

Graph Neural Network Symposium 2021

# Graph Contrastive Learning Recent Progress and Outlook

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# Outline

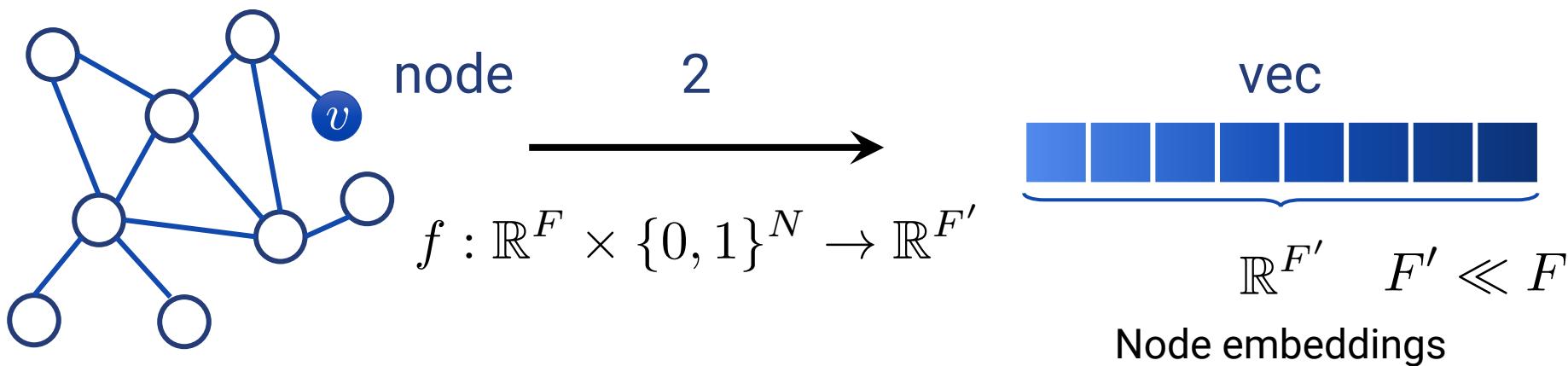
1. Preamble
2. Deep Graph CL: GRACE
3. Adaptive Augmentation: GCA
4. Experiments
5. Concluding Remarks

# Outline

- 1. Preamble**
2. Deep Graph CL: GRACE
3. Adaptive Augmentation: GCA
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# Representation Learning on Graphs

- Goal: efficient feature learning for machine learning on graphs
- Low-dimensional node embeddings encode both structural and attributive information.



# Self-supervised learning comes to rescue!

- Most GNN models are established in a supervised manner.
  - It is often expensive to obtain high-quality labels at scale in real world.
  - Supervised models learn the inductive bias encoded in labels, instead of **reusable, task-invariant knowledge**.

*“Labels are the opium of the machine learning researcher.”*

— Jitendra Malik

*“The future is self-supervised!”*

— Yann LeCun

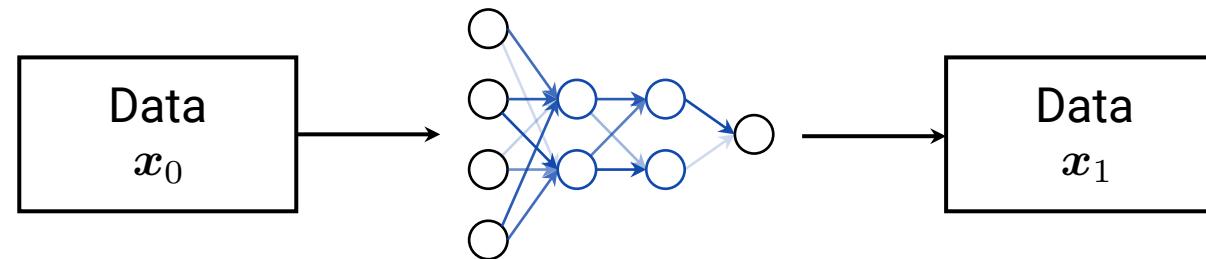
# Self-supervised learning comes to rescue!

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  - It is often expensive to obtain high-quality labels at scale in real world.
  - Supervised models learn the inductive bias encoded in labels, instead of **reusable, task-invariant knowledge**.
- Self-supervised methods employ **proxy tasks** to guide learning the representations.
  - The proxy task is designed to predict any part of the input from any other observed part.
  - Typical proxy tasks for visual data include corrupted image restoration, rotation angle prediction, reorganization of shuffled patches, etc.

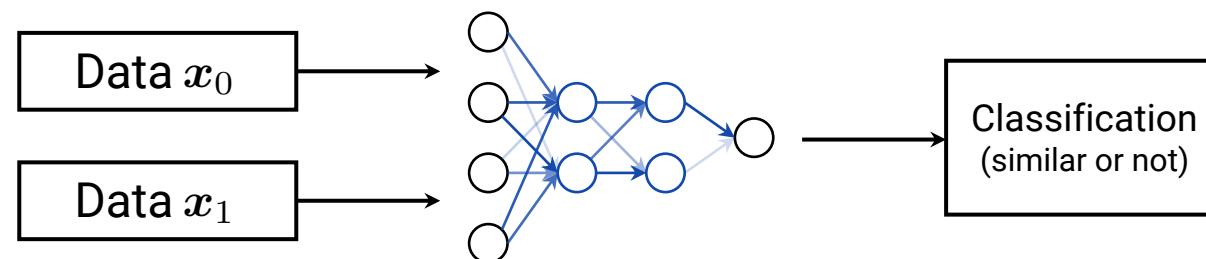
[Jing et al., 2020] L. Jing and Y. Tian, Self-supervised Visual Feature Learning with Deep Neural Networks: A Survey, *TPAMI*, 2020.

