Bounded Model Checking

COMP6210 - Automated Software Verification

ECS, University of Southampton

ROBDDs and Symbolic Model Checking (Review)

ROBDD is a *canonical form* to represent Boolean functions:

- often more compact than traditional normal forms,
- can be manipulated efficiently.

Reachable state space is represented as an **ROBDD**.

Property verification uses iterative computations on the reachable state space.

Some Limitations in Working with ROBDDs

ROBDDs are fantastic, but they come with a few problems:

- they can often become large,
- variable ordering must be uniform along paths,
- selecting the "right" variable ordering is crucial,
- in some cases *no space-efficient variable ordering exists*.

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Are there any alternatives in model checking?

Yes! SAT-based methods.

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E.g. is the following Boolean (propositional) formula satisfiable?

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That is, does there exist an assignment of truth values to the Boolean variables x_1, x_2, x_3, x_4 that makes the formula True?

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Yes! $x_1 =$ False, $x_2 =$ True, $x_3 =$ True, $x_4 =$ False.

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The SAT problem was the first example of an **NP-Complete** problem (*Cook-Levin theorem*, 1971).

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Problem: *can we factorise integers using SAT?*

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Problem: *can we factorise integers using SAT?*

Yes! Model a *multiplier circuit* as a Boolean formula, and ask if there are inputs to the circuit that result in a given number (e.g. 21, or 10101 in binary) on the output wires.

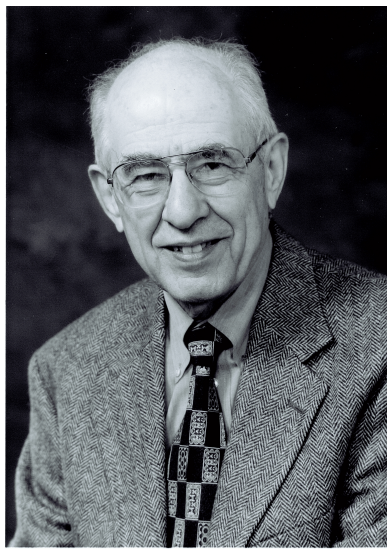
This is equivalent to asking "is 21 a prime number?".

DPLL Algorithm

Davis–Putnam–Logemann–Loveland (DPLL) Algorithm

- Introduced in 1962 as a refinement of an earlier 1960 algorithm by Davis and Putnam.
- A backtracking-based algorithm for solving the **SAT problem**.
- Worst case running time is exponential.
- Serves as the backbone of modern **SAT solvers** (e.g. MiniSAT, Z3).
- Major practical applications in *hardware verification*.

Martin Davis



Hilary Putnam

DPLL and SAT Solvers

DPLL can take an **exponential** amount of time, however it can often find solutions to SAT relatively quickly.

The **DPLL** algorithm works by

1. Converting the Boolean formula into CNF (*Conjunctive Normal Form*)
2. Applying simplification rules
3. Splitting

Conjunctive Normal Form

A *literal* is a atomic formula or its negation (e.g. a propositional variable x_1, or $\neg x_1$).

A *clause* is a disjunction of literals (e.g. $x_1 \vee \neg x_3 \vee x_2$)

A propositional formula is in Conjunctive Normal Form (CNF) if it is a conjunction of one or more clauses, e.g.

$$(x_1 \vee \neg x_3 \vee x_2) \wedge (\neg x_4 \vee \neg x_1 \vee x_3) \wedge (\neg x_5)$$

Any given propositional formula can be converted into CNF by applying transformations

- eliminating double negations (i.e. $\neg\neg\phi$ becomes ϕ)
- applying De Morgan's laws,
- applying the distributive law.

Conjunctive Normal Form

Caveat: Converting a formula into CNF can result in an exponentially large formula.

In **DPLL**, each clause is treated as a *set of literals*, e.g. $(x_1 \vee \neg x_3 \vee x_2)$ is treated as $\{x_1, \neg x_3, x_2\}$.

The overall CNF formula $C_1 \wedge C_2 \wedge \cdots \wedge C_l$, where C_i are clauses, is also thought of as a *set*

$$\{C_1, C_2, \ldots, C_l\}$$

Caveats: an empty clause $\{\}$ is unsatisfiable, whereas an empty *set of clauses* (which is equivalent to True) is considered satisfiable.

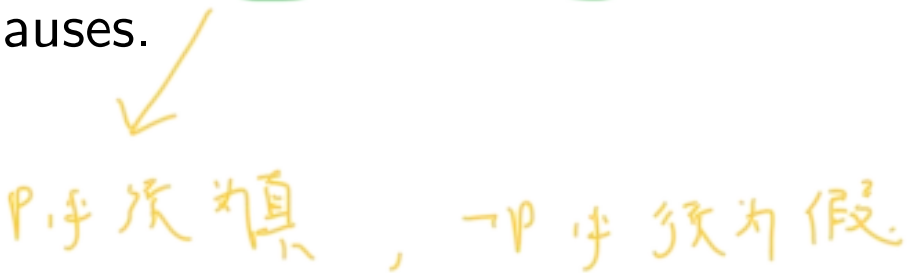
空子句是不可接受的(没有文字的子句.) → 所有对值都为假

空集合是可接受的(没有子句) → 任何对值都为真

DPLL Simplification Rules

There are two main rules

1. *Unit propagation (one literal rule)*: when there is a unit clause (a clause with just one element p), remove $\neg p$ from all the other clauses.



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2. *Affirmative-negative rule*: if p occurs either only negated, or only unnegated, then remove all clauses containing p.

肯否原则

p只以一种形式出现，
则移除所有含p的项

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Both simplification rules *preserve satisfiability*.

DPLL Splitting

The simplification rules by themselves do not generally allow us to check satisfiability.

To make further progress, we must consider *case splits*.

For a set of clauses Γ, choose a propositional variable x and consider two cases:

1. $\Gamma \cup \{x\}$ ("set x to True"),
2. $\Gamma \cup \{\neg x\}$ ("set x to False").

Then apply the simplification rules (unit propagation and affirmative-negative rule) as before.

Applying these steps recursively one obtains a backtracking strategy for solving the SAT problem.
[*If an empty set of clauses is found, the formula is satisfiable.*]

Current State of SAT Solving

Modern implementations of SAT solvers improve on the basic ideas in DPLL.

SAT solvers are capable of solving practical SAT problems with *millions* of propositional variables.

Enormous progress has been made in SAT solving technology over the past 20 years, spurred on by the SAT competition.

http://www.satcompetition.org/

Bounded Model Checking

Safety verification: *can we find a bad state in k steps?*

Verification of temporal properties (LTL, CTL):
can we find a counterexample to the property in k steps?

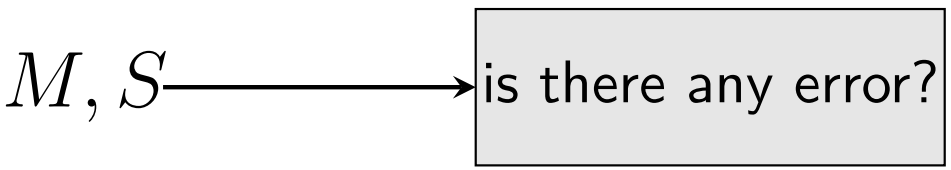
反例

Why bounded?

- Model Checking is supposed to be exhaustive, i.e. the *whole state space* of the system is explored. 全面的
- In real world systems, the number of states is usually **too big** (*state space explosion problem*).
- We can explore only a subset of the state space
  ... → Bounded Model Checking (BMC).

