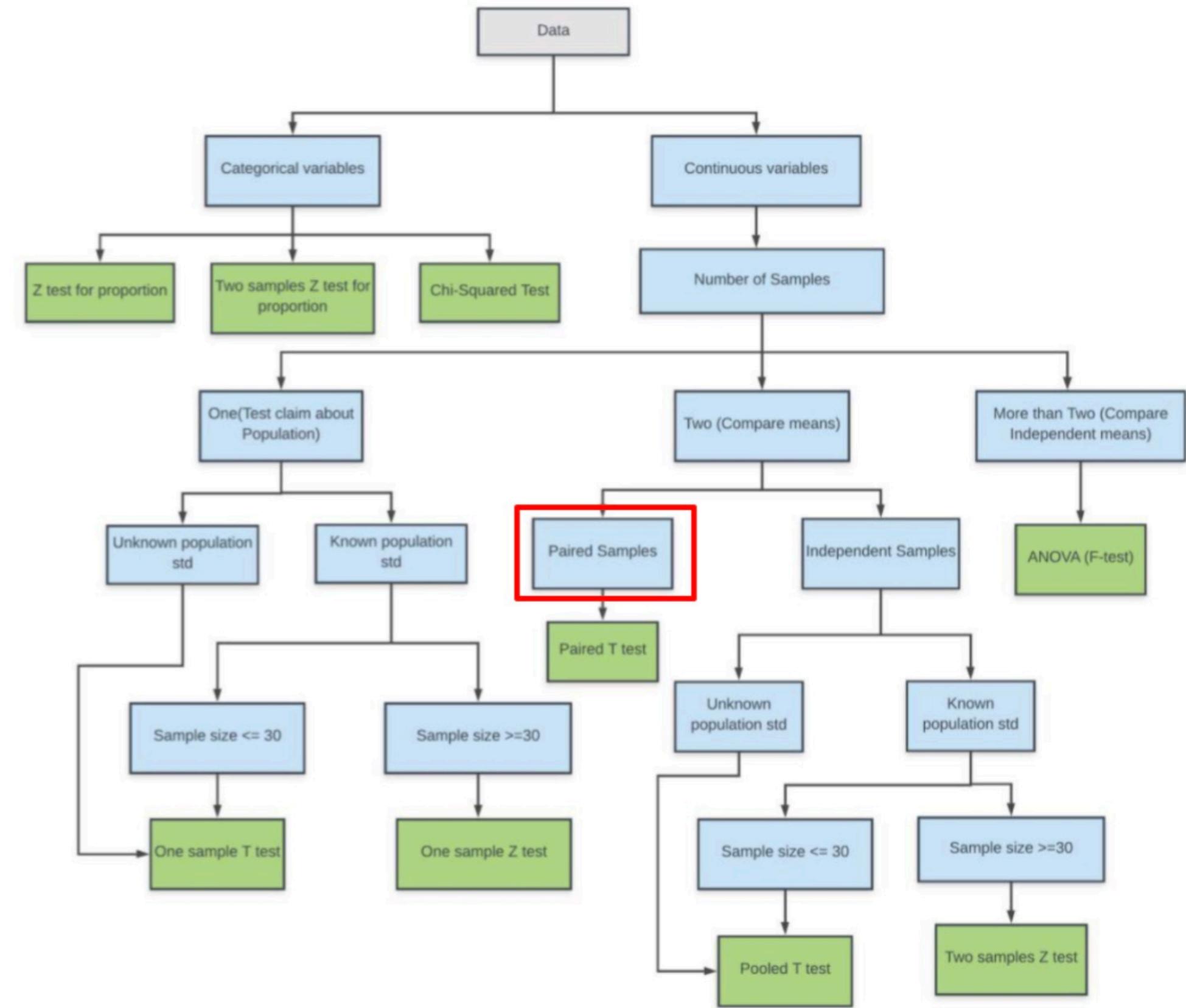


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# SOLUTION



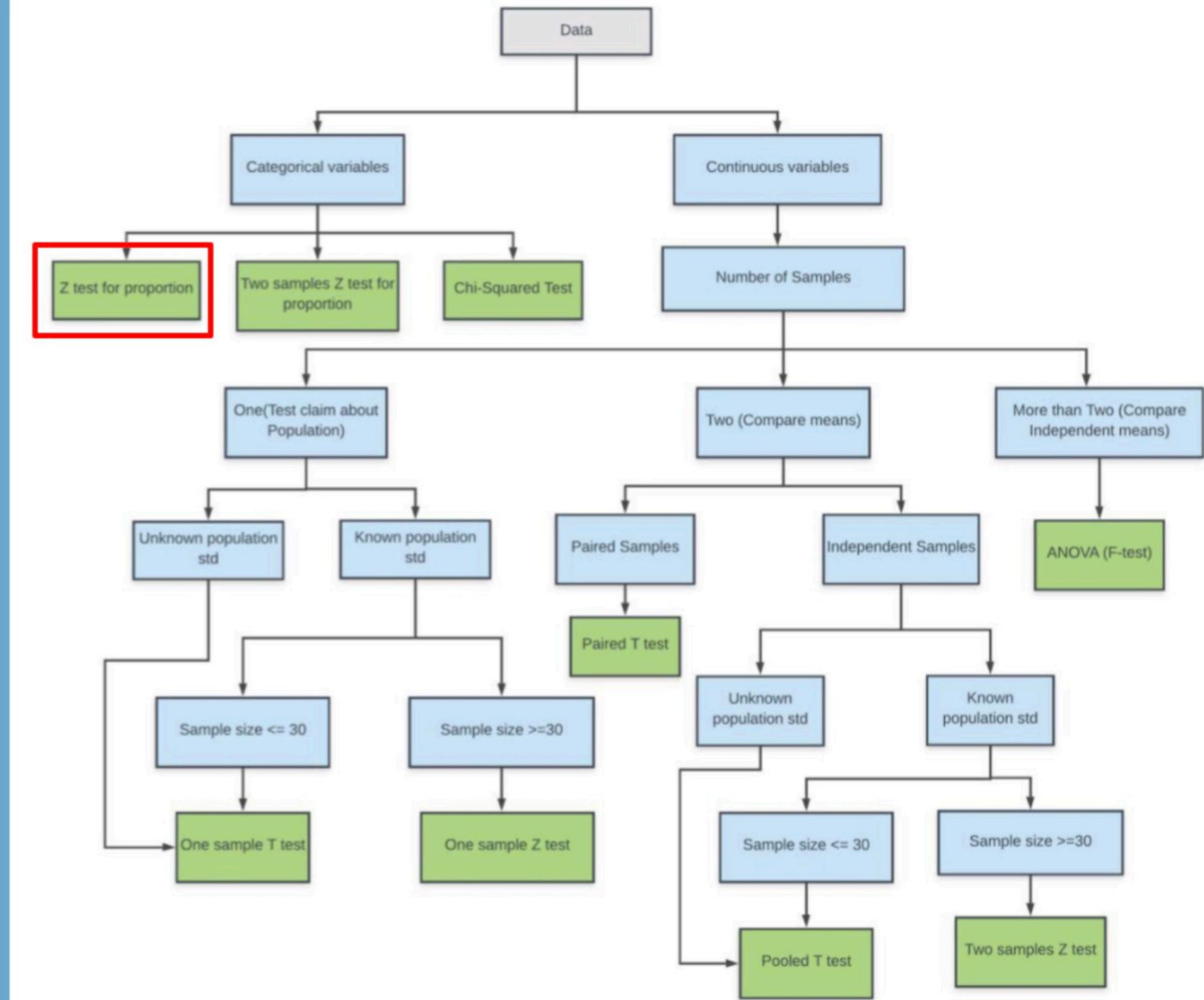


## LETS PRACTICE

**Q:** Ted Copper of the local police unit is concerned about the safety of drivers along Interstate 93. He would like your help in deciding which statistical test might be used for each of his questions. Read each of the research questions in parts (a) through (e) below and then write the appropriate statistical test:

- The police are interested in seeing how fast drivers typically go. The speed limit is 75 mph, but the officer suspects that they go faster.
- A large sample of male and female drivers is taken and each driver is asked the maximum mph over the speed limit they have ever gone. The police suspect that males will have driven faster than females, but want to test their theory.
- A sample of drivers is asked how fast they drove on Friday (max speed) and how fast they drove on Thursday. The police want to investigate if drivers head to their destinations faster on the weekend.
- A sample of 60 drivers is taken and each person is asked if they have ever used a cell phone while driving. Police are curious as to whether or not the rate has increased over a 60% rate from two years ago.
- A large sample of young, middle and older drivers is taken and each driver is asked how fast they drove. The police suspect that young drivers are faster than the others, but want to test their theory.