# Taxonomy of Self-Supervised Learning

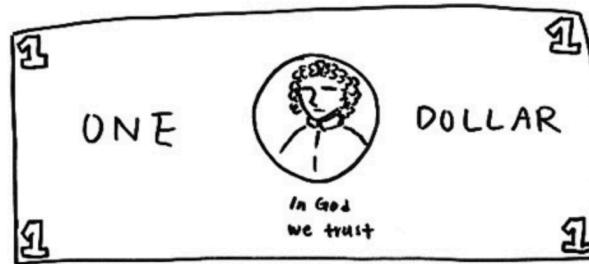
- (a) Generative/predictive: loss measured in the output space



- (b) Contrastive: loss measured in the latent space



# An analogy to brain's memory...



Drawing of a dollar bill from memory



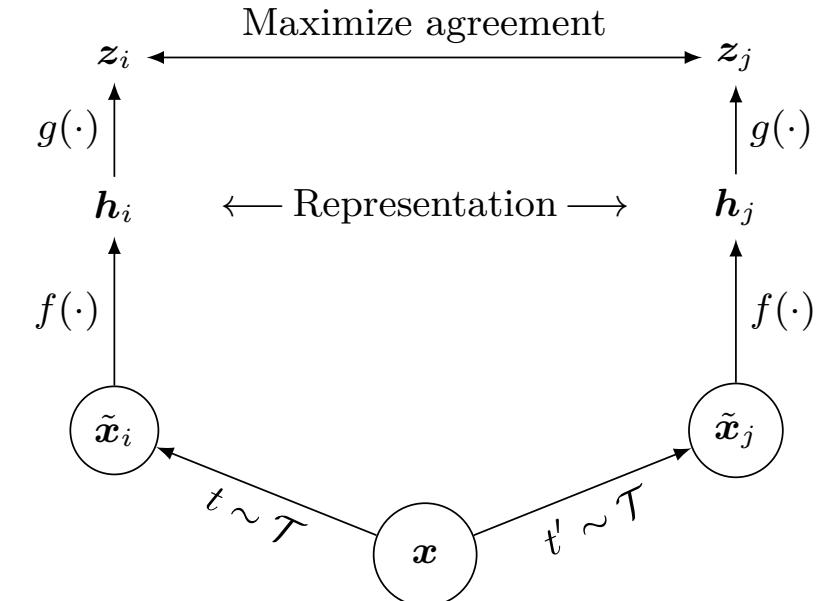
Drawing subsequently made with a dollar bill present

- We only need to retain several features to distinguish one bill from others!
  - Similarly, representation learning algorithms do not need to concentrate on **pixel-level details**. Encoding **high-level features** is sufficient enough to distinguish different objects.

[Epstein, 2016] R. Epstein, Your brain does not process information and it is not a computer, Aeon, 2016.

# The Contrastive Learning Paradigm

- Contrastive learning aims to maximize the agreement of latent representations under stochastic data augmentation.
- Three main components:
  - Data augmentation pipeline  $\mathcal{T}$
  - Encoder  $f$  and representation extractor  $g$
  - Contrastive objective  $\mathcal{L}$



[Chen et al., 2020] T. Chen, S. Kornblith, M. Norouzi, and G. E. Hinton, A Simple Framework for Contrastive Learning of Visual Representations, in *ICML*, 2020.

# Contrastive Learning Objectives

- A common pattern:

$$s(f(\mathbf{x}), f(\mathbf{x}^+)) \gg s(f(\mathbf{x}), f(\mathbf{x}^-))$$

- $f(\cdot)$  is the encoder, e.g., CNN and GNN.
- $s(\cdot, \cdot)$  measures similarity between two embeddings.
- Usually implemented with an  $n$ -way softmax function:

$$\mathcal{L} = -\mathbb{E}_X \left[ \log \frac{\exp(s(\mathbf{x}, \mathbf{x}^+))}{\exp(s(\mathbf{x}, \mathbf{x}^+)) + \sum_{j=1}^{n-1} \exp(s(\mathbf{x}, \mathbf{x}_j))} \right]$$

- Commonly referred to as **the InfoNCE loss**.
- The critic function can be simply implemented as  $s(\mathbf{x}, \mathbf{y}) = g(\mathbf{x})^\top g(\mathbf{y})$ .

[Oord et al., 2018] A. van den Oord, Y. Li, and O. Vinyals, Representation Learning with Contrastive Predictive Coding, arXiv.org, vol. cs.LG. 2018.