Model Checking versus Bounded Model Checking

Model Checking:



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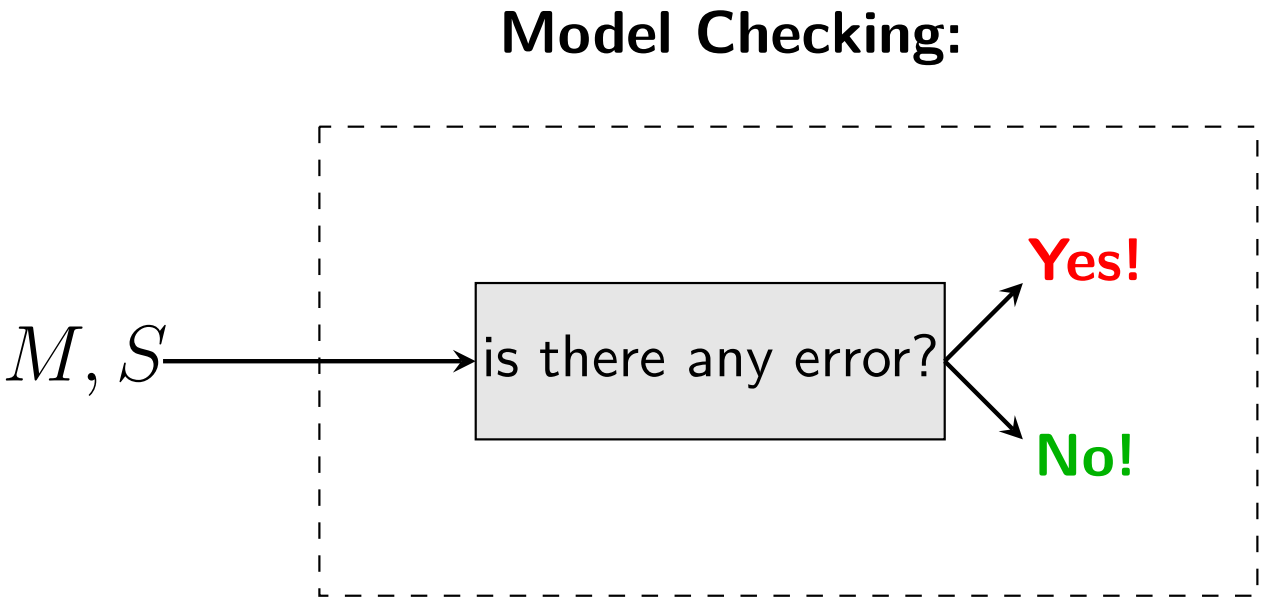

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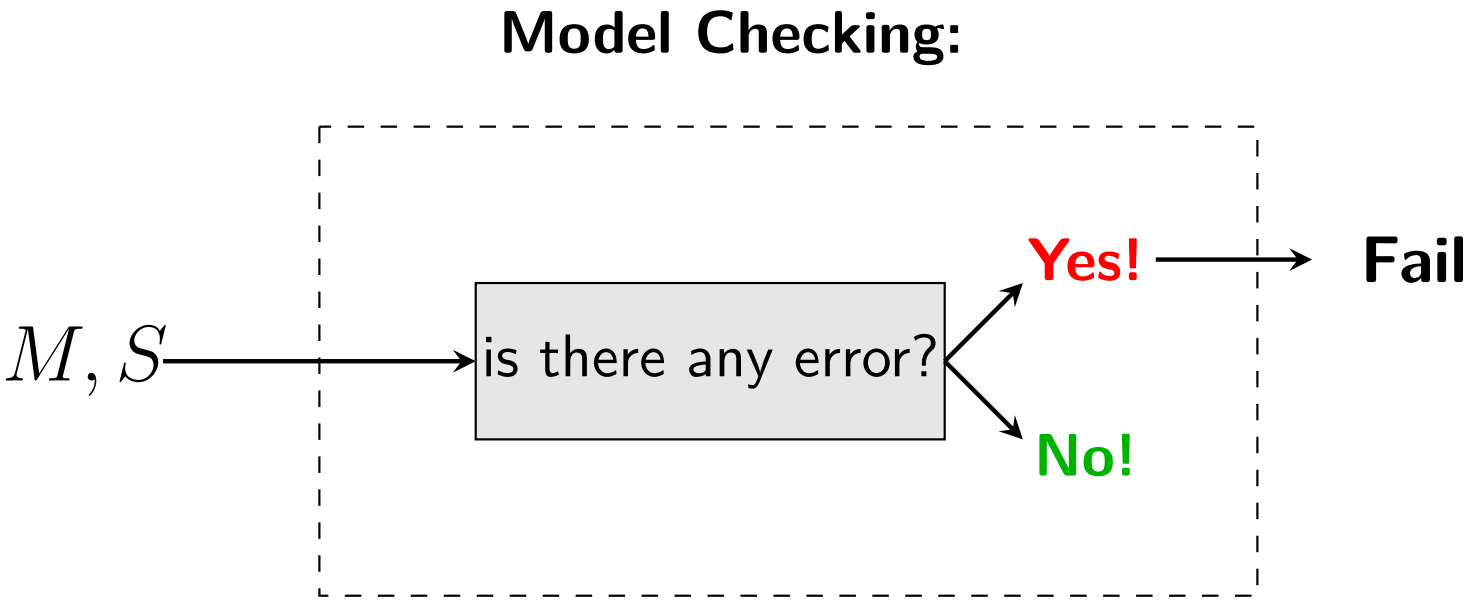

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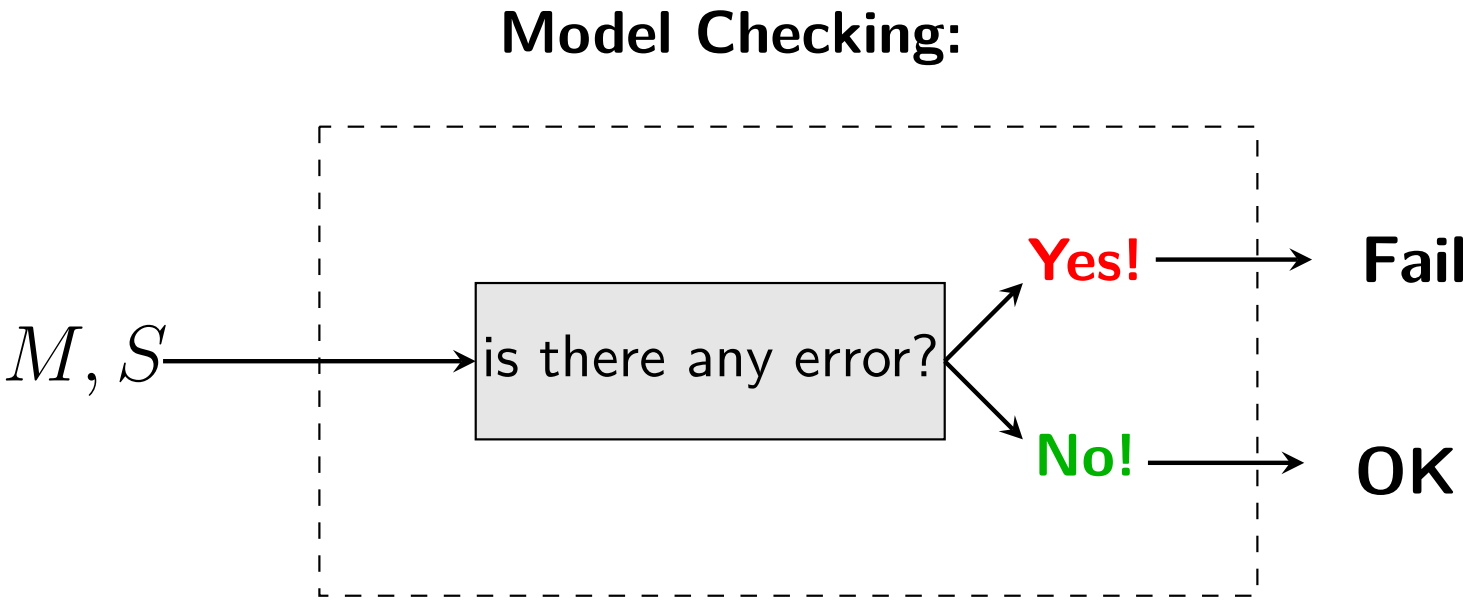

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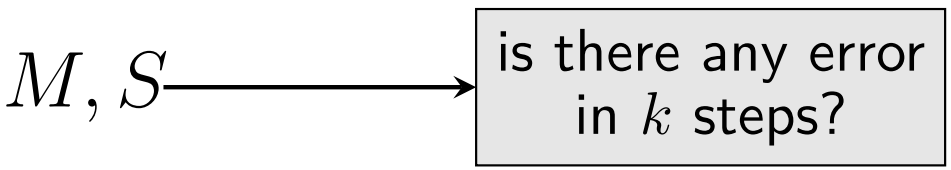