# SOLUTION



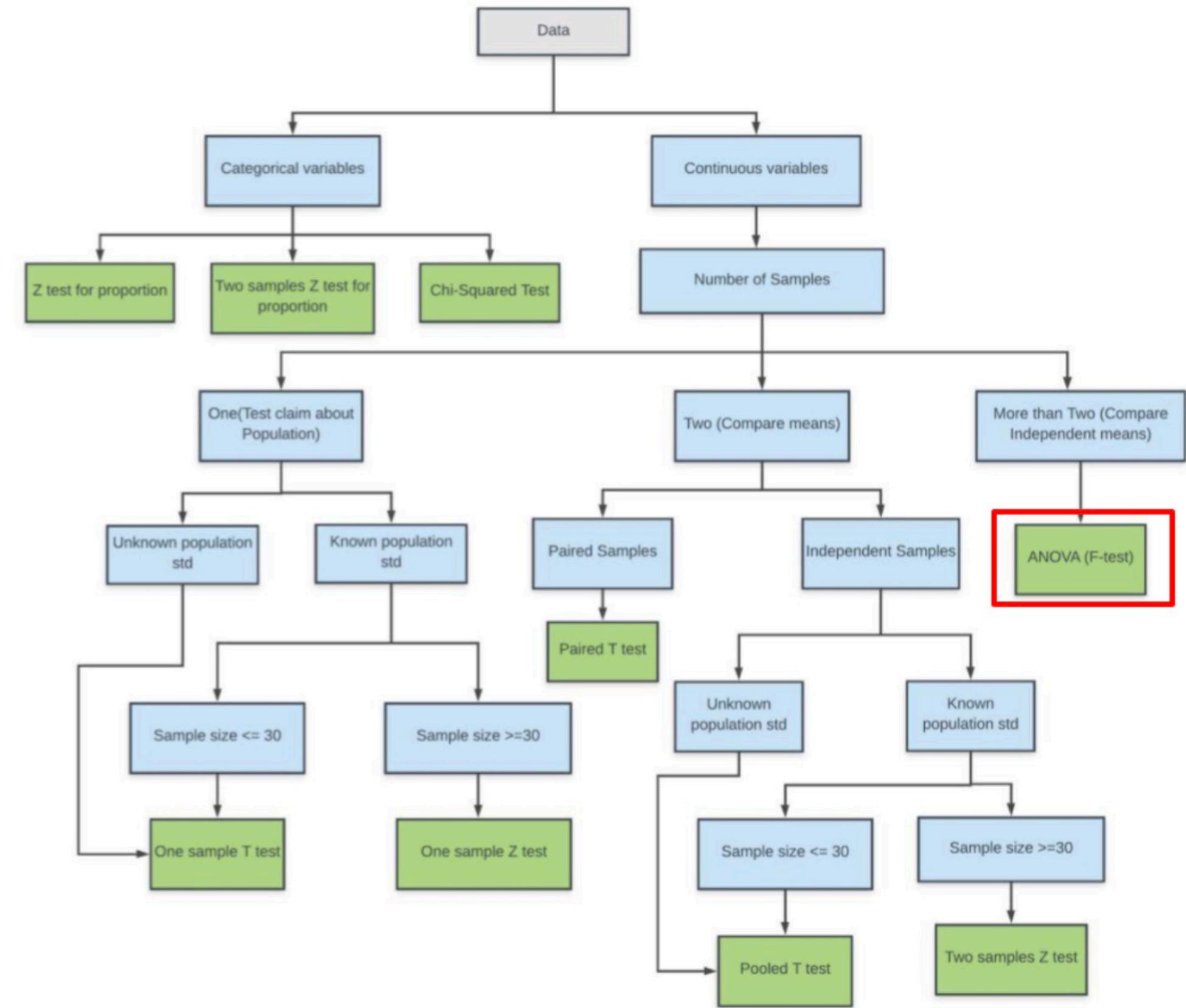


## LETS PRACTICE

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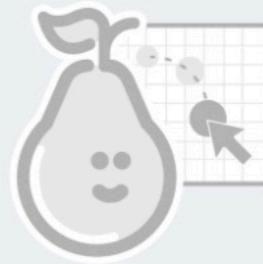
Have you understood the Hypothesis Tests?



Students, drag the icon!



Pear Deck Interactive Slide  
Do not remove this bar



No Draggable™ Response  
You didn't answer this question

NEXT SLIDE



# THANKS!

## Any questions?

You can find me at:

- ▶ [jason@clarusway.com](mailto:jason@clarusway.com)