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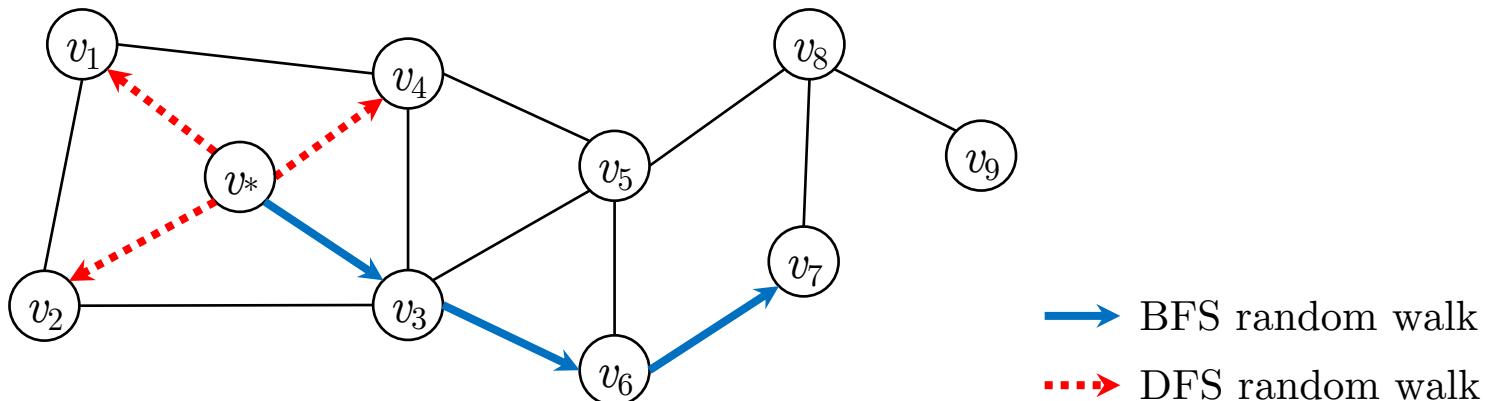
$$\mathcal{L} = -\mathbb{E}_X \left[ \log \frac{\exp(s(\mathbf{x}, \mathbf{x}^+))}{\exp(s(\mathbf{x}, \mathbf{x}^+)) + \sum_{j=1}^{n-1} \exp(s(\mathbf{x}, \mathbf{x}_j))} \right]$$

Distinguish a pair of representations from two augmentations of the same sample (**positives**) apart from  $(n - 1)$  pairs of representations from different samples (**negatives**).

[Oord et al., 2018] A. van den Oord, Y. Li, and O. Vinyals, Representation Learning with Contrastive Predictive Coding, arXiv.org, vol. cs.LG. 2018.

# Traditional Graph Contrastive Learning

- Traditional work of network embedding inherently follows a contrastive paradigm originated in the skip-gram model.
  - Nodes appearing on the same random walk are considered as positive samples and are encouraged to share similar embeddings.
  - Network embedding schemes could be regarded as reconstructing a preset graph proximity matrix, having difficulty of leveraging attributes.



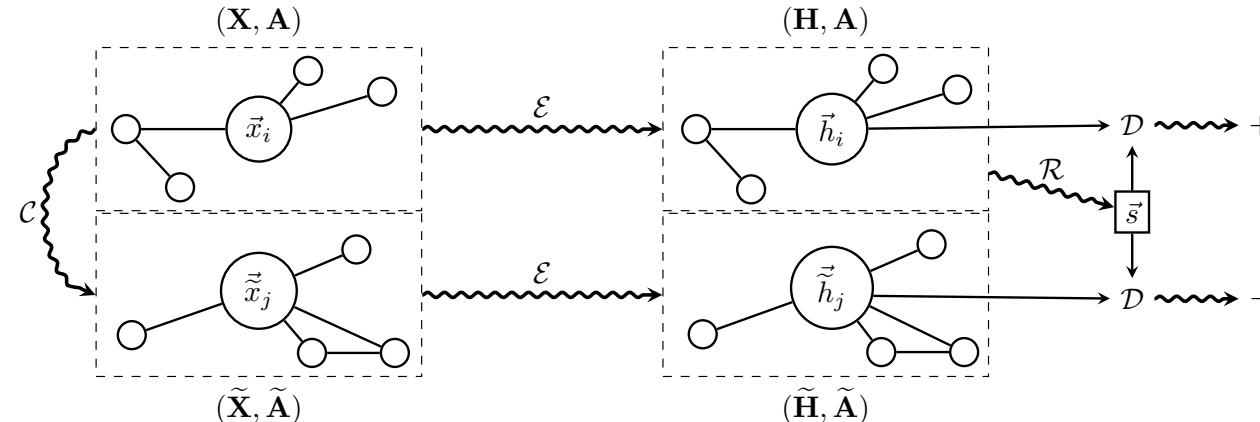
[Grover et al., 2016] A. Grover and J. Leskovec, node2vec: Scalable Feature Learning for Networks, in *KDD*, 2016.

# Deep Graph Contrastive Learning

- GNNs employ more powerful encoders for learning representations by aggregating information from neighborhood.
- GNN-based contrastive learning studies are in their infancy. Existing work primarily differs in **contrastive objectives** and **data augmentation** techniques.
  - Contrastive objective: defines which embeddings to **pull together** or **push apart**.
  - Data augmentation: transforms the original graphs to **congruent** counterparts.