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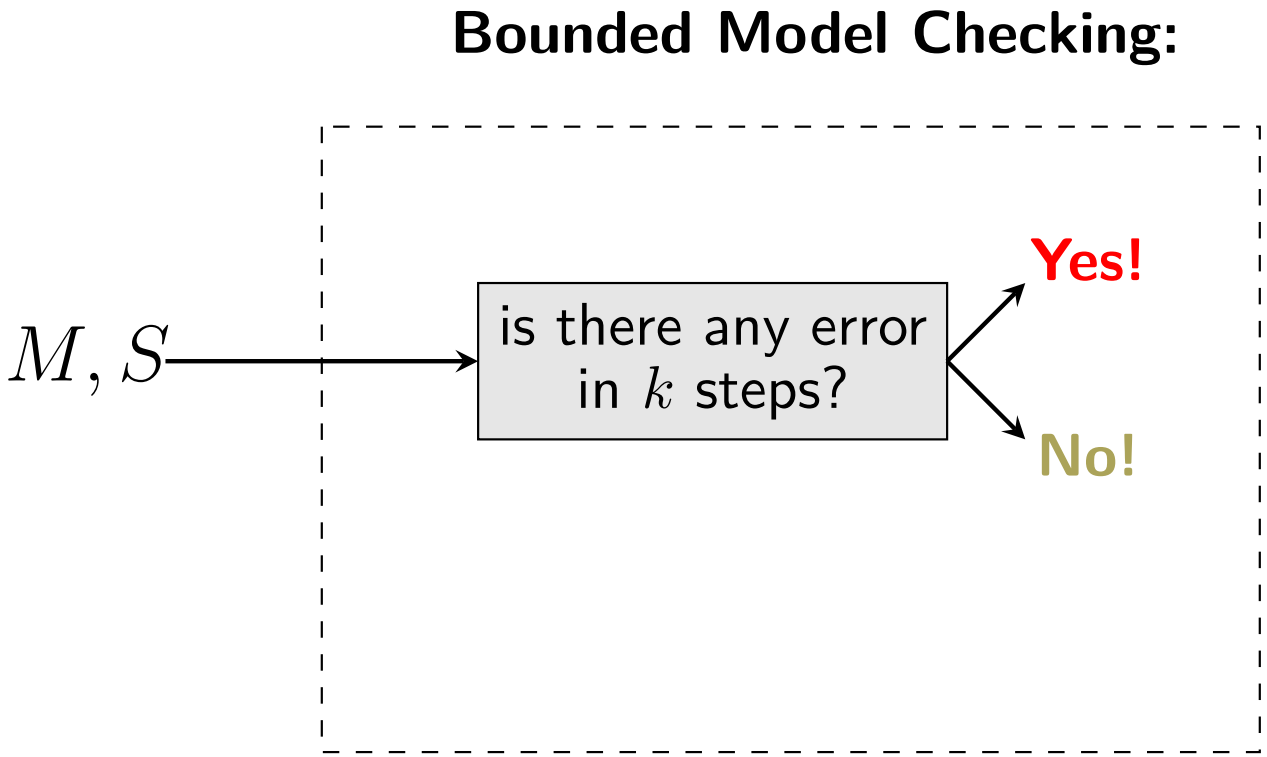

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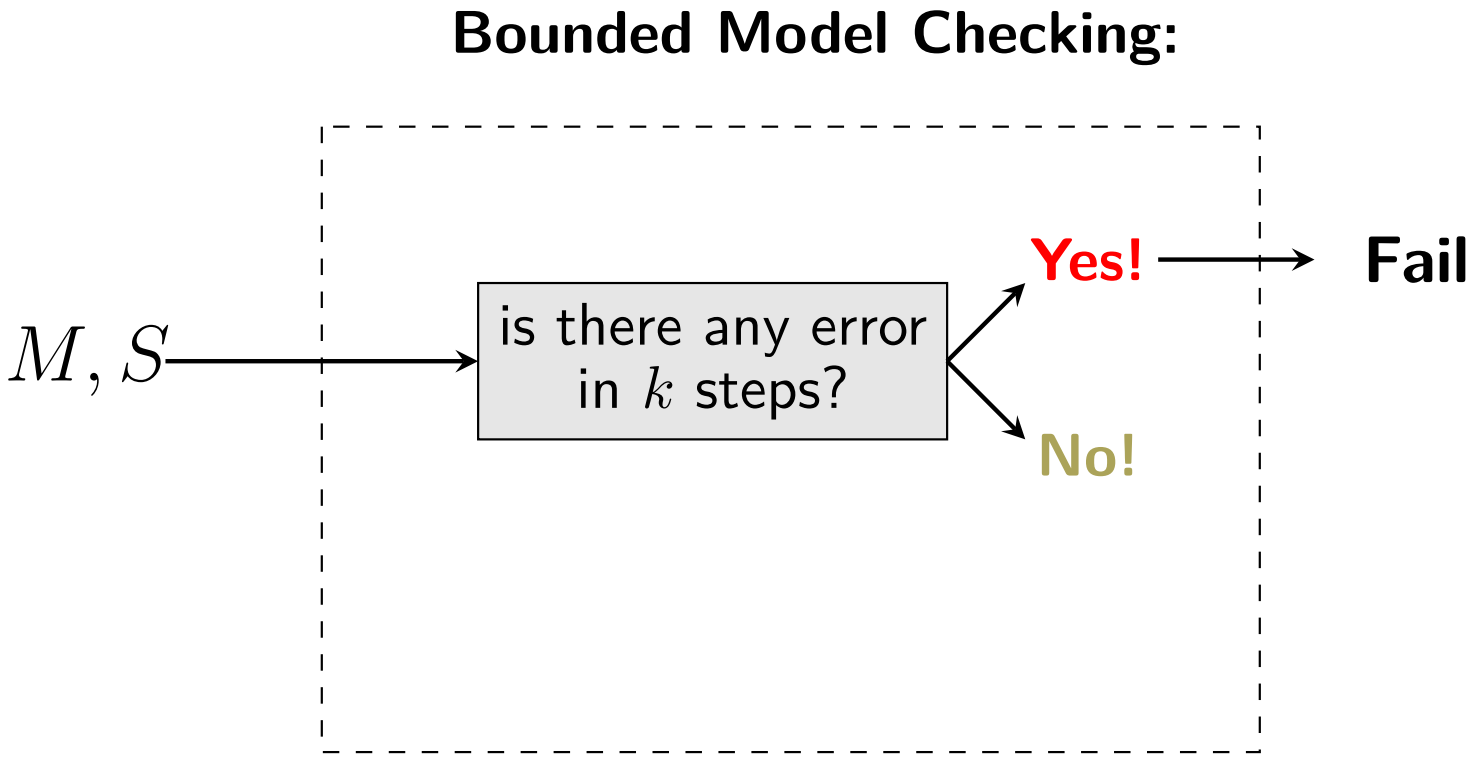

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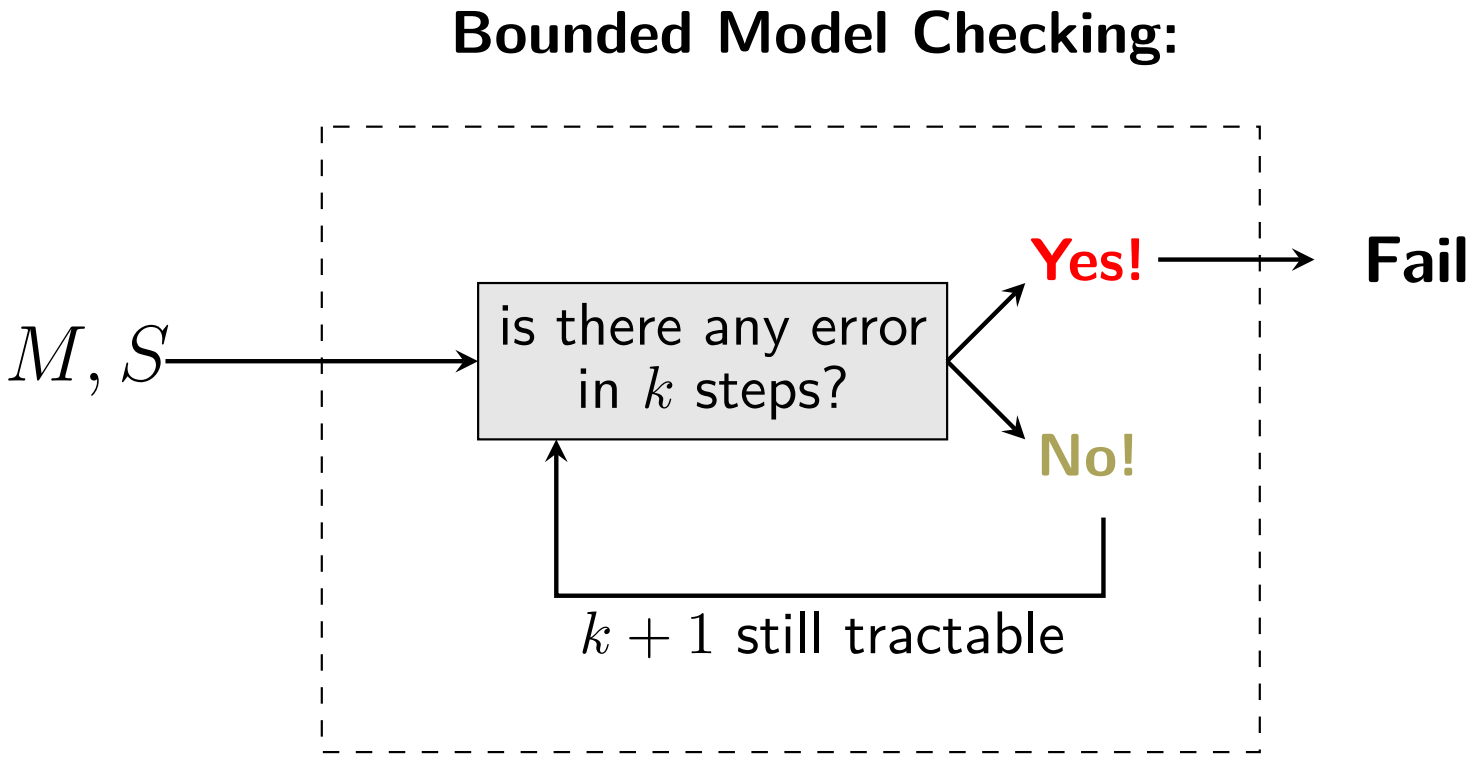