# Deep Graph Infomax

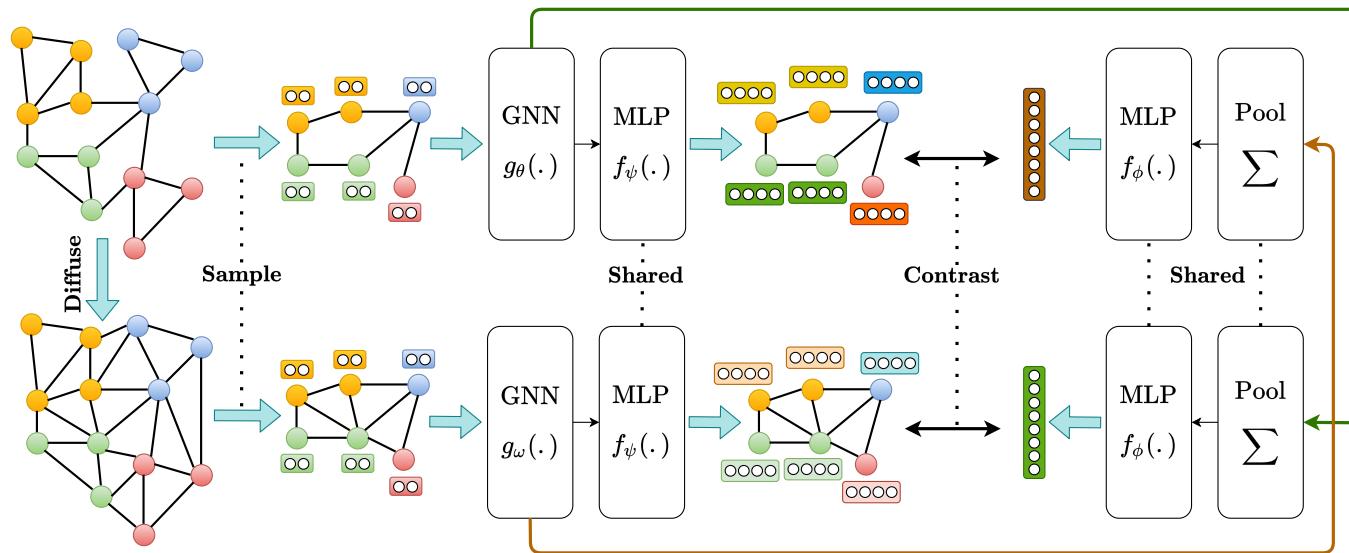
- Capture similarities using mutual information  $I(\mathbf{h}_i; \mathbf{s})$ .
  - $\mathbf{s}$  is a graph summary vector.
- Optimize MI using a binary classifier  $\mathcal{D}$ .
  - Generate negative examples (nodes not belonging to the graph) via an explicit corruption function  $\mathcal{C}$  which randomly shuffles the adjacency matrix.



[Veličković et al., 2019] P. Veličković, W. Fedus, W. L. Hamilton, P. Liò, Y. Bengio, and R. D. Hjelm, Deep Graph Infomax, in *ICLR*, 2019.

# Multiview Graph Contrastive Learning

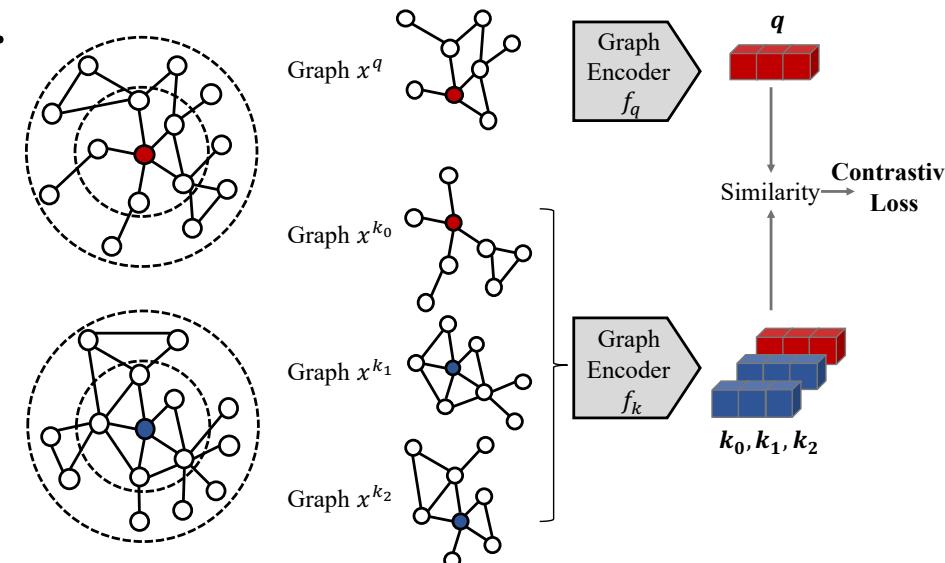
- Generate another view by applying diffusion kernels.
- Simultaneously optimize embeddings in two views to distill more global information.



[Hassani et al., 2020] K. Hassani and A. H. Khasahmadi, Contrastive Multi-View Representation Learning on Graphs, in *ICML*, 2020.

# Graph Contrastive Coding

- Generate a series of positives and negatives by **sampling subgraphs** within a batch.
- Employ feature engineering techniques for **transferable** node representations.



[Qiu et al., 2020] J. Qiu, Q. Chen, Y. Dong, J. Zhang, H. Yang, M. Ding, K. Wang, and J. Tang, GCC: Graph Contrastive Coding for Graph Neural Network Pre-Training, in *KDD*, 2020.

# Interlude: Contrastive Objectives

- Global-local contrastive objective:
  - DGI [Veličković et al., 2019] and MVGRL [Hassani et al., 2020] maximize the agreement between node- and graph-level representations.
  - The graph readout function should be **injective** [Xu et al., 2019], which is hard to fulfill. Otherwise, it is not guaranteed to distill enough information from node-level embeddings.
- Local-local contrastive objective:
  - Follow-up work GCC [Qiu et al., 2020], GRACE [Zhu et al., 2020], and GraphCL [You et al., 2020] eschew the need of an injective readout function and directly maximize the agreement of node embeddings across two augmented views.

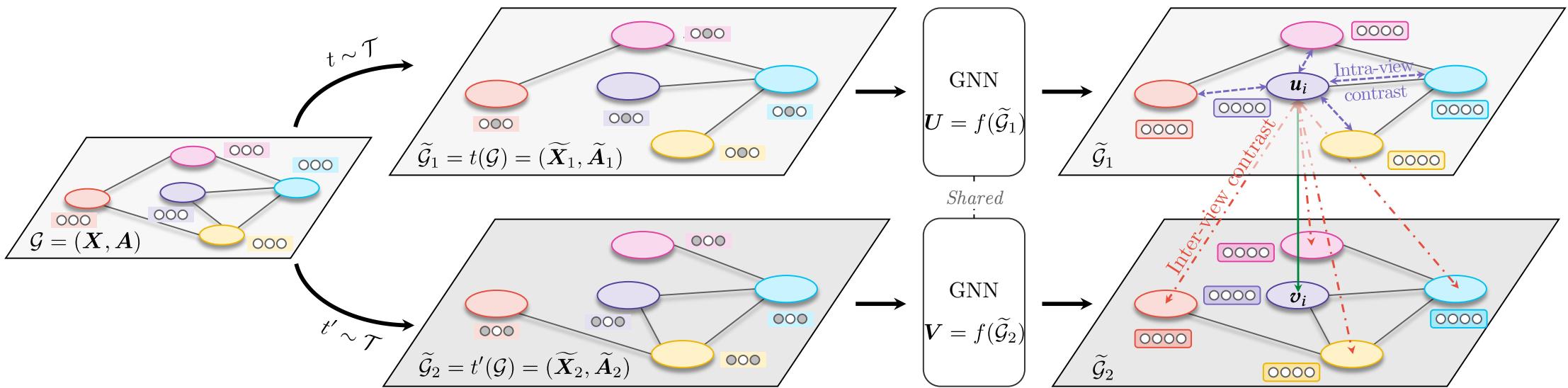
# Interlude: Augmentation for Graph CL

- Existing studies mostly adopt a bi-level augmentation scheme:
  - Structure-level augmentation
    - Shuffling the adjacency matrix [Veličković et al., 2019]
    - Adding / dropping edges [You et al., 2020; Zhu et al., 2020]
    - Sampling subgraphs [Hassani et al., 2020; Qiu et al., 2020; You et al., 2020]
    - Generating global view via diffusion kernels [Hassani et al., 2020]
    - ...
  - Attribute-level augmentation
    - Dropping / masking features [You et al., 2020; Zhu et al., 2020]
    - Adding Gaussian noise
    - ...

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# The Proposed Approach

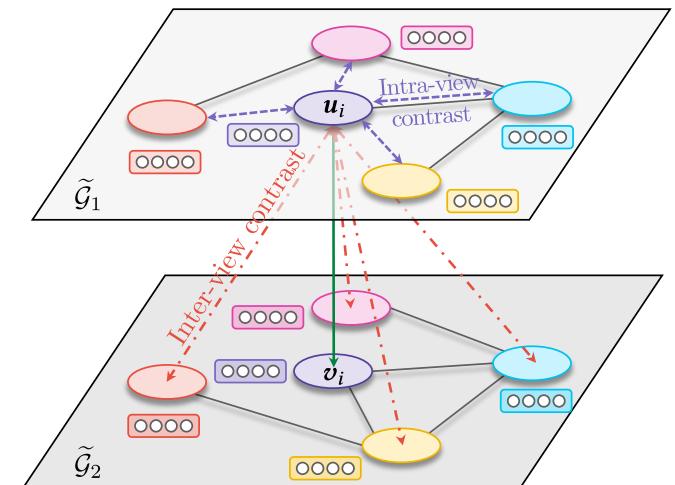


# Graph Contrastive Learning Across Views

- Firstly, we generate two correlated graph views by randomly augmenting the structure and features.
- Then, we train the model using a contrastive loss to maximize the **agreement** between node embeddings in the latent space.

$$\ell(\mathbf{u}_i, \mathbf{v}_i) = \log \frac{e^{\theta(\mathbf{u}_i, \mathbf{v}_i)/\tau}}{e^{\theta(\mathbf{u}_i, \mathbf{v}_i)/\tau} + \sum_{k \neq i} e^{\theta(\mathbf{u}_i, \mathbf{v}_k)/\tau} + \sum_{k \neq i} e^{\theta(\mathbf{u}_i, \mathbf{u}_k)/\tau}}$$

positives                                  inter-view negatives                          intra-view negatives



# Hybrid Graph View Generation

- We corrupt the original graph at both structure and attribute levels to construct diverse node contexts.
- **Removing edges:** randomly remove a portion of edges in the original graph.

$$\begin{aligned}\tilde{\mathbf{R}}_{ij} &\sim \text{Bern}(1 - p_r), \quad \text{if } \mathbf{A}_{ij} = 1, \\ \tilde{\mathbf{A}} &= \mathbf{A} \circ \tilde{\mathbf{R}}.\end{aligned}$$

- **Masking node features:** randomly mask a fraction of dimensions with zeros in node features.

$$\begin{aligned}\tilde{m}_i &\sim \text{Bern}(1 - p_m), \quad \forall i, \\ \widetilde{\mathbf{X}} &= [\mathbf{x}_1 \circ \widetilde{\mathbf{m}}; \ \mathbf{x}_2 \circ \widetilde{\mathbf{m}}; \ \dots; \ \mathbf{x}_N \circ \widetilde{\mathbf{m}}]^\top.\end{aligned}$$

# Theoretical Groundings

## Definition 1. Mutual Information (MI).

- Mutual information  $I(X; Y)$  is a measure of the mutual dependence between the two random variables  $X$  and  $Y$ , determining how different the joint distribution of the pair  $P(X, Y)$  is to the marginal  $P(X)P(Y)$ .

## Definition 2. InfoMax Principle.

- A function that maps a set of input values  $I$  to a set of output values  $O$  should be learned so as to maximize the MI between  $I$  and  $O$ .

[Linsker, 1998] R. Linsker, Self-Organization in a Perceptual Network, *IEEE Computer*, 1988.