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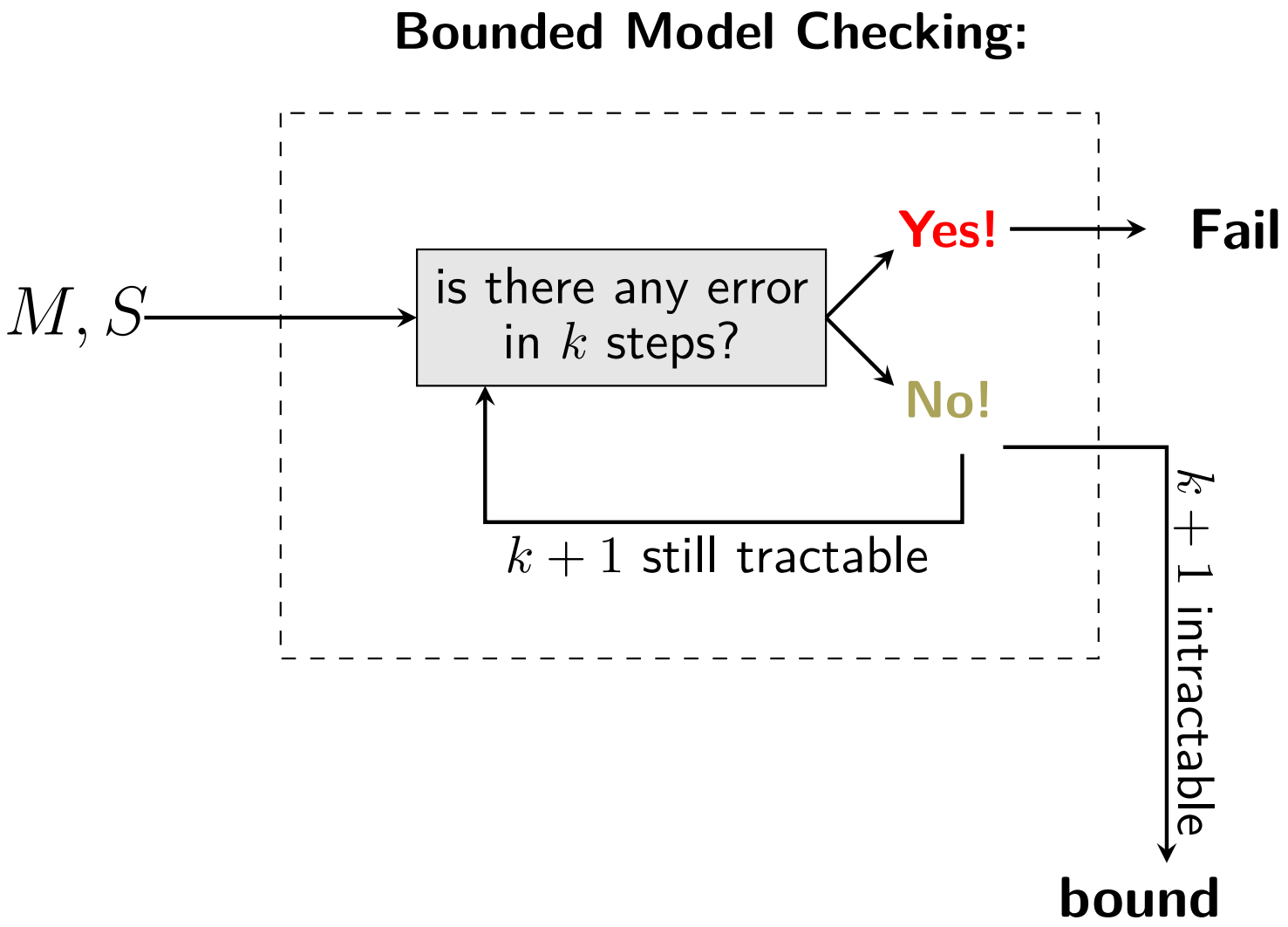

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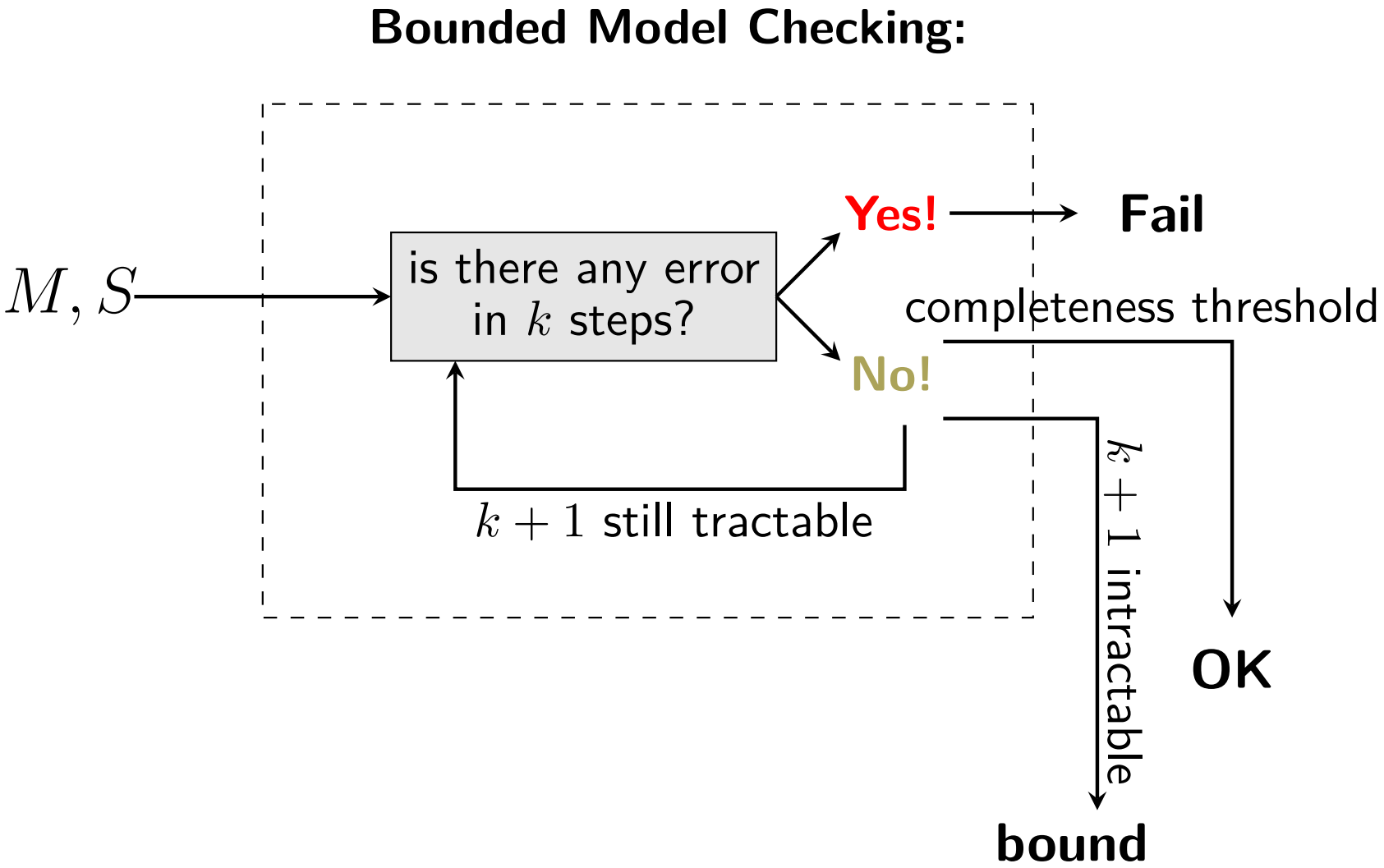
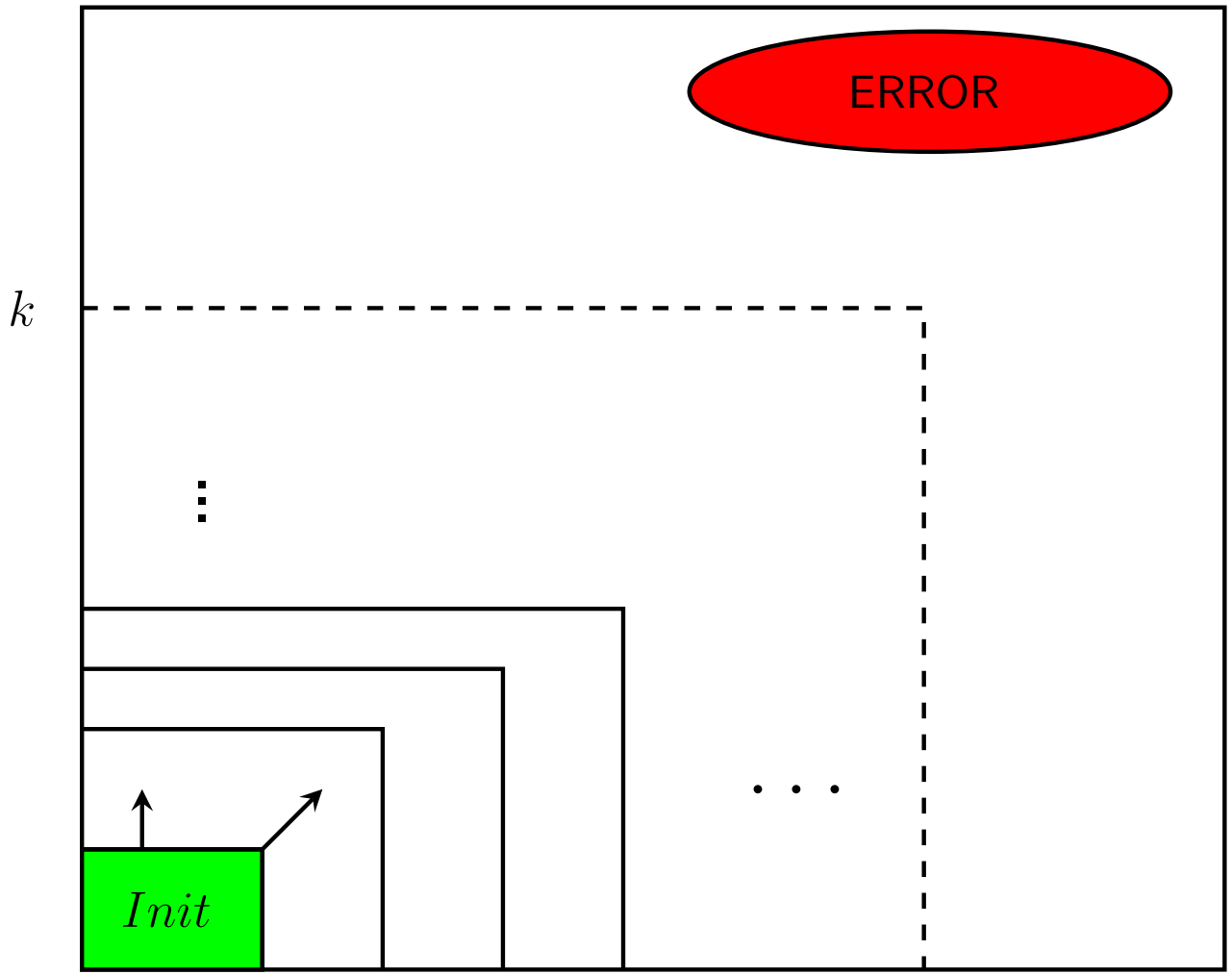