# Theoretical Groundings (cont.)

## Theorem 1. Connections to MI maximization.

- Let  $\mathbf{X}_i = \{\mathbf{x}_k\}_{k \in \mathcal{N}(i)}$  be the neighborhood of node  $v_i$  that collectively maps to its output embedding, where  $\mathcal{N}(i)$  denotes the set of neighbors of node  $v_i$  specified by GNN architectures, and  $\mathbf{X}$  be the corresponding random variable with a uniform distribution  $p(\mathbf{X}) = 1/N$ .
- Given two random variables  $\mathbf{U}, \mathbf{V} \in \mathbb{R}^{F'}$  being the embedding in the two views, with their joint distribution denoted as  $P(\mathbf{U}, \mathbf{V})$ , our objective  $\mathcal{J}$  is a lower bound of MI between input  $\mathbf{X}$  and node representations in two graph views  $\mathbf{U}, \mathbf{V}$ :

$$\mathcal{J} \leq I(\mathbf{X}; \mathbf{U}, \mathbf{V}).$$

# Theoretical Groundings (cont.)

## Theorem 2. Connections to the triplet loss.

- When the projection function  $g$  is the identity function, and we measure embedding similarity by simply taking the inner product, i.e.  $s(\mathbf{u}, \mathbf{v}) = \mathbf{u}^\top \mathbf{v}$ , and further assuming that positive pairs are far more aligned than negative pairs, i.e.  $\mathbf{u}_i^\top \mathbf{v}_k \ll \mathbf{u}_i^\top \mathbf{v}_i$  and  $\mathbf{u}_i^\top \mathbf{u}_k \ll \mathbf{u}_i^\top \mathbf{v}_i$ , minimizing the pairwise objective  $\ell(\mathbf{u}_i, \mathbf{v}_i)$  coincides with maximizing the triplet loss, as given in the sequel

$$-\ell(\mathbf{u}_i, \mathbf{v}_i) \propto 4N\tau + \sum_{j \neq i} (\|\mathbf{u}_i - \mathbf{v}_i\|^2 - \|\mathbf{u}_i - \mathbf{v}_j\|^2 + \|\mathbf{u}_i - \mathbf{v}_i\|^2 - \|\mathbf{u}_i - \mathbf{u}_j\|^2).$$

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# Adaptive Augmentation in CL

- How to integrate augmentation schemes into graph CL is still an empirical choice.
- In essence, CL seeks to learn representations that are **insensitive** to perturbation induced by augmentation schemes.
  - Simple random augmentation in either structural or attribute domain is not sufficient.
  - Discrepancy in the impact of nodes and edges exists. Augmentation should preserve **important structural and attribute information** of graphs.

[Wu et al., 2020] M. Wu, C. Zhuang, M. Mosse, D. Yamins, and N. Goodman, On Mutual Information in Contrastive Learning for Visual Representations, arXiv.org, vol. cs.LG. 27-May-2020.

[Xiao et al., 2020] T. Xiao, X. Wang, A. A. Efros, and T. Darrell, What Should Not Be Contrastive in Contrastive Learning, arXiv.org, vol. cs.CV. 13-Aug-2020.

# Adaptive Augmentation on Graphs

- Data augmentation should be **adaptive** to the given graph.
  - We propose to keep important structures and attributes unchanged and perturb possibly unimportant links and features.
- Bi-level augmentation: remove edges (topology-level) and mask features (attribute-level)
  - Removing or masking probabilities are skewed for unimportant edges or features.
  - From an amortized perspective, we emphasize important structures and attributes over randomly corrupted views.

# Topology-level Augmentation

- We sample a modified edge subset  $\tilde{\mathcal{E}}$  with probability

$$P\{(u, v) \in \tilde{\mathcal{E}}\} = 1 - p_{uv}^e.$$

- In network science, **node centrality**  $\varphi_c(\cdot)$  is a widely-used measure that quantifies the influence of a node in the graph.
- The **edge importance**  $w_{uv}^e$  for edge  $(u, v)$  can be defined based on the centrality of two connected nodes.
  - Directed graphs:  $w_{uv}^e = \varphi_c(v)$
  - Undirected graphs:  $w_{uv}^e = (\varphi_c(u) + \varphi_c(v))/2$

[Newman, 2018] M. E. J. Newman, Networks: An Introduction (Second Edition), Oxford University Press, 2018.

# Topology-level Augmentation (cont.)

- Alleviate the nodes with heavily dense connections:

$$s_{uv}^e = \log w_{uv}^e.$$

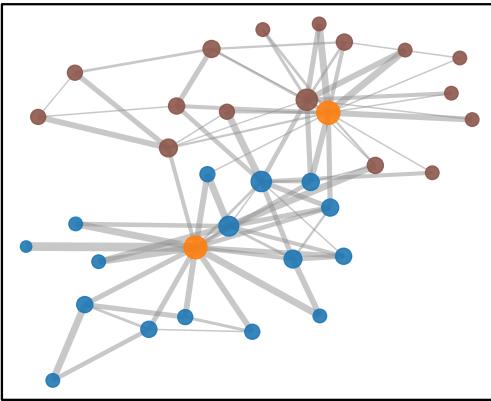
- Normalize to avoid overly high removal probabilities:

$$p_{uv}^e = \min \left( \frac{s_{\max}^e - s_{uv}^e}{s_{\max}^e - \mu_s^e} \cdot p_e, \ p_\tau \right),$$

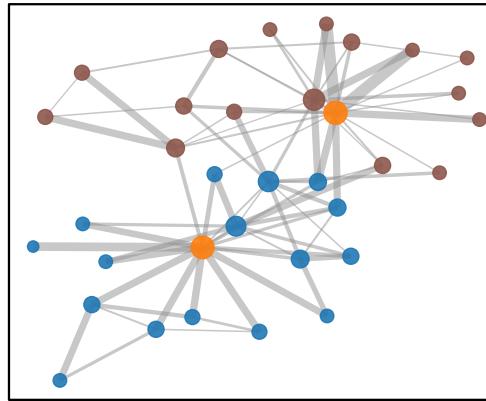
- $p_e$  is a hyperparameter that controls the overall removing probability.
- $s_{\max}^e$  and  $\mu_s^e$  is the maximum and average of  $s_{uv}^e$ .
- $p_\tau < 1$  is a cut-off probability.

# Centrality Measures

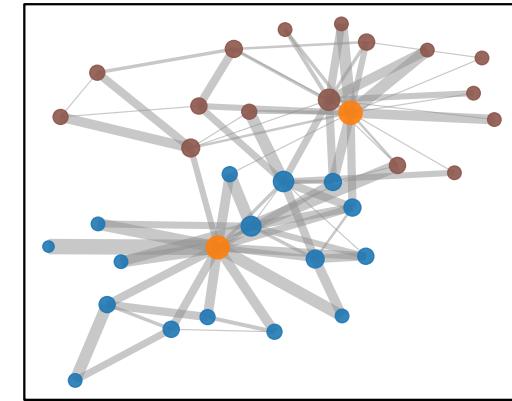
- We consider three widely-used centrality measures:



(a) Degree



(b) Eigenvector



(c) PageRank

- Visualized on the Karate club dataset.
- The three measures all highlight connection around the central nodes (two coaches) and exhibit negligible performance difference.

# Attribute-level Augmentation

- We add noise to node attributes via randomly masking a fraction of dimensions with zeros in node features:

$$\begin{aligned}\tilde{m}_i &\sim \text{Bern}(1 - p_i^f), \quad \forall i, \\ \widetilde{\mathbf{X}} &= [\mathbf{x}_1 \circ \widetilde{\mathbf{m}}; \ \mathbf{x}_2 \circ \widetilde{\mathbf{m}}; \ \cdots; \ \mathbf{x}_N \circ \widetilde{\mathbf{m}}]^\top.\end{aligned}$$

- The importance for each dimension of node features can be derived from node centrality scores.
  - Assumption: feature dimensions frequently appearing in influential nodes should be important.

$$w_i^f = \sum_{u \in \mathcal{V}} x_{ui} \cdot \varphi_c(u),$$

- $x_{ui} \in \{0, 1\}$  indicate the occurrence of dimension  $i$  in node  $u$ .

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# Datasets

Dataset	#Nodes	#Edges	#Features	#Classes
Wiki-CS	11,701	216,123	300	10
Amazon-Computers	13,752	245,861	767	10
Amazon-Photo	7,650	119,081	745	8
Coauthor-CS	18,333	81,894	6,805	15
Coauthor-Physics	34,493	247,962	8,415	5

# Baselines

- Network embedding methods:
  - DeepWalk [Perozzi et al., 2014] and node2vec [Grover et al., 2016]
- Unsupervised GNNs:
  - Recontraction-based methods: GAE, VGAE [Kipf et al., 2016], and GraphSAGE [Hamilton et al., 2017]
  - Contrastive learning methods: DGI [Veličković et al., 2019], GMI [Peng et al., 2020], and MVGRL [Hassani et al., 2020]
- Supervised GNNs:
  - GCN [Kipf et al., 2017] and GAT [Veličković et al., 2018]

# Experimental Configurations

- Linear evaluation: unsupervised training followed by employing a simple  $\ell_2$ -regularized logistic regression model.
- Evaluation metrics: node classification accuracy.
- Base model: we employ a two-layer GCN as the encoder for all baselines.

$$\text{GC}_i(\mathbf{X}, \mathbf{A}) = \sigma \left( \hat{\mathbf{D}}^{-\frac{1}{2}} \hat{\mathbf{A}} \hat{\mathbf{D}}^{-\frac{1}{2}} \mathbf{X} \mathbf{W}_i \right),$$

$$f(\mathbf{X}, \mathbf{A}) = \text{GC}_2(\text{GC}_1(\mathbf{X}, \mathbf{A}), \mathbf{A}).$$

# Overall Performance

Method	Training Data	Wiki-CS	Computers	Photo	CS	Physics
Raw features	$\mathbf{X}$	71.98	73.81	78.53	90.37	93.58
node2vec	$\mathbf{A}$	71.79	84.39	89.67	85.08	91.19
DeepWalk	$\mathbf{A}$	74.35	85.68	89.44	84.61	91.77
DeepWalk + features	$\mathbf{X}, \mathbf{A}$	77.21	86.28	90.05	87.70	94.90
GAE	$\mathbf{X}, \mathbf{A}$	70.15	85.27	91.62	90.01	94.92
VGAE	$\mathbf{X}, \mathbf{A}$	75.63	86.37	92.20	92.11	94.52
DGI	$\mathbf{X}, \mathbf{A}$	75.35	83.95	91.61	92.15	94.51
GMI	$\mathbf{X}, \mathbf{A}$	74.85	82.21	90.68	OOM	OOM
MVGRL	$\mathbf{X}, \mathbf{A}$	77.52	87.52	91.74	92.11	95.33
<b>GRACE</b>	$\mathbf{X}, \mathbf{A}$	78.19	86.25	92.15	92.93	95.26
<b>GCA-DE</b>	$\mathbf{X}, \mathbf{A}$	78.30	<b>87.85</b>	92.49	<b>93.10</b>	95.68
<b>GCA-PR</b>	$\mathbf{X}, \mathbf{A}$	<b>78.35</b>	87.80	<b>92.53</b>	93.06	95.72
<b>GCA-EV</b>	$\mathbf{X}, \mathbf{A}$	78.23	87.54	92.24	92.95	<b>95.73</b>
GCN	$\mathbf{X}, \mathbf{A}, \mathbf{Y}$	77.19	86.51	92.42	<u>93.03</u>	<u>95.65</u>
GAT	$\mathbf{X}, \mathbf{A}, \mathbf{Y}$	<u>77.65</u>	<u>86.93</u>	<u>92.56</u>	92.31	95.47