Bounded Model Checking (Main Idea)

BMC: check if a property holds for a subset of states.



Bounded Model Checking for Reachability

是否能在k步内到达目标状态

Given a *positive integer* k, is the **Target** reachable in k steps?
(*Think of* **Target** *as defining bad states*.)

We encode a **BMC** problem as a **SAT** problem.

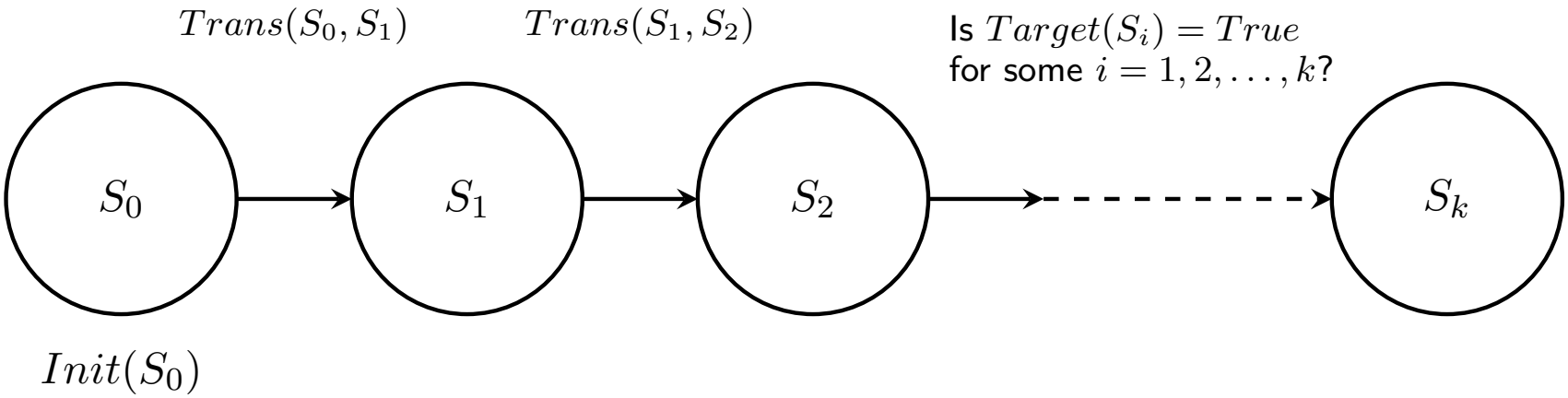
The Bounded Model Checking problem

$$X,\ Init(X),\ Target(X),\ Trans(X, X'),\ k$$

is transformed into a Boolean formula Φ_k

Φ_k is satisfiable iff **Target** is reachable from $Init$ within k steps following $Trans$.

Bounded Model Checking for Reachability



状态变量的集合

- $S_0, S_1, S_2, \ldots, S_k$ are copies of $X = (x_1, x_2, \ldots, x_n)$:
  - $S_0 = ({x_1}^0, {x_2}^0, \ldots, {x_n}^0)$,
  - $S_1 = ({x_1}^1, {x_2}^1, \ldots, {x_n}^1)$,
  - $S_2 = ({x_1}^2, {x_2}^2, \ldots, {x_n}^2)$,
  - $\vdots$
  - $S_k = ({x_1}^k, {x_2}^k, \ldots, {x_n}^k)$.

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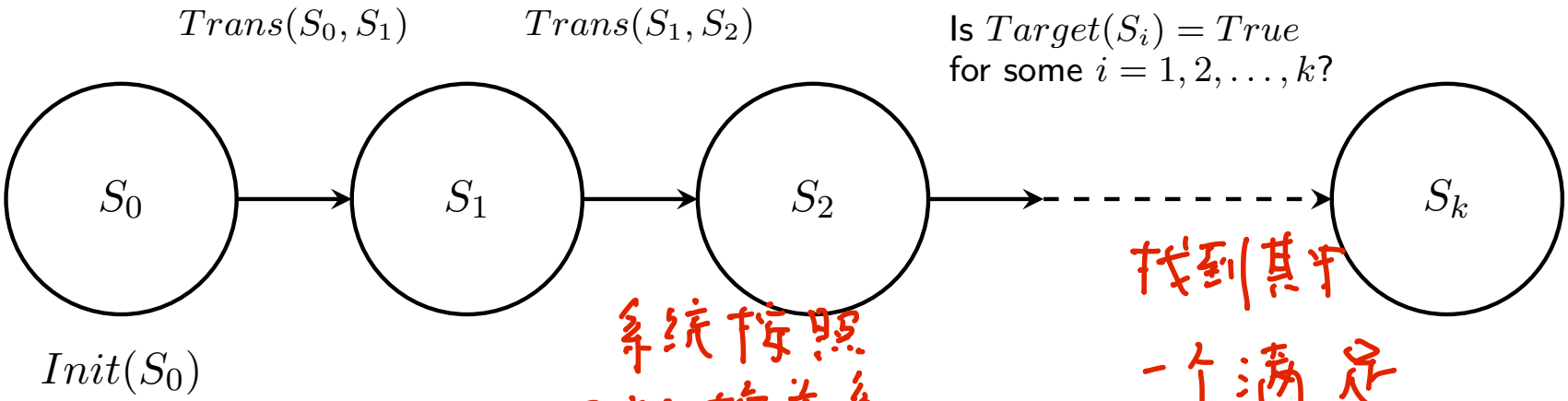
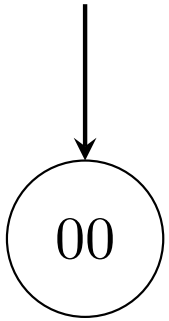


$$\Phi_k = Init(S_0) \wedge \left(\bigwedge_{i=0}^{k-1} Trans(S_i, S_{i+1}) \right) \wedge \left(\bigvee_{i=0}^{k} Target(S_i) \right)$$

Φ_k is satisfiable if and only if Target is reachable within k steps.