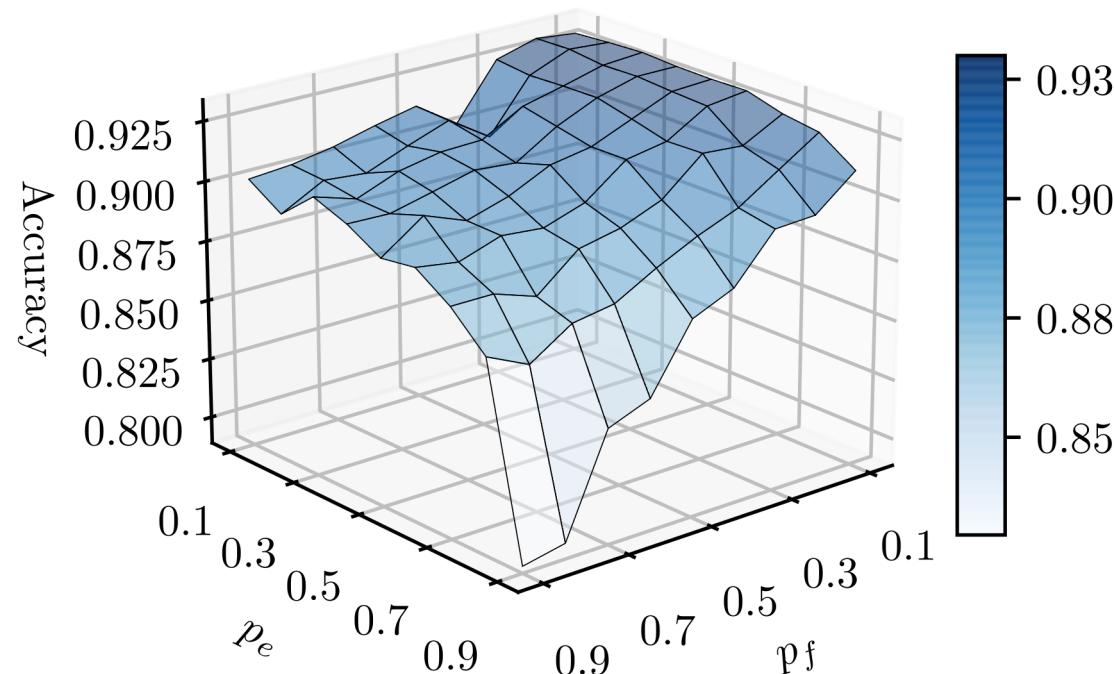
# Ablation Studies

- GCA-T-A (GRACE): uniform augmentation.
- GCA-T and GCA-A: substitute the topology and the attribute augmentation scheme with uniform sampling respectively.

Variant	Topology	Attribute	Wiki-CS	Computers	Photo	CS	Physics
GCA-T-A	Uniform	Uniform	78.19	86.25	92.15	92.93	95.26
GCA-T	Uniform	Adaptive	78.23	86.72	92.20	93.07	95.59
GCA-A	Adaptive	Uniform	78.25	87.66	92.23	93.02	95.54
<b>GCA</b>	Adaptive	Adaptive	<b>78.30</b>	<b>87.85</b>	<b>92.49</b>	<b>93.10</b>	<b>95.68</b>

# Sensitivity Analysis

- Vary the removal and masking probabilities from 0.1 to 0.9 to see the robustness under different magnitudes of perturbation.



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# Wrapping Up

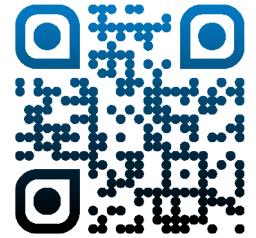
1. We have developed a novel graph CL framework GRACE and its extension GCA with adaptive augmentation.
2. Local-local contrastive objective enables better distillation of node-level representations.
3. Augmentation schemes on both structural and attributive levels are critical for graph CL.
4. Important nodes/attributes, identified using centrality measures, should be preserved during augmentation to force the model learn intrinsic patterns of graphs.
5. Our proposed method achieves SOTA performance and bridges the gap between unsupervised and supervised learning.

# Graph SSL: Retrospect and Prospect

- Graph self-supervised learning (SSL) is a promising way to learn graph embeddings without human annotations.
- Graph CL stems from traditional network embedding approaches and has established a new paradigm for unsupervised representation learning on graphs.
- However, the development of graph CL remains nascent, yet calls for a principled understanding of it.
  - Utilization of both topology and attribute spaces
  - Data augmentation and positive/negative sampling on graphs
  - Contrastive objectives
  - ...

# Useful Resources

- A curated list of must-read papers, survey, and talks
  - <http://bit.ly/GraphSSL>
- Graph contrastive learning library for PyTorch
  - To be released in late March
  - <http://bit.ly/GraphCL>



# Bibliographies (1/3)

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# THANKS

A photograph of a long, straight asphalt road stretching into the distance under a clear blue sky. The road is marked with a yellow center line and white edge lines. It is flanked by dry, gravelly desert terrain and low, dark mountains on both sides. The perspective of the road creates a sense of depth and vanishing point.