(The size of Φ_k is $|Init| + k|Trans| + k|Target|$.)

Example: Two Bit Counter



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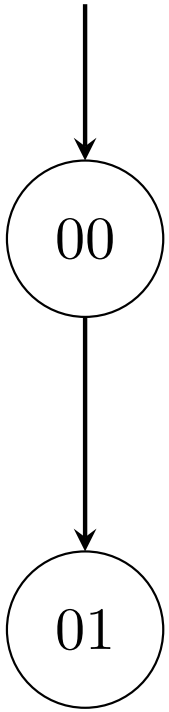

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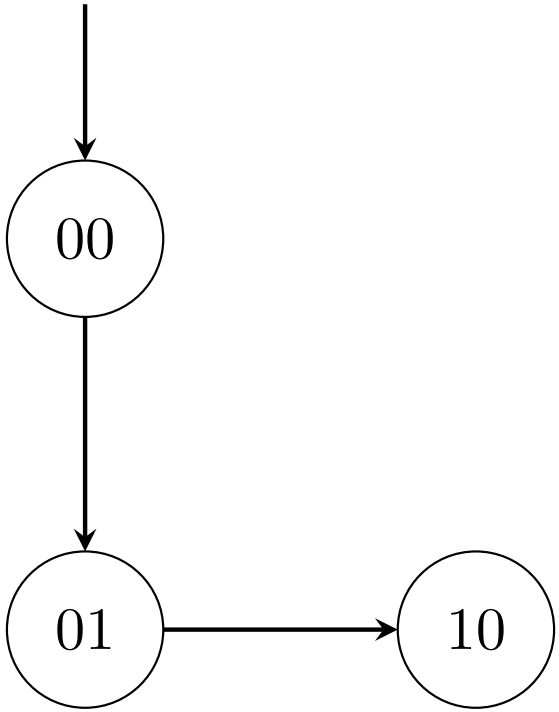

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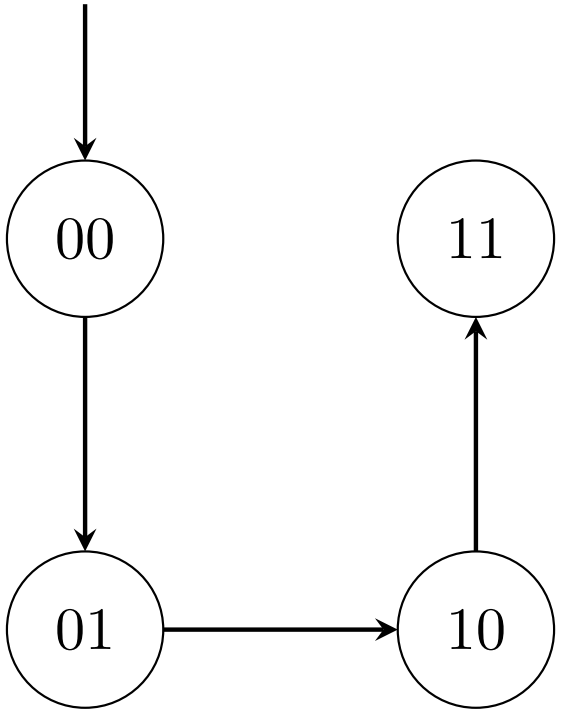

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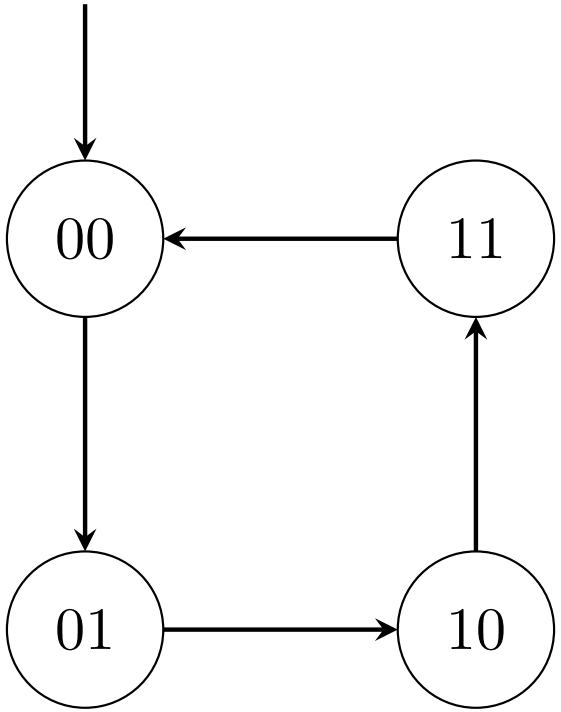

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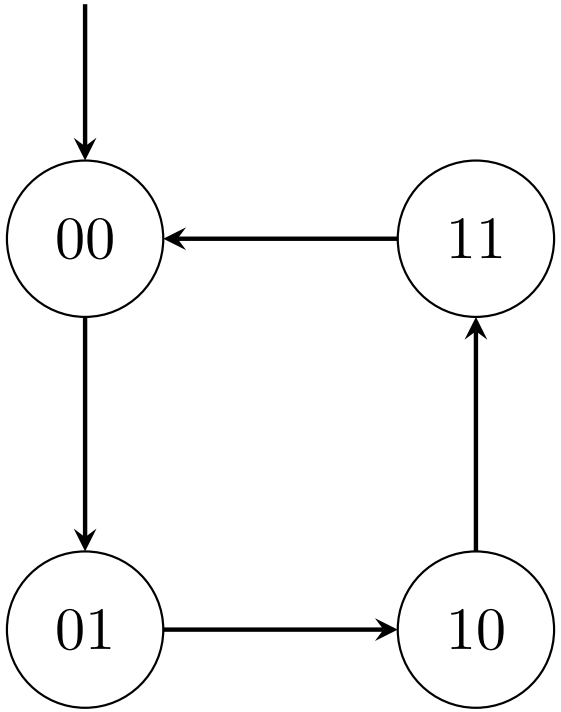

Two Boolean variables l and r (for the left and right bit).

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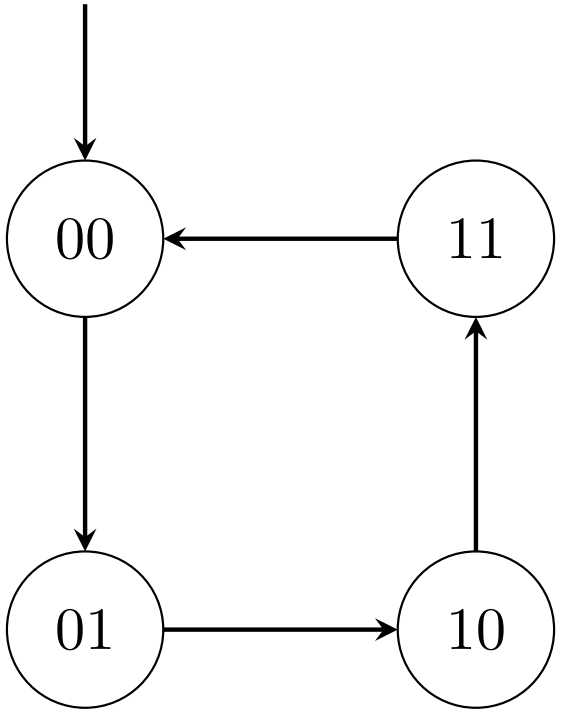

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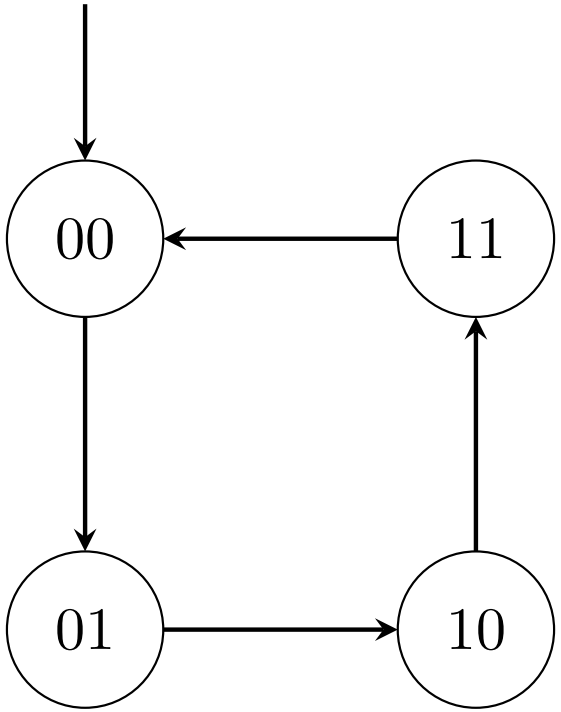

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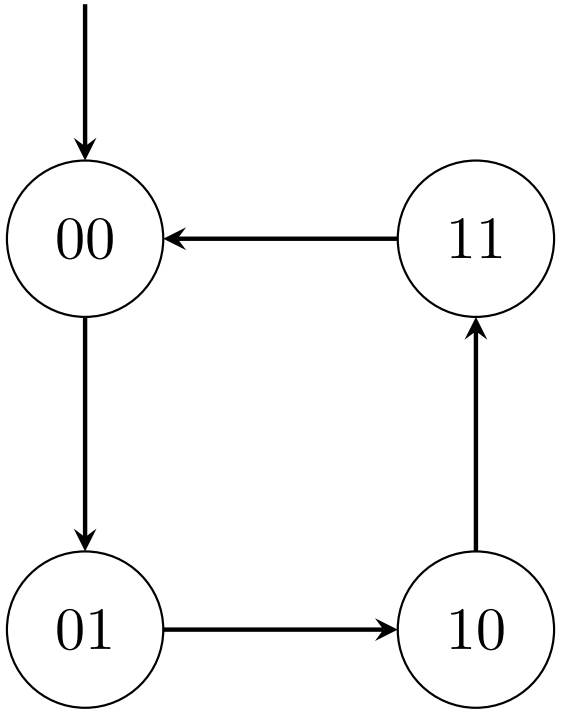

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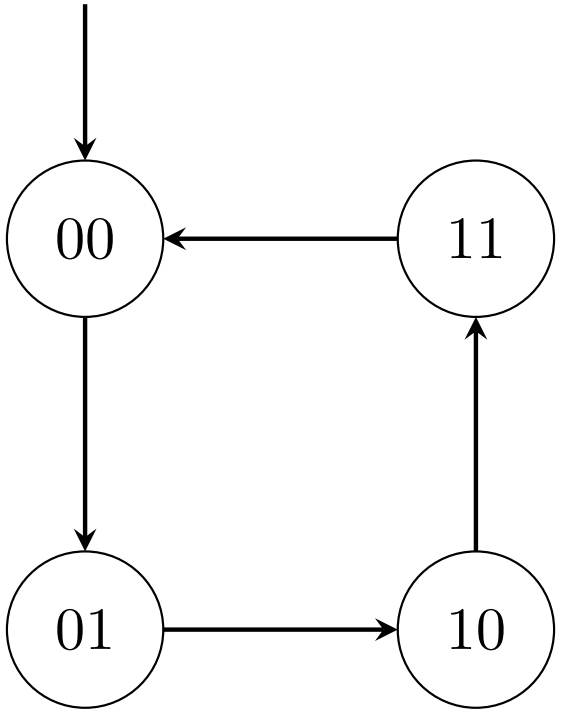

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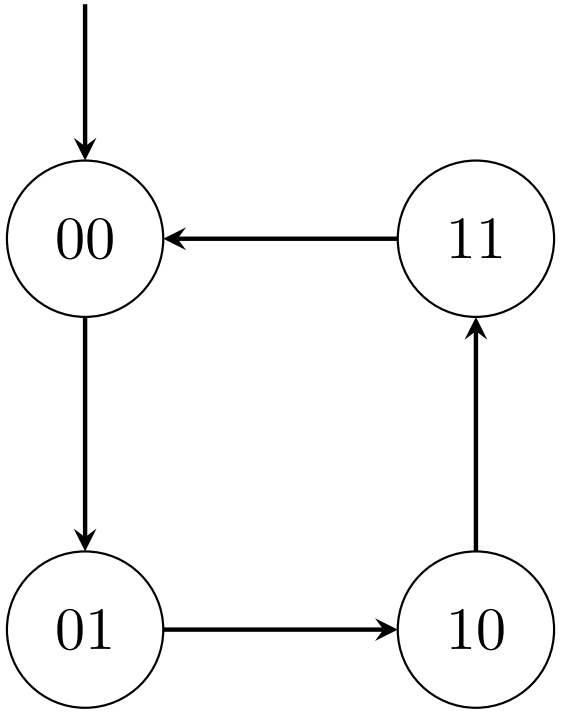

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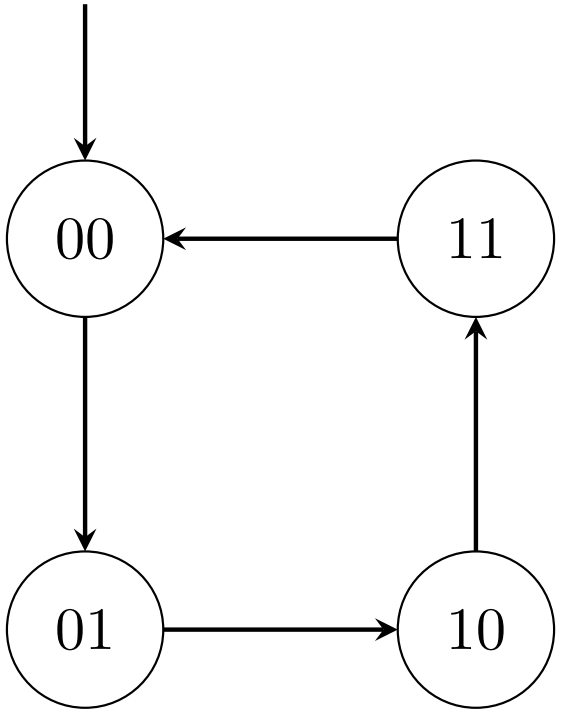

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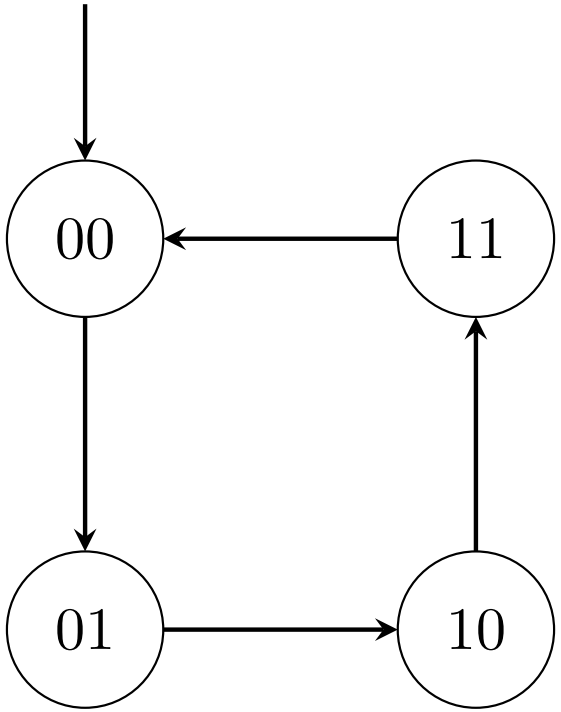

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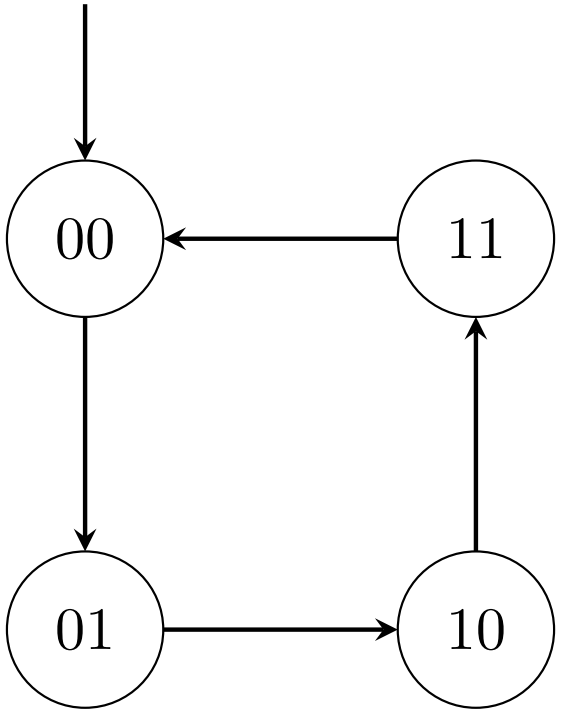
$Init \equiv \neg l \wedge \neg r$. $Trans \equiv (r' = \neg r$

Example: Two Bit Counter

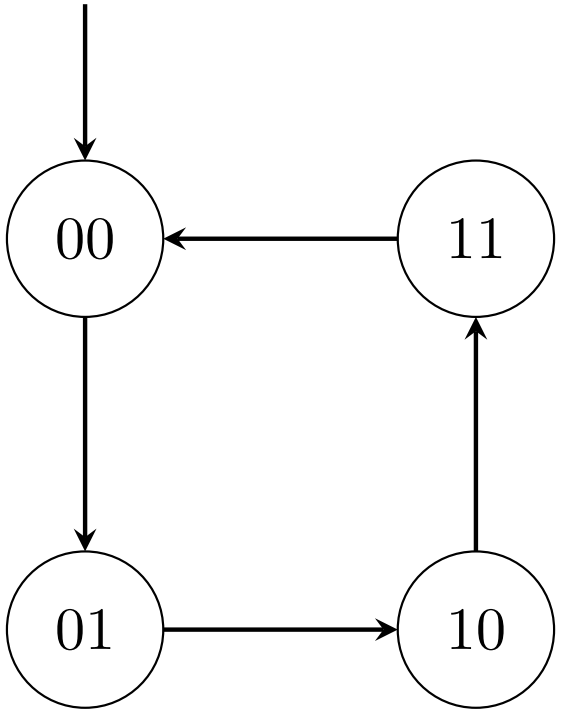


Two Boolean variables l and r (for the left and right bit).

Safety property: $G(\neg l \vee \neg r)$. Therefore $Target \equiv l \wedge r$.

$Init \equiv \neg l \wedge \neg r$. $Trans \equiv (r' = \neg r \wedge l' =$

Example: Two Bit Counter

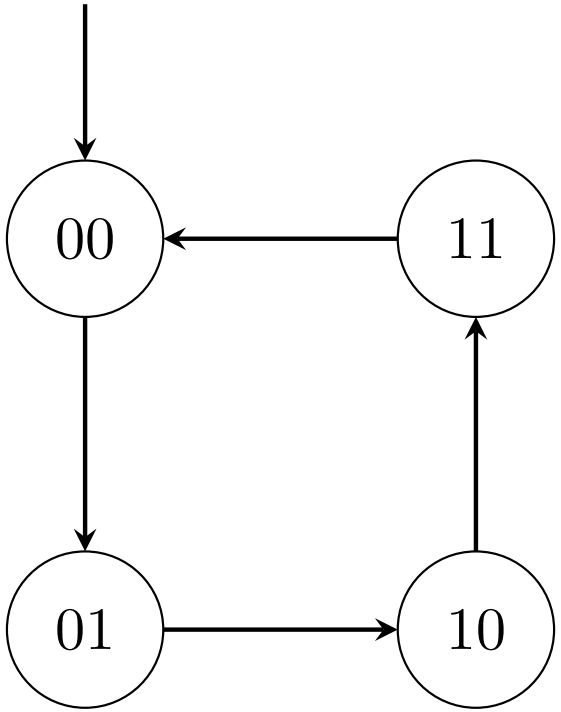


Two Boolean variables l and r (for the left and right bit).

Safety property: $G(\neg l \vee \neg r)$. Therefore $Target \equiv l \wedge r$.

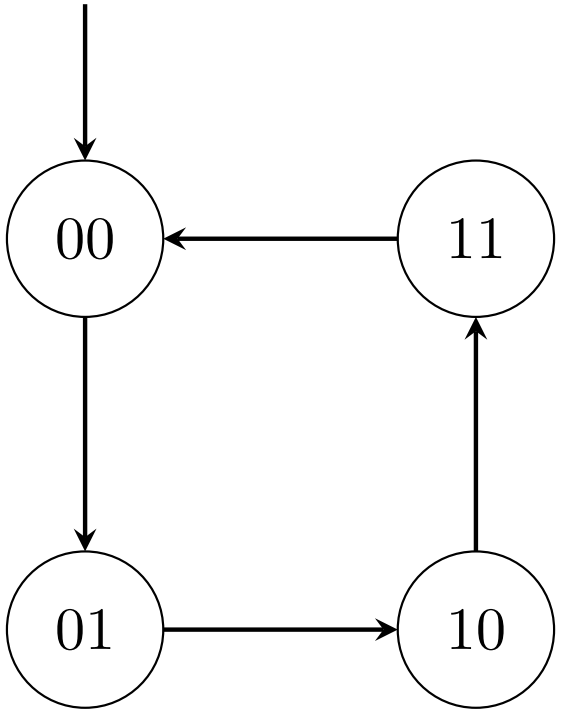
$Init \equiv \neg l \wedge \neg r$. $Trans \equiv (r' = \neg r \wedge l' = (l \neq r))$.

Example: Two Bit Counter



$$\Phi_2 = (\neg l_0 \wedge \neg r_0) \wedge \begin{pmatrix} l_1 = (l_0 \neq r_0) \wedge r_1 = \neg r_0 \wedge \\ l_2 = (l_1 \neq r_1) \wedge r_2 = \neg r_1 \end{pmatrix} \wedge \begin{pmatrix} (l_0 \wedge r_0) \vee \\ (l_1 \wedge r_1) \vee \\ (l_2 \wedge r_2) \end{pmatrix}$$

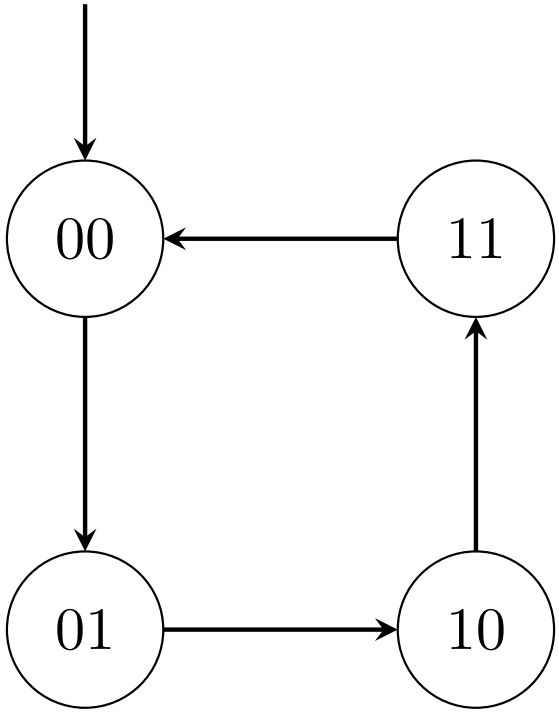
Example: Two Bit Counter



$$\Phi_2 = (\neg l_0 \wedge \neg r_0) \wedge \begin{pmatrix} l_1 = (l_0 \neq r_0) \wedge r_1 = \neg r_0 \wedge \\ l_2 = (l_1 \neq r_1) \wedge r_2 = \neg r_1 \end{pmatrix} \wedge \begin{pmatrix} (l_0 \wedge r_0) \vee \\ (l_1 \wedge r_1) \vee \\ (l_2 \wedge r_2) \end{pmatrix}$$

Φ_2 is **unsatisfiable** (hence, no proof of violation of the safety property).

Example: Two Bit Counter



$$\Phi_2 = (\neg l_0 \wedge \neg r_0) \wedge \begin{pmatrix} l_1 = (l_0 \neq r_0) \wedge r_1 = \neg r_0 \wedge \\ l_2 = (l_1 \neq r_1) \wedge r_2 = \neg r_1 \end{pmatrix} \wedge \begin{pmatrix} (l_0 \wedge r_0) \vee \\ (l_1 \wedge r_1) \vee \\ (l_2 \wedge r_2) \end{pmatrix}$$

Φ_2 is **unsatisfiable** (hence, no proof of violation of the safety property). Φ_3 **is satisfiable** (so the safety property is violated).

Further Reading:

- **Biere, Armin, et al.** "*Symbolic model checking without BDDs.*" International conference on tools and algorithms for the construction and analysis of systems. Springer 1999. https://link.springer.com/content/pdf/10.1007/3-540-49059-0_14.pdf

Optional :

- **Järvisalo, Matti, et al.** "*The international SAT solver competitions*." AI Magazine 33.1 (2012): 89-92. https://www.aaai.org/ojs/index.php/aimagazine

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